INELASTIC ANALYSIS OF

COMPOSITE BEAMS WITH PARTIAL CONNECTION

INELASTIC ANALYSIS OF COMPOSITE BEAMS WITH PARTIAL CONNECTION

BY

K. P. MA

A PROJECT

SUBMITTED TO THE SCHOOL OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS

FOR THE DEGREE

MASTER OF ENGINEERING

McMASTER UNIVERSITY

October 1973

MASTER OF ENGINEERING (1973) McMASTER UNIVERSITY

Civil Engineering and Engineering Mechanics Hamilton, Ontario.

TITLE: INELASTIC ANALYSIS OF COMPOSITE BEAMS WITH PARTIAL CONNECTION

AUTHOR: Kock Ping Ma, B.Sc.,
Chinese University of Hong Kong, Hong Kong, 1970.

SUPERVISOR: Dr. H. Robinson

NUMBER OF PAGES: xvi, 142

SCOPE AND CONTENTS:

The project aims at developing a method to be used for the inelastic analysis of single span composite beams. As an integral part of the project, a computer program is also presented, which works for both shored and unshored beams with or without cover plate for symmetrical loading cases. After successful testing of the program against experimental results, computations were performed to determine the lower limit to the degree of partial connection permissible in practical composite beams.

ACKNOWLEDGEMENTS

This project was perpared under the supervision of Dr. H. Robinson. The author wishes to express his gratitude for the guidance and valuable advice received.

Particular thanks are also due Mr. A. A. Baig and Mr. I. Elkholy, graduate students in the Department of Civil Engineering, whose previous computer programs form an integral part of the computer program developed in this project.

Finally the author gratefully acknowledges the National Research Council of Canada and the Department of Civil Engineering and Engineering Mechanics, McMaster University for the financial support on this project.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF REFERENCES

(1) Arden, B.W. and Astill, K.N., "Numerical Algorithms: Orgins and Application", Reading, Mass.: Addison-Wesley, 1970.

(2) Dai, P.K.H., "Analytical Study of Composite Beams with Inelastic Shear Connector", Ph.D. Thesis, University of Illinois, Urbana, Illinois, May 1963; also published as Structural Research Series No. 267, Civil Engineering Studies, University of Illinois, May 1963 by Dai, P.K.H. and Siess, C.P.

(3) Robinson, H. and Wallace, I.W., "Composite Beams with Partical Connection", Civil Eng. & Eng. Mechanics Report 71-14, June 1971.

(4) Thiruvengadam, T.R., "A Method for Inelastic Analysis of Single Span Composite Beams", Ph.D. Thesis, University of Illinois, Urbana, Illinois, 1969.

(5) Viest, I.M., Fountain, R.S. and Singleton, R.C., "Composite Construction in Steel and Concrete for Bridges and Buildings", McGraw-Hill Book Company, INC., New York, 1958.

(6) Yam, L.C.P. and Chapman, J.C.,"The Inelastic Behaviour of Simply Supported Composite Beams of Steel and Concrete", Paper No.7111, Proc. I.C.E., December 1968.

(7) Canadian Sheet Steel Building Institute Composite Beam Manual for the Design of Steel Beams with Concrete Slab and Cellular

Steel Floor, 1968 edition, first printing.

(8) CSA Standard S16-1968.

(9) Specification for the Design, Fabrication and Erection of Structural Steel for Buildings AISC, 1969.

(10) Gurfindel, G. and Robinson, A. R., "Determination of Strain Distribution and Curvature in a Reinforced Concrete Section Subjected to Bending Moment and Longitudinal Load," Journal of the American Concrete Institute, Proceedings, V.64, No. 7, pp. 398-403, July 1967.

(11) Hildebrand, F. B., "Introduction to Numerical Analysis," McGraw-Hill, 1956.

NOTATION

A_b = cross-sectional area of steel beam

A_i = area of the ith element in a subdivision

$A_{(i)}$ = ith element of matrix [A]

A_s = cross-sectional area of slab

B_{ij} = an element of matrix [B] (Eq.(1-13))

B_{2i} = an element of matrix [B] (Eqs.(2-18),(2-19) and (2-20))

b = effective width of slab

c_b, c_s = distances between the respective centroidal axes of the beam and the slab, and the interface between the beam and slab

$D_{(i)}$ = ith element of matrix [D]

d = intercept on the slip axis of the tangent to load-slip curve

d_o = distance between force centers in beam and slab

d_i = d for connector i

E = modulus of elasticity of steel or concrete

E_b, E_s = moduli of elasticity of the materials for beam and slab, respectively

F = absolute value of the force produced by composite action acting on the centroid of beam or slab

F_b, F_s = F with sign acting at beam and slab respectively

$F_{(i)}$ = force F at the (i)th panel

H = length of portion of the slab in which the strain is negative

h = portion of H in which the magnitude of strain is smaller than the yield value

I_b, I_s = moments of inertia of beam and slab respectively

k = slop of the load-slip curve when the section of the curve considered approximated as a straight line

k_i = k for connector i

M = magnitude of external or total internal moment

M_A = ultimate moment computed from stress block

M_B = moment at which the first connector reaches its ultimate load

M_{DL} = moment produced by dead load

M_b = moment in the beam

M_C = moment at which the average connector force is equal to the ultimate connector load

$M_{(i)}$ = M at the middle of panel i

$M_{(ij)}$ = M at the middle of subpanel j within panel i

M_s = moment in the slab

$M_{b(i)}, M_{s(i)}$ = M_b and M_s at the middle of panel i respectively

N.A. = neutral axis

r = slip of connector

Q = connector force

Q_i = Q at connector i

s = spacing of connector

$s_{(i)}$ = length of panel (i)

t = thickness of solid part of slab

t_t = total thickness of slab

$\bar{y}$ = distance from the centroid of an area to a reference line

$\bar{y}_i$ = $\bar{y}$ for ith element

Z = distance between the centroidal axes of slab and beam

$= c_s + c_b$

λ = parameter for the cross-section of composite beam

$= \left(\frac{1}{E_b A_b} + \frac{1}{E_s A_s} + \frac{Z^2}{\Sigma EI} \right)$

$\lambda_{(i)}$ = for panel (i)

$\beta = \frac{Z}{\Sigma EI}$

$\beta_{(i)}$ = β for panel (i)

$\beta_{(ij)}$ = β for subpanel (j) within panel (i)

ϕ = curvatures

ϕ_b, ϕ_s = curvatures of beam and slab respectively

ϵ = general representation for strain

$\epsilon_b, \epsilon_{bb}$ = strains at top and bottom fibres of steel beam

$\epsilon_s, \epsilon_{ss}$ = strains at bottom and top fibres of slab

ϵ_s' = strain at bottom fibre of the solid part of slab

ϵ_y = yield strain of beam or slab

$\sum EI = E_b I_b + E_s I_s$

σ_b = stress at top fibre of beam

σ_s = stress at bottom fibre of slab

σ_y = yield stress of the material for beam or slab

INTRODUCTION

In the field of composite structures, much work has been done to analyze single span composite beams. Several successful ways have been established for the analysis within the elastic limit. However, when the external loading exceeds a certain value, the strains in the steel beam or in the concrete slab exceed their respective yield limits, these methods appropriate for the elastic analysis are no longer valid. This causes difficulties in the study of the properties of the composite beam, since many of the features will not manifest themselves until the composite beam is well into the inelastic stage. So far, most of the research into this aspect has been done mainly by experiments which are either inconvenient or insufficent for many purposes. For this reason, an inelastic analysis must be developed.

The term 'analysis' used here is explained as the study of the deflection of the beam, forces at the connectors, the moment — carrying capacity, as well as many other quantities related to the behaviour of composite beams at various loading stages. This project aims at developing a method capable of carrying out the above analysis.

Because of the non-linear nature of the behaviour of the composite beam in the inelastic zone, some nonlinear equations are encountered and are difficult to solve. To overcome this difficulty,

an essentially iterative method is employed. This can be accomplished conveniently with the aid of a computer.

There are three distinguishable parts included in this project. Developing a method, and writing a computer program are the first two parts. Determining a practical lower limit to the degree of connection which should be permitted in composite beam forms the third part.

CHAPTER 1

BASIC EQUATIONS

The analysis in this project is close to that given by Thiruvengadam (4) in his Ph.D. thesis at University of Illinois. The project initially started with setting up a computer program using Thiruvengadam's method. However when the program was tested by computing some examples, many difficulties were encountered. They were found to be directly related to the iterative techniques, taken from Thiruvengadam and stated in articles 2.7.4 and 2.7.5 in this project. In order to overcome the difficulties, the analysis was firstly modified in many ways, but without much success. It was found that only the local modifications themselves could not remove the difficulties effectively. It requires a new approach adopting a smooth load-slip curve and without necessarily solving the finite difference equations. The development of this method forms a primary part of the project. Since it is one of the discrete methods, some useful techniques applicable to discrete methods and described in Thiruvengadam's thesis were adopted. To attain a clear picture of the origin of the numerical difficulties, it may be appropriate to first introduce his method, and subsequently give the necessary changes.

In this chapter, most of the equations are fundamental to any discrete approach. Some equations, which are used in Thiruvengadam's method and will not be used in this project, will be mentioned in the appropriate places.

1.1 General Definition

The composite beam, shown in fig.(1.1), is a single-span composite beam with n panels and n+1 shear connectors. The space between connector i and i+1 is referred to as the ith panel. If parentheses are not used, the letter 'i' denotes the ith connector in stead of the panel. In the analysis, the connector forces are defined as positive when they act in the direction shown in fig. (1.2). Tensile normal forces and moments producing tension in the bottom fibre are defined as positive.

1.2 Basic Assumptions

The basic assumptions used throughout this analysis are the following:

1) Concrete slab and steel beam deflect equally at all points along the span.

2) The strain distribution along the depth of slab and steel beam is linear.

3) The shear connection between the slab and beam is provided by shear connectors placed at discrete points along the span of the beam.

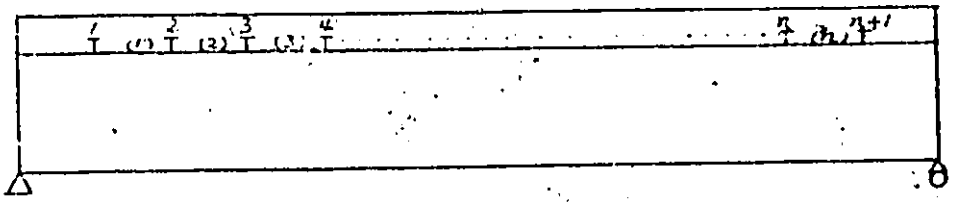


Fig. (1.1) A Composite Beam Showing the Position of Connectors and Panels

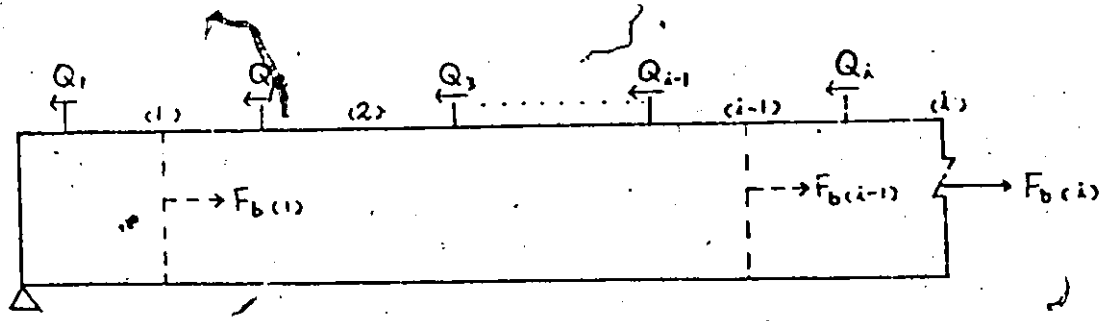


Fig. (1.2) Positive Direction of the Connector Forces

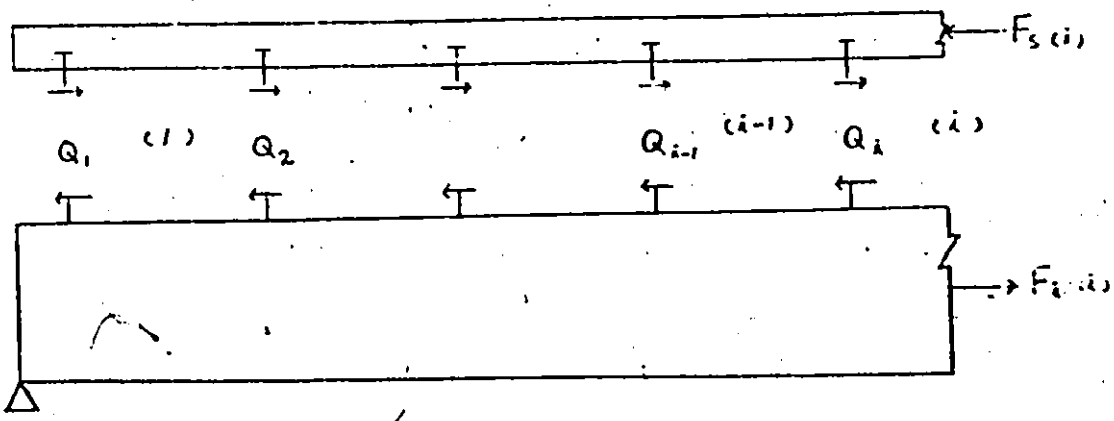


Fig. (1.3) Free Body Diagram of a Slab and a Steel Beam with Horisontal Forces Only

1.3 Equilibrium Condition

The condition required for the equilibrium of forces at a section in the middle of panel (i), as shown in fig.(1.3), is given by

$$F_{b(i)} = -F_{s(i)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}1)$$

The external moment $M_{(i)}$ is equal to the internal moment in magnitude, which can be found by referring to fig.(1.4).

In the figure, the stress distribution at a cross-section through a point at panel (i) is shown.

All the forces represent their magnitudes without sign in the following derivation.

$C_{(i)}$ = compressive resultant force in the slab

$F_{1(i)}$ = the resultant force from negative stress in the beam

$F_{2(i)}$ = the resultant force from positive stress in the beam

$T_{(i)}$ = the resultant force in the beam

$= F_{2(i)} - F_{1(i)}$

Other quantities are clearly shown in the figure. The moment can be found by using one of the following methods:

(A) One couple method

$$M_{(i)} = C_{(i)} d_o = T_{(i)} d_o \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}2a)$$

(B) Two couple method

$$M_{(i)} = C_{(i)} e + F_{1(i)} e' \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}2b)$$

(C) The method in terms of the moments of the beam and slab

From eq.(1-2b), one obtains

$$M_{(i)} = C_{(i)}(z_c + Z + z_2) + F_{1(i)} e'$$

$$M_{(i)} = C_{(i)}z_c + C_{(i)}z_2 + F_{1(i)}(d'+z_2) + C_{(i)}Z$$

$$= C_{(i)}z_c + C_{(i)}z_2 + F_{1(i)}z_2 + F_{1(i)}d' + C_{(i)}Z$$

$$= C_{(i)}z_c + (C_{(i)}+F_{1(i)})z_2 + F_{1(i)}d' + C_{(i)}Z$$

$$= M_{s(i)} + M_{b(i)} + F_{(i)}Z \quad \text{.....} \quad (1\text{-}2c)$$

The last step is obtained if we define

$M_{s(i)} = C_{(i)}z_c$, the force in the slab times the distance from the force center to the neutral axis of the slab

$M_{b(i)} = F_{2(i)}z_2 + F_{1(i)}d'$

$= T_{(i)}z_b$, the resultant force times the distance between the force center and the neutral axis of the steel beam

$F_{(i)} = F_{b(i)} = C_{(i)}$, the magnitude of the force due to interaction. The derivation gives the meanings of the terms, $M_{s(i)}$ and $M_{b(i)}$. Eq.(1-2c) can also be obtained simply by referring to fig.(1.5) and applying the equation for equilibrium of moments.

The moment $M_{s(i)}$ or $M_{b(i)}$ is the sum of the products obtained by multiplying the force at each elemental area at the cross-section with the corresponding distance to the neutral axis. In the elastic case, the force at each elemental area can be separated into

two parts, the force produced by pure bending and the force due to interaction, since the latter part produces no net moment. The moment, $M_{s(i)}$ or $M_{b(i)}$, is that produced by pure bending, namely,

$$M_{b(i)} = \phi_{b(i)} E_b I_b \text{ and } M_{s(i)} = \phi_{s(i)} E_s I_s \quad \cdots\cdots\cdots (1\text{-}3)$$

In the inelastic case, eq.(1-2c) is still valid. Because the superposition principle fails, the moments can not be obtained from the pure bending formula.

Now turning back to fig.(1.3); from the equilibrium of horizontal forces acting on the beam, one obtains

$$F_{b(i)} - F_{b(i-1)} = Q_i \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots (1\text{-}4)$$

and

$$F_{b(i)} = \sum_{j=1}^{i} Q_j \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (1\text{-}5)$$

From now on $F_{(i)}$ is used to represents the magnitude of $F_{b(i)}$ or $F_{s(i)}$ without sign.

1.4 Compatibility Condition

The relative movement between the slab and the beam, called slip, is resisted by the shear connectors placed discretely along the span. The connectors deform to permit slip when interaction exists between the beam and the slab. Fig.(1.6) shows the deformed shape of a composite beam.

The strain of the slab and beam ϵ_s and ϵ_b, and the slip of the connectors, r, satisfy the following relationship:

$$s_{(i)} + \int_{s_{(i)}} \epsilon_s ds + r_{i+1} = r_i + s_{(i)} + \int_{s_{(i)}} \epsilon_b ds$$

or

$$r_{i+1} - r_i = \int_{s_{(i)}} (\epsilon_b - \epsilon_s) ds \quad \cdots\cdots\cdots\cdots (1\text{-}6)$$

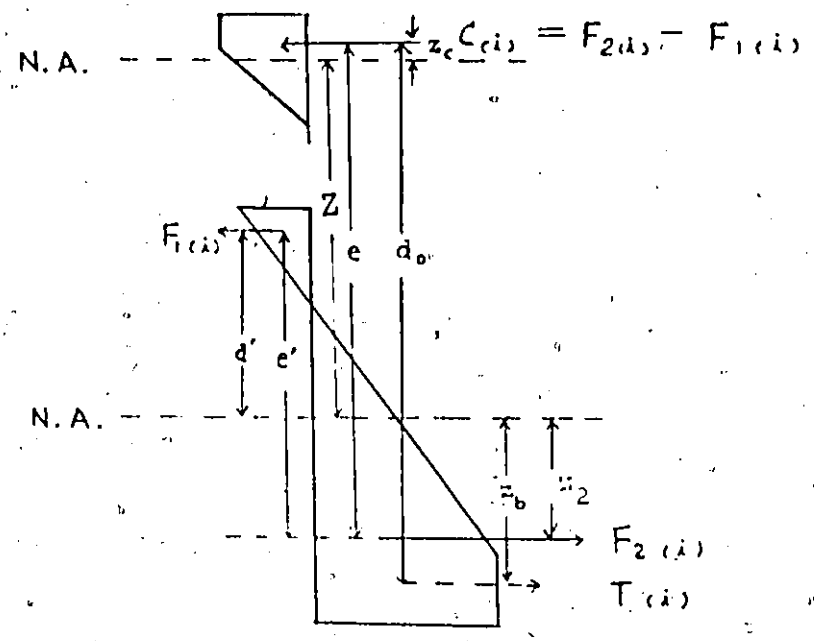


Fig. (1.4) Forces in the Slab and Beam at a Cross-Section

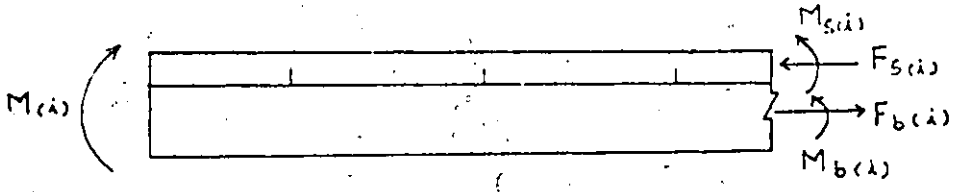


Fig. (1.5) The Relationship Between the Total Bending Moment and the Moments in the Slab and Beam

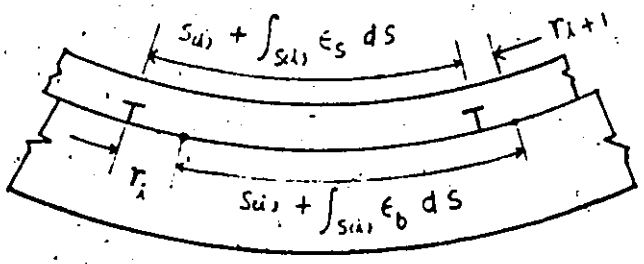


Fig.(1.6) Bending of a Panel with Slip

or $$r_{i+1} = r_i + \int_{s(i)} (\epsilon_b - \epsilon_s) ds \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}7)$$

Eq.(1-7) is the basic equation for the method outlined in this project.

The assumption of equal deflection gives equal curvature if the deflection is small and the difference in curvature is negligible. Then

$$\phi_b = \phi_s = \phi \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}8)$$

1.5 Finite Difference Equations

If the load-slip curve of a connector is assumed to be linearly elastic, the load-slip relationship may be expressed as

$$Q = Kr \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}9)$$

where K is the slope of the load-slip curve.

Substitution of eq.(1-9) in eq.(1-6) yields

$$\frac{Q_{i+1}}{k_{i+1}} - \frac{Q_i}{k_i} = \int_{s(i)} (\epsilon_b - \epsilon_s)\, ds \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}10)$$

Substitution of eq.(1-4) in eq.(1-10) yields

$$\frac{F_{(i-1)}}{k_i} - \left(\frac{1}{k_i} + \frac{1}{k_{i+1}}\right) F_{(i)} + \frac{F_{(i+1)}}{k_{i+1}} = \int_{s(i)} (\epsilon_b - \epsilon_s) ds$$

$$\ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}11)$$

Eq.(1-11) is written for panel (i) . For a composite beam having n panels, there are n such equations resulting in a set of simultaneous equations as given below

$$\frac{F_{(0)}}{k_1} - \left(\frac{1}{k_1} + \frac{1}{k_2}\right) F_{(1)} + \frac{F_{(2)}}{k_2} = \int_{s_{(1)}} (\epsilon_b - \epsilon_s) ds$$

$$\frac{F_{(1)}}{k_2} - \left(\frac{1}{k_2} + \frac{1}{k_3}\right) F_{(2)} + \frac{F_{(3)}}{k_3} = \int_{s_{(2)}} (\epsilon_b - \epsilon_s)\, ds$$

.....

$$\frac{F_{(i-1)}}{k_i} - \left(\frac{1}{k_i} + \frac{1}{k_{i+1}}\right) F_{(i)} + \frac{F_{(i+1)}}{k_{i+1}} = \int_{s_{(i)}} (\epsilon_b - \epsilon_s) ds$$

.....

$$\frac{F_{(n-1)}}{k_n} - \left(\frac{1}{k_n} + \frac{1}{k_{n+1}}\right) F_{(n)} + \frac{F_{(n+1)}}{k_{n+1}} = \int_{s_{(n)}} (\epsilon_b - \epsilon_s) ds$$

..... (1-12)

The boundary conditions are

$$F_{(0)} = 0 \quad \text{and} \quad F_{(n+1)} = 0$$

The set of simultaneous equations for n panels can be written in a compact form by using matrix notation:

$$[B]\,[F] = [A] \quad \text{.....} \quad (1\text{-}13)$$

[B] is a symmetric tridiagonal matrix whose components are:

$$B_{11} = -\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$B_{12} = \frac{1}{k_2}$$

$$B_{nn} = -\left(\frac{1}{k_n} + \frac{1}{k_{n+1}}\right)$$

$$B_{n(n-1)} = \frac{1}{k_n}$$

for $1 < i < n$ $\qquad B_{i(i-1)} = \frac{1}{k_i}$

$$B_{ii} = -\left(\frac{1}{k_i} + \frac{1}{k_{i+1}}\right)$$

$$B_{i(i+1)} = \frac{1}{k_{i+1}}$$

all other $B_{ij} = 0$

$[F]$ is a vector whose ith term is $F_{(i)}$.

$[A]$ is a vector whose ith term, $A_{(i)}$, is

$$A_{(i)} = \int_{s_{(i)}} (\epsilon_b - \epsilon_s)\, ds \quad \ldots\ldots\ldots\ldots \quad (1\text{-}14)$$

The eq.(1-11) is derived for linearly elastic connectors. For inelastic shear connectors, the equation should be modified. Moreover, if the stress-strain relationship remains in the elastic region, a simpler form may be obtained. All these modifications are given in the following paragraphs.

Eq.(1-11) and its matrix representation eq.(1-13) are referred to as the basic equation in Thiruvengadam's method.

1.6 Modification of Terms in Basic Equations

All of the following modifications are used in Thiruvengadam's method. Only those stated in 1.6.1 are used in this project.

1.6.1 Elastic Case

When the beam and the slab are both elastic, the strain in the beam or the slab can be expressed in terms of force and moment

$$\epsilon_b = \frac{1}{E_b}\left(\frac{F}{A_b} - \frac{M_b c_b}{I_b}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}15a)$$

$$\epsilon_s = \frac{1}{E_s}\left(\frac{-F}{A_s} + \frac{M_s c_s}{I_s}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}15b)$$

where A_b, I_b and E_b refer to the area moment of inertia and modulus of elasticity of the material of the beam respectively, and A_s, I_s and E_s refer to the corresponding quantities for the slab.

Substitution of eq.(1-15) in eq.(1-14) yields

$$A_{(i)} = \int_{s(i)} \left(F\left(\frac{1}{E_b A_b} + \frac{1}{E_s A_s}\right) - \left(\frac{M_b c_b}{E_b I_b} + \frac{M_s c_s}{E_s I_s}\right)\right) ds \quad \ldots \quad (1\text{-}16)$$

The combination of eqs.(1-8) and (1-2c) gives

$$\frac{M_b}{E_b I_b} = \frac{M_s}{E_s I_s} = \frac{M_b + M_s}{E_b I_b + E_s I_s} = \frac{M - ZF}{E_b I_b + E_s I_s} \quad \ldots\ldots\ldots\ldots \quad (1\text{-}17)$$

Substitution of eq.(1-17) in eq.(1-16) yields

$$A_{(i)} = \int_{s(i)} \left(F\left(\frac{1}{E_b A_b} + \frac{1}{E_s A_s} + \frac{Z^2}{\sum EI}\right) - \frac{MZ}{\sum EI}\right) ds$$

where $\sum EI = E_b I_b + E_s I_s$

If $$\alpha = \frac{1}{E_b A_b} + \frac{1}{E_s A_s} + \frac{Z^2}{\sum EI}$$

and $$\beta = \frac{Z}{\sum EI}$$

$A_{(i)}$ can be expressed as

$$A_{(i)} = F_{(i)} \alpha_{(i)} s_{(i)} - \int_{s(i)} \beta M ds \quad \ldots\ldots\ldots\ldots\ldots \quad (1\text{-}18)$$

In the last step, F is taken out of the integration sign, as F is a constant through the length of the panel. By using eq.(1-18), eq.(1-11) can then be written as

$$\frac{F_{(i-1)}}{k_i} - \left(\frac{1}{k_i} + \frac{1}{k_{i+1}} + \alpha_{(i)} S_{(i)} \right) F_{(i)} + \frac{F_{(i+1)}}{k_{i+1}} = - \int_{s_{(i)}}^{\beta} M ds \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}19)$$

Eq.(1-19) reduces to the same matrix notation as given by eq.(1-13) except for the following changes:

for $1 \leq i \leq n$

$$B_{ii} = \frac{1}{k_i} + \frac{1}{k_{i+1}} + \alpha_{(i)} S_{(i)} \quad \ldots\ldots\ldots\ldots \quad (1\text{-}20)$$

and

$$A_{(i)} = - \int_{s_{(i)}}^{\beta} M ds \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1\text{-}21)$$

This is a simplified form of the basic equation in Thiruvengadam's method.

If the modified form of $A_{(i)}$ shown in eq.(1-18) is used to express the integration of the differential strains in the beam and the slab along the interface of panel (i), a convenient form of the integration is obtained

$$\int_{s_{(i)}} (\epsilon_b - \epsilon_s) ds = F_{(i)} \alpha_{(i)} S_{(i)} - \int_{s_{(i)}}^{\beta} M ds \quad \ldots \quad (1\text{-}22)$$

By virtue of eq.(1-22), eq.(1-7) can be written as

$$r_{i+1} = r_i + F_{(i)} \alpha_{(i)} S_{(i)} - \int_{s_{(i)}}^{\beta} M ds \quad \ldots\ldots\ldots \quad (1\text{-}23)$$

Eq.(1-23) is a convenient form of the basic equation used in this project for the elastic case.

1.6.2 Inelastic Shear Connectors

In deriving equation (1-11), it was assumed that the connector load-slip relationship is linear; thus

$$r = \frac{Q}{k}$$

However, if the load-slip curve is non-linear, then the load-slip relationship is expressed in the following modified form (see fig.(1.7))

$$r = \frac{Q}{k} + d \qquad \text{..... (1-24)}$$

where k is the slope of the tangent at a point corresponding to Q and r, and d is the intercept with sign of the tangent on the axis representing slip. The compatibility condition is the same as before:

$$r_{i+1} - r_i = \int_{s(i)} (\epsilon_b - \epsilon_s) ds \qquad \text{..... (1-6)}$$

Substitution of eq.(1-24) into eq.(1-6) yields

$$\left(\frac{Q_{i+1}}{k_{i+1}} + d_{i+1} \right) - \left(\frac{Q_i}{k_i} + d_i \right) = \int_{s(i)} (\epsilon_b - \epsilon_s) ds$$

or $$\frac{Q_{i+1}}{k_{i+1}} - \frac{Q_i}{k_i} = \int_{s(i)} (\epsilon_b - \epsilon_s) ds + (d_i - d_{i+1}) \qquad \text{..... (1-25)}$$

Equation (1-25), simplified in a manner similar to equation (1-11) and expressed in matrix notation yields

$$[B]\,[F] = [A] + [D]$$

where $D_{(i)} = d_i - d_{i+1}$

1.7 A Brief Discussion of the Solution for the Term $\int_{s(i)} (\epsilon_b - \epsilon_s) ds$ for the Inelastic Beam and Slab

If the stress-strain relationship for the material of the slab or beam is non-linear, the strain difference at the interface, that is, $(\epsilon_b - \epsilon_s)$ can no longer be expressed as a linear combination of F and M as was done with the elastic case (eq.(1-15)). It is possible, of course, to express $(\epsilon_b - \epsilon_s)$ as a non-linear function of F and M.

However, this nonlinearity causes difficulty in solving the basic equation. Furthermore, for different stress- strain curves and different cross-sectional shapes, different non-linear functions of F and M must be derived. For this reason, no attempt has been made to express $(\epsilon_b - \epsilon_s)$ as a function of F and M. Since the computations are performed using a digital computer; it is preferable to use numerical techniques to determine $(\epsilon_b - \epsilon_s)$ corresponding to the given F and M. These techniques are described in detail in chapter 2.

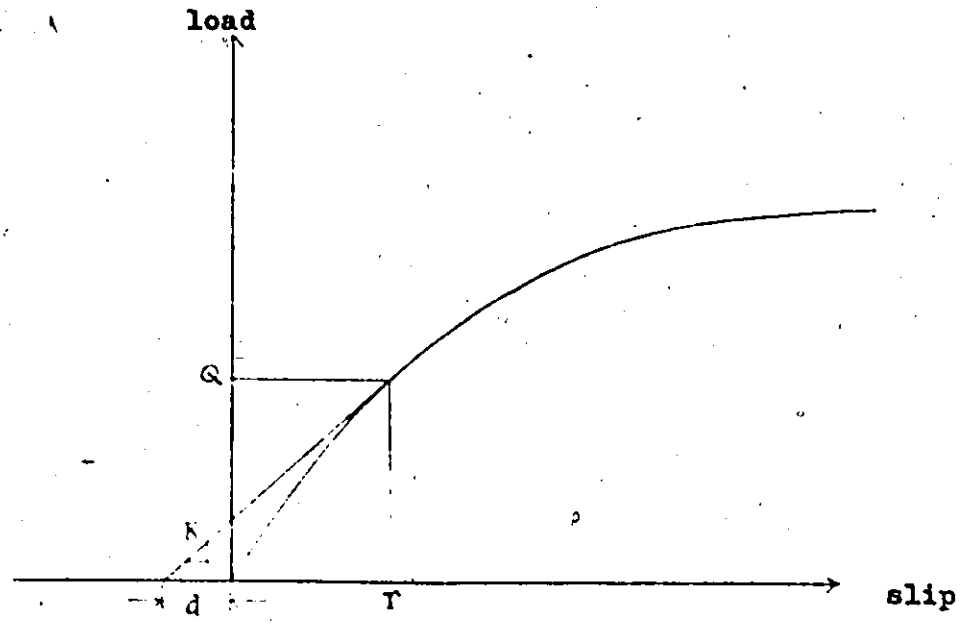


Fig. (1.7) Load Slip Curve for a Connector

CHAPTER 2

METHOD OF ANALYSIS

The method used here is close to Thiruvengadam's method, but a great change and modification have been made to remove the numerical difficulties and reduce the computer time. The distinguishable difference is that the method in this project does not necessarily need to solve the finite difference equations, and, hence, can accommodate a smooth load-slip curve (even with negative slips if necessary) while Thiruvengadam's method must use a series of straight lines to approximate a smooth curve.

2.1 Shored and Unshored Beams

The method is devised to study both shored and unshored beams. In practice, generally the beam is placed on the supports and then the concrete is cast on tep to form a slab. This kind of composite beam is called an unshored beam. For an unshored beam, the dead load is carried by the steel beam alone; an initial strain and curvature due to dead load exists. The slab has no strain and the connectors subject to no force before the live load is added on. When live load exists, it will be carried by the composite section. The shored beam case is quite different from that above. It is the composite section which carries

the total loading from the very beginning.

2.2 The Slab with Cellular Deck

In the computation of the force and moment in the slab with cellular deck, the solid portion alone is used. However the strain, ϵ_s, in eq.(1-6) or in any equation derived from equation (1-6) must take into account the value of strain of the slab at the interface between the slab and the beam.

2.3 Cracking

The concrete is assumed as not being able to support tension. When the tensile stress appears, the portion of the slab with tensile stress is said to be cracked. The effective area for computing force and moment in the slab is reduced to the area of the portion having compressive stress only.

2.4 Load-Slip Curve for the Connector

Thiruvengadam used a series of straight lines to approximate a smooth load-slip curve.

In this project, a smooth curve defined analytically is used to represent the load-slip curve derived from push-out tests. For the composite beam with cellular deck, a hyperbola in the form of

$$Q = \frac{C}{r-A} + B \quad \text{.....} \quad (2\text{-}1a)$$

is preferred to represent the load-slip relationship. A, B and C are three constants to be determined from the values given by three points

which are to be passed by the curve. One of them is the origin denoted as (0,0). The other two are properly chosen to define the curve. They are denoted as (r_2,Q_2) and (r_3,Q_3). By using these three sets of values, eq.(2-1a) is solved for the three constants. The expressions obtained are

$$B = \frac{r_2Q_2Q_3 - r_3Q_3Q_2}{r_2Q_3 - r_3Q_2} \qquad \text{..... (2-1b)}$$

$$C = \frac{Br_2Q_2 - B^2r_2}{Q_2} \qquad \text{..... (2-1c)}$$

$$A = \frac{C}{B} \qquad \text{..... (2-1d)}$$

In choosing a curve, a hyperbolic representation gives more freedom than an exponential representation which is in the form $Q = A\,(1-e^{-Br})$ * with A and B as two constants determined by the values of the two points to be passed by the curve. In the latter case, it is necessary to choose the two points under the condition that the slip of one point is two times of the slip of the other. The hyperbolic representation also gives a better result for the composite beam having a cellular deck.

The hyperbola used in the computation of all the examples in this project is shown in fig.(2.1).

2.5 Stress-Strain Curve for the Steel Beam

The analysis can accomodate either linear or smooth stress-strain curves. The curve shown in fig.(2.2) is generally used in this project.

---

* This form was suggested in Reference (6).

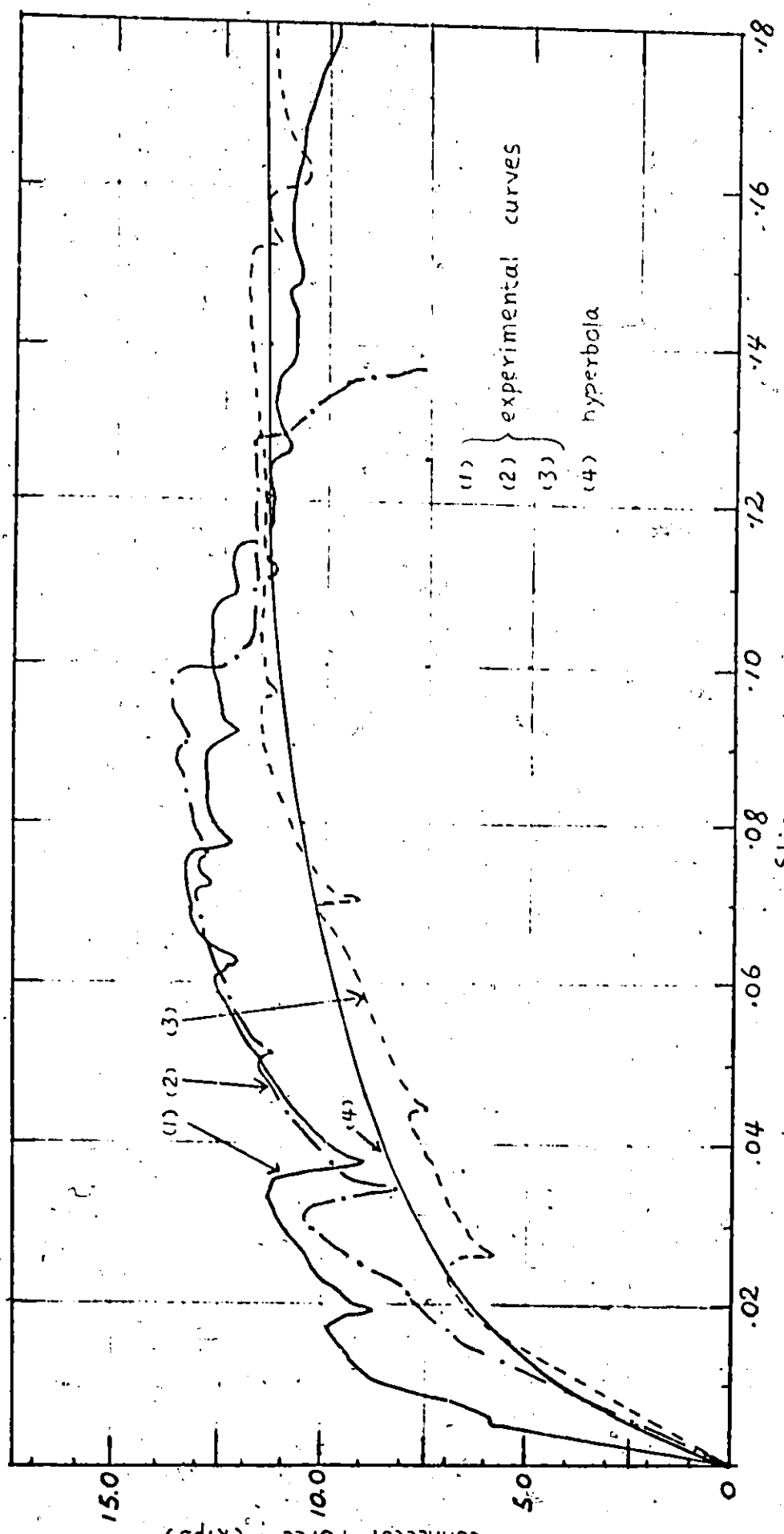


Fig. (2.1) Experimental Load–Slip Curves and the Hyperbola Representation

2.6 Stress-Strain Curve for the Slab

Similar to the case of the beam, the choice of stress-strain curve for the slab can be very flexible, but for simplicity, the curve shown in fig.(2.3) is used in all the examples.

2.7 Method of Computation

Though the method stated below is applicable for both elastic and inelastic cases, it will not be used in the elastic case, since a much simpler method is available.

2.7.1 Strain Distribution

If the top or bottom fibre strain and the curvature are given, the strain distribution is given. Referring to fig.(2.4) (b); ϕ_b and ϵ_{bb}, for example, are known. The strain in the steel beam at any point with a distance y from the bottom fibre is

$$\epsilon = \epsilon_{bb} - \phi_b y \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}2)$$

The strain in the slab is computed in a similar way by replacing ϕ_b by ϕ_s and the quantities for the beam by that for the slab. ϕ_b and ϕ_s are not two independent quantities; they are related by the expression

$$\phi_b = \phi_s + \phi_{DL}$$

For a shored beam $\phi_{DL} = 0$, while for an unshored beam $\phi_{DL} = \frac{M_{DL}}{E_b I_b}$

2.7.2 Computation of Force and its Location in the Slab from a Known Strain Distribution

The term 'location' used here means the force center or the point of application of the resultant force.

If the forces in the beam and in the slab and their locations are known, the moment of the composite beam can be found by applying

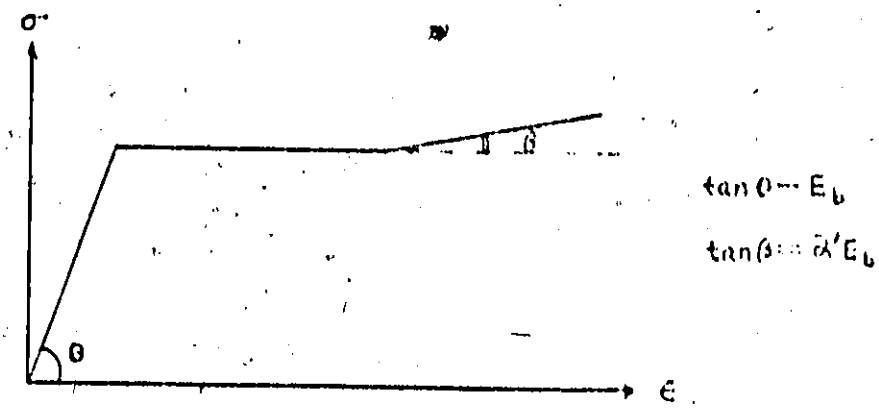


Fig. (2.2) The Stress-Strain Curve Used for Steel

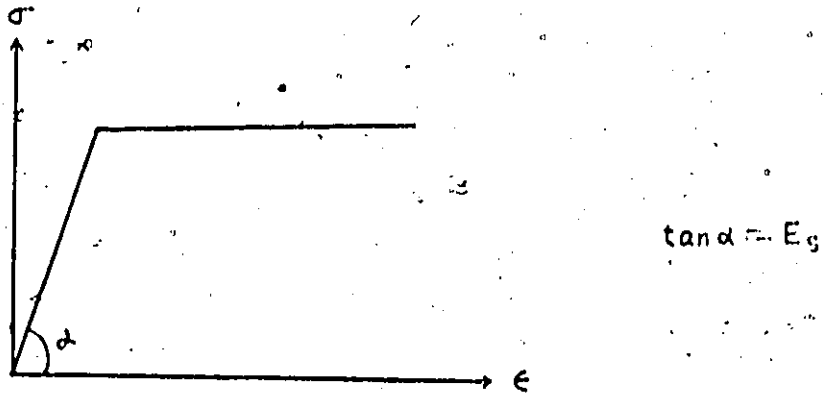
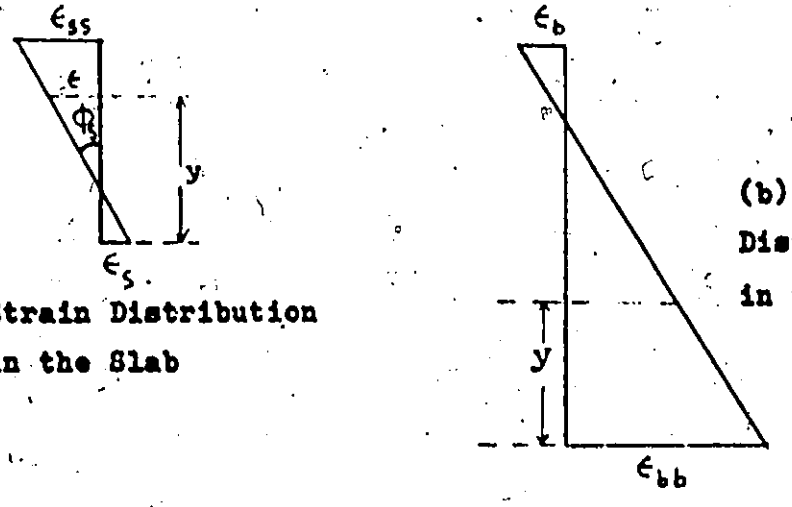


Fig. (2.3) The Stress-Strain Curve Used for Concrete



(a) Strain Distribution in the Slab

(b) Strain Distribution in the Beam

Fig. (2.4) Strain Distribution for a Composite Beam

eq.(1-2).

If the slab has a rectangular shape and if the curve shown in fig.(2.3) is taken for the stress-strain curve of the concrete, an exact calculation can be made very easily. In the case where a smooth stress-strain curve for the concrete is represented by a series of straight lines, it is more convenient to use the method of subdivision which was the only method used in Thiruvengadam's analysis. The exact method is given in the following paragraph. The method of subdivision applied to a slab is very similar to that applied to a beam. To avoid repetition, it is not given in this article.

Refer to fig.(2.5) which includes all the possible cases of stress and strain distributions in the slab. The strain distributions are shown in fig.(2.5(a)) and the stress distributions in fig.(2.5(b)). Fig.(2.5(c)) is drawn to show the quantities used in the computation of the force and its location. Though the strain and stress distribution shown in fig.(2.5(c)) is only one of the cases, it serves to derive a general equation for any of the cases.

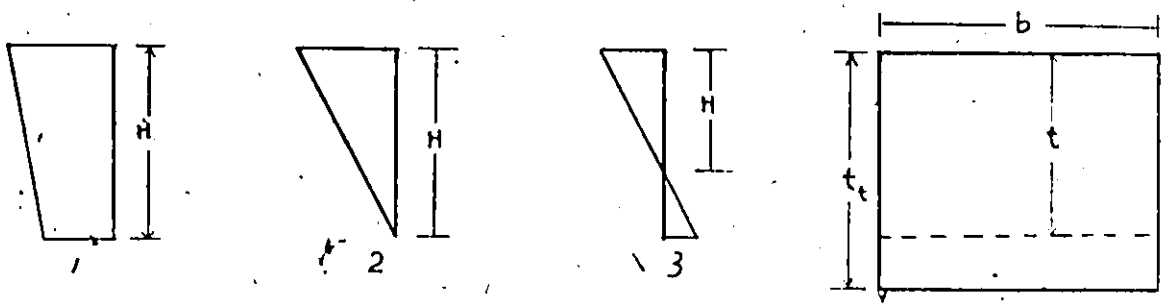
In fact, the distribution 2 is the limiting case of 1 or 3, and case 6 is the limiting case of case 4 or 8. Similarly case 7 is that of 5 or 9.

However the computation is general for any case.

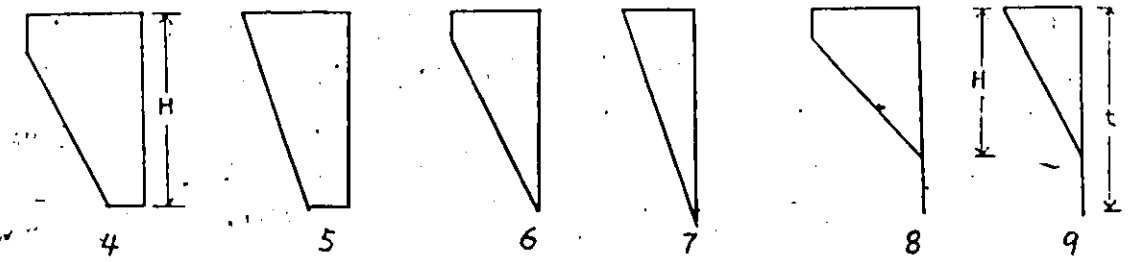
The depth of the uncracked portion is

$$H = \frac{|\epsilon_{ss}|}{\phi_s} \quad (\text{if} \quad \frac{|\epsilon_{ss}|}{\phi_s} > t \ , \ H=t)$$

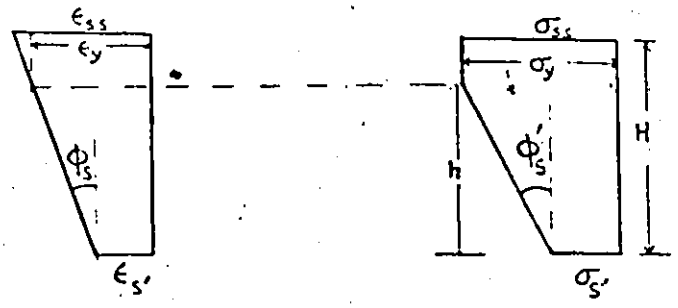
In the following derivation , for simplicity, the values of force, moment, strain and stress are their absolute values, that is the



(a) Strain Diagram

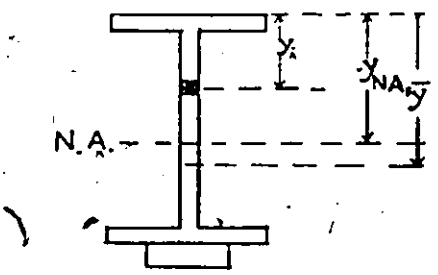


(b) Stress Diagram

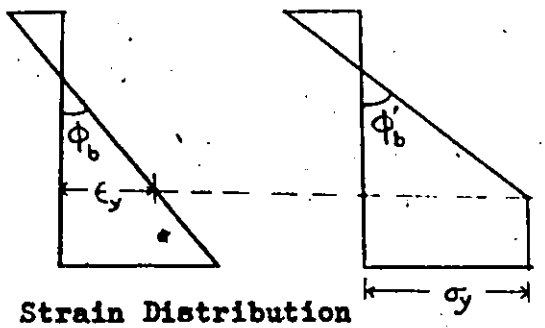


(c) A Typical Diagram Drawn for Computation

Fig. (2.5) Possible Cases of Strain and Stress Distribution in the Slab



(a) Cross-Sectional Shape



(b) Strain Distribution

(c) Stress Distribution

Fig. (2.6) A Typical Strain and Stress Distribution in the Steel Beam.

magnitudes without sign.

$$\sigma_{ss} = \begin{cases} \epsilon_{ss} E_s \\ \sigma_{sy} \end{cases} \quad \text{if} \quad \epsilon_{ss} \geq \epsilon_{sy}$$

Replace ϵ_{ss} by $\epsilon_{s'}$, $\sigma_{s'}$ is obtained in a similar way.

$$\Delta\sigma = \sigma_{ss} - \sigma_{s'}$$

$$h = \begin{cases} \dfrac{\epsilon_y - \epsilon_{s'}}{\phi_s} \text{ (if } \dfrac{\epsilon_y - \epsilon_{s'}}{\phi_s} < H) \\ H \text{ otherwise} \end{cases}$$

$$F_1 = \sigma_{ss} H b$$

$$F_2 = \frac{\Delta\sigma}{2} h b$$

$$F = F_1 - F_2 = \sigma_{ss} H b - \frac{\Delta\sigma}{2} h b \quad \ldots\ldots\ldots\ldots\ldots \quad (2\text{-}3)$$

The point of application of force F measured from the lower edge of the uncracked portion of the slab is

$$\bar{y} = \left(\frac{F_1 H}{2} - \frac{F_2 h}{3} \right) / \left(F_1 - F_2 \right) \quad \ldots\ldots\ldots\ldots \quad (2\text{-}4)$$

If needed, the moment M_s is

$$M_s = \left(\frac{H}{2} - \bar{y} \right) F \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}5)$$

2.7.3 Computation of the Force and its Location in the Steel Beam from a Known Strain Distribution

As the cross-sectional shape of the steel beam is not so regular and the stress-strain curve may not be a simple curve formed by a series of very few straight lines, it is much more convenient to use the method of subdivision for the computation. Unlike the exact

method used in the slab, this method can be easily modified to accommodate various shapes and various kinds of stress-strain curve.

First, the cross-section is subdivided into sufficient number of small elements as shown in fig.(2.6a).

$$F = \sum F_i, \quad F_i = \begin{cases} E_b \epsilon_i A_i & \text{if } \epsilon_i \leq \epsilon_{yi} \\ \sigma_{yi} A_i & \text{if } \epsilon_i \geq \epsilon_{yi} \end{cases}$$

The point of application from the top of the steel beam is

$$\bar{y} = \frac{\sum F_i y_i}{\sum F_i} \qquad \text{..... (2-7)}$$

If one employs eq.(1-2c) for total moment, the term M_b is obtained as

$$M_b = (\bar{y} - y_{NA}) F$$

In the calculation above, a question may arise concerning an unshored beam. One may ask which of the strain distributions should be used, the strain distribution from live load or the total strain distribution produced by the dead load and live load? It seems logical to argue in this way: the force exists only when live load exists, hence, the force can be obtained from the strain distribution produced by the live load alone taking into account the effect of dead load by noting that when the sum of the steel strains exceeds the yield point value, the corresponding stress should take the yield point stress. According to this, one may compute the difference between the total strain and the dead load strain in order to get the live load strain for the computation of the force. However this is only partially correct. Because the principle of superposition applies only within the elastic limit, outside of which the system no longer retains its linear properties, the principle

of superposition fails.

It is very simple to solve this problem by realizing that the force required is the actual force existing in the steel beam due to the total strain distribution. They are directly related; namely, the total strain distribution always gives the total force at the beam, even though the live load may not have been added. An alternative way of visualizing this problem is that the total strain provides the force which arises only when live load exists, because the dead load, generates a force-free symmetrical initial strain.

2.7.4 Computation of the Extreme Fibre Strain in the Slab for Given Force and Curvature

This problem requires a strain distribution such that it promises a fixed given force, since after the extreme fibre strain is found, together with the given curvature, a unique strain distribution corresponding to the given force will be established. For this purpose, a numerical technique, developed by Gurfinkel and Robinson(10), and first applied to composite structures by Thiruvengadam, may be conveniently used.

As just stated, the given force is uniquely determined by a strain distribution, which , in term, is determined by the curvature ϕ and one of the extreme fibre strains. After fixing ϕ, F is a fuction of ϵ only. Setting ϕ to the given value ϕ_o and writing

$$F = F(\epsilon, \phi_o)$$

and using Taylor's expansion to express the force F, retaining only the linear term, one obtains

$$F(\epsilon,\phi_o) \doteq \bar{F}(\bar{\epsilon},\phi_o) + \left.\frac{\partial F(\epsilon,\phi_o)}{\partial \epsilon}\right|_{\epsilon=\bar{\epsilon}} d\epsilon \quad \ldots\ldots\ldots \quad (2\text{-}8)$$

where $\bar{\epsilon}$ is the known strain corresponding to a known force $\bar{F}$.

ϕ_o is the given curvature.

ϵ is the extreme fibre strain required which together with ϕ_o gives the strain distribution resulting in a force equal to the given force, $F(\epsilon,\phi_o)$.

$F(\epsilon,\phi_o)$ is the given force.

Eq.(2-8) provides a good approximation only when $F(\epsilon,\phi_o)$ is close to $\bar{F}(\bar{\epsilon},\phi_o)$, that is, $F(\epsilon,\phi_o)$ lies in the small neighbourhood of $\bar{F}(\bar{\epsilon},\phi_o)$. If $d\epsilon$ can be found, the required strain is

$$\epsilon = \bar{\epsilon} + d\epsilon \quad \ldots\ldots\ldots \quad (2\text{-}9)$$

In order to find $d\epsilon$, the term $\left.\frac{\partial F(\epsilon,\phi_o)}{\partial \epsilon}\right|_{\epsilon=\bar{\epsilon}}$ in eq.(2-8) must first be computed. For this purpose, a small increment of ϵ, called $\Delta\epsilon$, is added to $\bar{\epsilon}$. The sum is called ϵ', that is

$$\epsilon' = \bar{\epsilon} + \Delta\epsilon$$

From the known curvature ϕ_o and strain ϵ', the force $F(\epsilon',\phi_o)$ can be computed using the method in article 2.7.2. Then the force-strain gradient is approximated as:

$$\left.\frac{\partial F}{\partial \epsilon}\right| \doteq \frac{F(\epsilon',\phi_o) - \bar{F}(\bar{\epsilon},\phi_o)}{\Delta\epsilon} \quad \ldots\ldots\ldots \quad (2\text{-}10)$$

The combination of eqs.(2-8), (2-9) and (2-10) gives the required strain:

$$\epsilon \doteq \bar{\epsilon} + \left(F(\epsilon,\phi_o) - \bar{F}(\bar{\epsilon},\phi_o)\right) \Big/ \frac{\partial F(\epsilon,\phi_o)}{\partial \epsilon}$$

$$\epsilon \doteq \bar{\epsilon} + \frac{\left(F(\epsilon,\phi_o) - \bar{F}(\bar{\epsilon},\phi_o)\right)\Delta\epsilon}{F(\epsilon',\phi_o) - \bar{F}(\bar{\epsilon},\phi_o)} \quad \ldots\ldots\ldots \quad (2\text{-}11)$$

Corresponding to the computed ϵ , the force is computed and compared with the given force $F(\epsilon,\phi_o)$. If both forces are sufficiently close, the computed ϵ is acceptable.

If $F(\epsilon,\phi_o)$ is not in the small neighbourhood of $\bar{F}(\bar{\epsilon},\phi_o)$, eq.(2-8) is a poor approximation, and the value of ϵ obtained will not provide the desired accuracy. Then the procedure should be repeated by taking the last ϵ computed as $\bar{\epsilon}$ and, correspondingly, the force F computed from ϵ and ϕ_o as $\bar{F}$. This process converges rapidly, especially when the first $\bar{\epsilon}$ is chosen properly. The required accuracy is usually obtained after not more than three cycles.

2.7.5 Computation of Strain Distribution for the Composite Beam for a Given Force and External Moment

The external moment and the force of interaction between the beam and the slab at a point along the span are given. The problem requires the strain distribution throughout the depth at a cross-section of the composite beam such that the force and total internal moment computed from the strain distribution equal the prescribed force and the external moment.

Before proceeding, let us examine what is the minimum number of independent variables required to find the moment and force of the composite beam. Four variables are apparently needed. Those are ϵ_b, the extreme fibre strain in the beam; ϵ_s, the extreme fibre strain in the slab; ϕ_b, the curvature of the beam and ϕ_s, the curvature in the slab. However this is an overestimation, since ϕ_b and ϕ_s are related by

$$\phi_b = \phi_s + \phi_{DL} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}12)$$

The curvature due to dead load ϕ_{DL} vanishes for a shored beam and has a known value for an unshored beam. Therefore ϕ_s can be eliminated. If ϵ_b and ϕ_b are known, the force F_b in the beam is also known by using the method in 2.7.3. Then employing eq.(1-1), F_s can be found.

After ϕ_s and F_s are obtained, with reference to the method described in 2.7.4, ϵ_s, the extreme fibre strain in the slab is found.

Therefore ϵ_s and ϕ_s are substantially two dependent variables. The only two acting as independent variables are ϵ_b and ϕ_b. Then one may write

$$F = F(\epsilon_b, \phi_b) \quad \text{and} \quad M = M(\epsilon_b, \phi_b)$$

Now, to find F and M, ϵ_b and ϕ_b should be known at first, other quantities are derivable in the intermediate steps. However, in the present case, problem arises. F and M are prescribed, while ϵ_b, ϕ_b and other dependent quantities have to be computed.

To start with, two known values of ϵ_b and ϕ_b are identified as $\bar{\epsilon}_b$ and $\bar{\phi}_b$. The corresponding force and moment are computed, and denoted as $\bar{F}$ and $\bar{M}$ respectively. Applying Taylor's expansion formula for the two dimensional case, again only the linear terms are retained. For simplicity, ϵ_b and ϕ_b are denoted as ϵ and ϕ in the remaining part of this article, 2.7.5. The expansion is

$$F(\epsilon, \phi) = \bar{F}(\bar{\epsilon}, \bar{\phi}) + \frac{\partial F}{\partial \epsilon}\Big|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} d\epsilon + \frac{\partial F}{\partial \phi}\Big|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} d\phi \quad \ldots (2\text{-}13)$$

$$M(\epsilon, \phi) = \bar{M}(\bar{\epsilon}, \bar{\phi}) + \frac{\partial M}{\partial \epsilon}\Big|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} d\epsilon + \frac{\partial M}{\partial \phi}\Big|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} d\phi \quad \ldots (2\text{-}14)$$

The computational procedure is very similar to the one dimensional problem in 2.7.4. It can be summarized in the following steps.

(A) Add a small increment of strain, $\Delta\epsilon$, to $\bar{\epsilon}$, obtain $\epsilon' = \bar{\epsilon} + \Delta\epsilon$, maintaining the curvature $\phi = \bar{\phi}$.

(B) Compute $F(\epsilon', \bar{\phi})$, $M(\epsilon', \bar{\phi})$ from ϵ' and $\bar{\phi}$.

a. The force and its location in the beam is found using the method in 2.7.3.

b. With the aid of eq.(2-12) $\phi_s = \phi_b - \phi_{DL}$, and $F_s = -F_b$, the method in 2.7.4 is applied to find the extreme fibre strain in the slab.

c. The force and its location in the slab are computed after knowing ϕ_s and ϵ_s using the method in 2.7.2.

d. One of the equations of (1-2) is used to compute the moment, M, of the composite beam.

(C) Add a small increment of curvature, $\Delta\phi$, to $\bar{\phi}$, to obtain $\phi' = \bar{\phi} + \Delta\phi$, maintaining $\epsilon = \bar{\epsilon}$.

(D) Compute $F(\bar{\epsilon}, \phi')$, $M(\bar{\epsilon}, \phi')$ from $\bar{\epsilon}, \phi'$. The computation is very similar to (B). The only difference is that the strain and curvature of the beam are now $\bar{\epsilon}$ and ϕ'.

(E) Compute the force and moment gradients by the following approximation:

$$\left.\frac{\partial F}{\partial \epsilon}\right|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} = \frac{F(\epsilon', \bar{\phi}) - F(\bar{\epsilon}, \bar{\phi})}{\Delta\epsilon} \qquad \text{(2-15a)}$$

$$\left.\frac{\partial M}{\partial \epsilon}\right|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} = \frac{M(\epsilon', \bar{\phi}) - M(\bar{\epsilon}, \bar{\phi})}{\Delta\epsilon} \qquad \text{(2-15b)}$$

$$\left.\frac{\partial F}{\partial \phi}\right|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} = \frac{F(\bar{\epsilon}, \phi') - F(\bar{\epsilon}, \bar{\phi})}{\Delta\phi} \qquad \text{(2-15c)}$$

$$\left.\frac{\partial M}{\partial \phi}\right|_{\substack{\epsilon=\bar{\epsilon} \\ \phi=\bar{\phi}}} = \frac{M(\bar{\epsilon},\phi') - M(\bar{\epsilon},\bar{\phi})}{\Delta\phi} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}15d)$$

(F) Solve equations (2-13) and (2-14) for $d\epsilon$ and $d\phi$:

$$d\epsilon = \frac{\frac{\partial F}{\partial \phi}\left(F(\epsilon,\phi)-\bar{F}(\bar{\epsilon},\bar{\phi})\right) - \frac{\partial F}{\partial \phi}\left(M(\epsilon,\phi)-\bar{M}(\bar{\epsilon},\bar{\phi})\right)}{\left(\frac{\partial F}{\partial \epsilon}\right)\left(\frac{\partial M}{\partial \phi}\right) - \left(\frac{\partial F}{\partial \phi}\right)\left(\frac{\partial M}{\partial \epsilon}\right)} \quad \ldots\ldots\ldots \quad (2\text{-}16)$$

$$d\phi = \frac{F - \bar{F} - \frac{\partial F}{\partial \epsilon} \cdot d\epsilon}{\frac{\partial F}{\partial \phi}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2\text{-}17)$$

In the expressions for $d\epsilon$ and $d\phi$ above, all the partial derivatives are evaluated at $\epsilon = \bar{\epsilon}$ and $\phi = \bar{\phi}$.

(G) ϵ and ϕ are then obtained as

$$\epsilon = \bar{\epsilon} + d\epsilon \quad \text{and} \quad \phi = \bar{\phi} + d\phi$$

which are used to compute F and M in a way similar to that in (B). The results are compared with the required F and M. If the computed values are satisfactorily close to the prescribed values, the computation is complete.

(H) Since eqs.(2-13) and (2-14) are a linear approximation, the required accuracy may not be reached in one trial. The same steps are repeated for the next cycle with the last values of ϵ and ϕ as $\bar{\epsilon}$ and $\bar{\phi}$, the corresponding F and M computed are taken as $\bar{F}$ and $\bar{M}$ respectively. This process converges rapidly; after a few cycles the desired accuracy can usually be reached.

It is not absolutely necessary to take ϵ_b and ϕ_b as the independent variables. If ϵ_s and ϕ_s are chosen as independent variables, other quantities become dependent variables. The analysis proceeds in similar fashion.

2.7.6 Information Required Prior to Computation

The following information is required before any computation can be made.

(A) The geometry of the steel beam and slab, length of span, the dimension of the cross-section of the beam and the slab.

(B) The weight of the steel beam and slab, or other information providing the value of dead load.

(C) The number of connectors and their spacing.

(D) The load slip curve for each connector, the stress-strain curves of the steel and the concrete.

(E) The loading and the support condition.

(F) Whether the beam is shored or unshored .

The following values may be obtained from the computation.

(A) The strain distribution at each panel, namely the curvatures and extreme fibre strains in the steel beam and the slab.

(B) The slip and force at each connector.

(C) The deflection at each panel.

2.7.7 Procedure for Thiruvengadam's Computation

(A) The terms in the basic equation

Thiruvengadam's basic equation for ith panel with the modification for consideration of the inelastic connectors can be written in the form below

$$B_{2i-1}F_{(i-1)} + B_{2i}F_{(i)} + B_{2i+1}F_{(i+1)} = A_{(i)} + D_{(i)} \quad \text{..} \quad (2\text{-}18a)$$

Let n be the number of panels, then

$$B_1 = B_{2n+1} = 0$$

If the ith panel is elastic, for $1<i<n$

$$B_{2i-1} = \frac{1}{k_i}$$

for $1 \leq i \leq n$

$$B_{2i} = -\left(\frac{1}{k_i} + \frac{1}{k_{i+1}} + \alpha_{(i)} S_{(i)} \right) \quad \text{.....} \quad (2\text{-}19)$$

When the ith panel is inelastic, the expression for B_{2i} is

$$B_{2i} = -\left(\frac{1}{k_i} + \frac{1}{k_{i+1}} \right) \quad \text{.....} \quad (2\text{-}20)$$

for $1 \leq i \leq n$. The basic equation for all panels expressed in matrix notation is

$$[B]\,[F] = [A] + [D] \quad \text{.....} \quad (2\text{-}18b)$$

To consider inelastic connectors, the ith term, $D_{(i)}$ of $[D]$ is

$$D_{(i)} = d_i - d_{i+1} \quad \text{.....} \quad (2\text{-}21)$$

where d_i is the intercept on the slip axis of the tangent to the load-slip curve for connector i at a point corresponding to the connector force, Q_i. The element $A_{(i)}$ of $[A]$ for the elastic case as derived in article 1.6.1 is

$$A_{(i)} = -\int_{s_{(i)}} \beta M ds \quad \text{..... (1-21)}$$

This integral can be evaluated directly or replaced by a summation. For the latter case, the panel is divided into the desired number of subpanels. If there are $N_{(i)}$ subpanels in panel (i) and $S_{(ij)}$ is the length of subpanel (j) within the panel (i) and $M_{(ij)}$ is the moment at the middle of subpanel (j), then

$$A_{(i)} = -\sum_{j=1}^{N_{(i)}} \beta_{(ij)} M_{(ij)} S_{(ij)} \quad \text{..... (2-22)}$$

When the panel (i) is inelastic, $A_{(i)}$ is

$$A_{(i)} = \int_{s_{(i)}} (\epsilon_b - \epsilon_s)\, ds \quad \text{..... (1-14)}$$

Because $(\epsilon_b - \epsilon_s)$ may vary nonlinearly within the panel (i), subdivision of panel (i) is made to increase accuracy. Then eq.(1-14) approximated in a manner similar to eq.(2-22) gives

$$A_{(i)} = \sum_{j=1}^{N_{(i)}} (\epsilon_b - \epsilon_s)_{(ij)} \Delta S_{(ij)} \quad \text{..... (2-23)}$$

where $(\epsilon_b - \epsilon_s)_{(ij)}$ is the differential slip strain evaluated at the mid-point of subpanel (j) within panel (i).

(B) Definition of elastic and inelastic treatment

Under the given loading, if the beam and the slab are elastic at all points in the composite beam, the problem is solved by the method derived specially for the elastic case. However, if the loading is such that the strains in either the beam or the slab at any point exceed the elastic limit, the problem is solved by the method for the inelastic case. The method by which the inelastic case is

solved is a general method and can be used to solve the elastic case also.

(C) Elastic case

The method of solution when the slab and the beam are elastic consists of the following steps:

(1) Compute the moment at the mid-points of each panel or subpanel.

(2) Suitable trial values of k are assumed for each of the connectors, if sufficient information is not available to estimate suitable values for k, the initial slope of the load-slip curves can be assumed for k of all connectors.

(3) From the trial values of k, the corresponding values of d are determined from the load-slip curves.

(4) From eq.(2-19), the terms in [B] are computed.

(5) From eq.(2-21), the terms in [D] are computed.

(6) Since [B], [D] and [A] are known, the unknown [F] can be solved by the Gauss-Seidel method.

(7) Since F is known, the connector loads, Q, are calculated from the relationship:

$$Q_i = F_{(i)} - F_{(i-1)} \quad \text{.....} \quad (1\text{-}4)$$

(8) The slope of the load-slip curve, K corresponding to the connector load, Q, just computed, is determined for each of the connectors from their respective load-slip curves. The calculated values are then compared with the trial valus assumed at step (2). If for any connector, the calculated k is different from the assumed k, the new value of k is taken as the next trial value and the

computations described in steps (3) to (8) are repeated until the trial values of k are equal to the calculated values.

(9) The curvature and strain distribution at any section of the composite beam are determined.

From one dimensional beam theory

$$\phi = \frac{M - ZF}{\Sigma EI} \qquad \text{..... (1-17)}$$

for the shored beam, M is the total moment due to dead load and live load, and $\phi_b = \phi_s = \phi$.

for the unshored beam, M is the moment due to live load alone, $\phi_b = \phi + \phi_{DL}$ and $\phi_s = \phi$.

For both the cases above, the following formulas can be used

$$\epsilon_s = -\frac{F}{E_s A_s} + \phi_s C_s \qquad \text{..... (2-24)}$$

$$\epsilon_{ss} = \epsilon_s - \phi_s D_s$$

$$\epsilon_b = \frac{F}{E_b A_b} - \phi_b C_b \qquad \text{..... (2-25)}$$

$$\epsilon_{bb} = \epsilon_b + \phi_b D_b$$

where D_s and D_b are the total depth of the slab and the beam respectively.

(10) If either the strain in the bottom fibre of the beam, ϵ_{bb}, or the strain in the top fibre of the slab, ϵ_{ss}, at a panel (i) exceeds its respective yield strain, then panel (i) is declared inelastic. The treatment of inelastic panels is given in article 2.7.7 (D).

(11) Since the curvature at any point is known from step (9), the deflection at any point is obtained by numerical integration of the curvatures along the length of the beam.

(D) Inelastic case

a. Method 1

This method is general and can be used to solve both symmetric and unsymmetric problems. It consists of the following steps:

(1) Suitable trial value of F are assumed for all the yield panels. To begin, the values of F in the previous loading stage can be assumed.

(2) Each of the yielded panels is divided into a suitable number of subpanels.

(3) The moments due to live load and dead load at mid-points of all subpanels are determined.

(4) For any subpanel (j) the force and moment at its center is known; therefore the strains, ϵ_b and ϵ_s, at the interface can be determined by the method described in article 2.7.5. The terms in [A] for all yielded panels are determined from eq.(2-23).

(5) The terms in [A] for all the elastic panels are computed from eq.(2-22).

(6) Steps (2) to (6) in the elastic method (article 2.7.7(c)) are then followed.

(7) The computed values of F for the yielded panels are compared with the assumed values in step (1). If the difference between the computed and assumed values of any F is greater than the allowable error limit, the computation described in steps (2) to (7) are repeated with the new values as the next trial values.

(8) If after two successive trials, it is found that the solution diverges, the δ^2 — method proposed by Aitkin(11) can be used

to get a new trial value. The method states that, if any three successive iterates, Z_K, Z_{K+1} and Z_{K+2} are known, the relationship

$$Z = \frac{Z_K Z_{K+2} - Z_{K+1}^2}{Z_{K+2} - 2Z_{K+1} + Z_K}$$

affords an extrapolation which may provide an improved estimate for the trial value Z.

(9) Steps (7) and (8) in the elastic method (article 2.7.7 (C)) are then followed.

(10) For all the elastic panels, step(9) in the elastic method (article 2.7.7(C)) is followed.

(11) For all inelastic panels, the extreme fibre strains and the curvatures of the beam and the slab were computed in step (4). The values computed in the last cycle are taken.

(12) The strains in the top fibre of the slab and the bottom fibre of the beam are checked to determine whether they have exceeded their respective yield limit. If any new panels have yielded, these panels are included in the inelastic method and steps(1) to (12) are repeated.

(13) The deflection at any point is obtained by numerical integration of the curvatures over the length of the span.

b. Method 2

In the case of symmetric problems, a comparatively rapid convergence is obtained by using this method.

The panel adjacent to the center of the span is designated as (m) and it is assumed that (i) is the leftmost yielded panel. Because of symmetry, only one-half the span need be considered.

The panels to the left of panel (i) are elastic and the panels from (i) to (m) are inelastic. Because of symmetry, the load on the central connector m+1, Q_{m+1}, is zero and the slip $r_{m+1} = 0$.

The method consists of the following steps:

(1) The yielded panels are subdivided into suitable number of subpanels and the moments due to live and dead loads are determined at the mid-points of all these subpanels.

(2) A trial value of force for the panel (i), $F_{(i)}$, is assumed.

(3) For a subpanel (j) in panel (i), the force and moment at the center being known, the strains ϵ_b and ϵ_s can be found by the method described in article 2.7.5.

(4) Step (3) is carried out for all subpanels in panel (i) and $A_{(i)}$ is computed from eq.(2-23).

(5) By assumption, (i) is the leftmost yielded panel. Since panels (1) to (i-1) are elastic, steps (1) to (8) of the elastic method (article 2.7.7(C)) are followed. Thus, forces $F_{(1)}$ to $F_{(i-1)}$ and connector loads Q_1 to Q_i are obtained corresponding to an assumed value of $F_{(i)}$.

(6) The slip of the connector i, r_i, corresponding to Q_i is determined from the load-slip curve of the ith connector.

(7) The slip of the connector i+1 is given by

$$r_{i+1} = r_i + A_{(i)}$$

(8) From the load-slip curve of connector i+1, the connector load Q_{i+1} corresponding to r_{i+1} is obtained.

(9) Then $F_{(i+1)}$ is given by

$$F_{(i+1)} = F_{(i)} + Q_{i+1}$$

(10) For the (i+1)th panel, $A_{(i+1)}$ is determined in the manner similar to that described in steps (3) and (4).

(11) The slip of the connector i+2 is obtained from

$$r_{i+2} = r_{i+1} + A_{(i+1)}$$

(12) Steps (8) to (11) are followed for successive panels until r_{m+1} is determined.

(13) If the first trial of $F_{(i)}$ is close to the correct value, then $r_{m+1} \doteq 0$. However, if the desired accuracy is not obtained, a new trial value of $F_{(i)}$ is assumed and the cycle of computations repeated. The convergence can be accelerated by using Regula Falsi or bisection techniques for determining successive trial values of $F_{(i)}$.

(14) Steps (9) to (13) in method 1 for inelastic case (article 2.7.7(D)a) are then followed.

CHAPTER 3

COMPUTATIONAL DIFFICULTIES

Several kinds of numerical difficulties may arise in Thiruvengadam's computation.

3.1 Very often the computation can not proceed at the stage of computing ϵ and ϕ from the given force and moment as stated in article 2.7.5.

Because the linear approximation for Taylor's expansion is used in the computation of ϵ and ϕ, the initial values for $\bar{\epsilon}_b$ and $\bar{\phi}_b$ must be close to the required values of ϵ_b and ϕ_b. If $\bar{\epsilon}$ and $\bar{\phi}$ chosen lie in the small neighbourhood of the required ϵ and ϕ, the computed values of ϵ and ϕ in the first iterative process, or first cycle, will be closer to the required values than the starting values of $\bar{\epsilon}$ and $\bar{\phi}$. Then the iterative process converges. If the starting values of $\bar{\epsilon}$ and $\bar{\phi}$ are not close enough to the required values of ϵ and ϕ, the values of ϵ and ϕ computed in a cycle may be farther away from the required values. Taking these values for $\bar{\epsilon}$ and $\bar{\phi}$ for next cycle, the divergent tendency becomes more severe. Finally the computation reaches the stage where $\bar{\epsilon}$ and $\bar{\phi}$ take excessively large values. The respective increments of $\Delta\epsilon$ of $\bar{\epsilon}$ and $\Delta\phi$ of $\bar{\phi}$ will result in the force and moment which do not differ from $\bar{F}$ and $\bar{N}$ respectively. Then all the force and

moment gradients, $\frac{\partial F}{\partial \epsilon}$, $\frac{\partial F}{\partial \phi}$, $\frac{\partial M}{\partial \epsilon}$ and $\frac{\partial M}{\partial \phi}$ become zero. The computation in eq.(2-16) or eq.(2-17) cannot be performed due to zero denominators. Taking the computed value of ϵ and ϕ from the previous loading stage as the value for $\bar{\epsilon}$ and $\bar{\phi}$, as suggested by Thiruvengadam, is not a good choice, since not only does it take more cycles to attain the desired accuracy, but also it may not give the values of $\bar{\epsilon}$ and $\bar{\phi}$ close enough to the required ones, at high loading stages. The numerical difficulties just mentioned are unavoidable.

3.2 Even if $\bar{\epsilon}$ and $\bar{\phi}$ are well chosen, some numerical difficulties may still arise in the computation stated in 2.7.5, especially when the method of subdivision is used to compute the force and its location in the slab from a known strain distribution, as was stated in article 2.7.2.

While computing the term $\frac{\partial M}{\partial \epsilon}$, as stated in articles 2.7.5(A) and (B), the force $F(\epsilon + \Delta\epsilon, \bar{\phi})$ in the beam and in the slab is first found. Then the extreme fibre strain in the slab is computed corresponding to the known curvature and force by using the iterative method in 2.7.4. If the subdivision in the slab is not fine enough, or the prescribed accuracy for the iterative computation is not great enough, or the strain increment $\Delta\epsilon$ is too small, or the combination of these, the strain distribution computed in the slab may not be accurate enough to give the required total bending moment $M(\epsilon + \Delta\epsilon, \bar{\phi})$. This results in the moment $M(\epsilon + \Delta\epsilon, \bar{\phi})$ which is not accurate, or worse, has the same value as $\bar{M}(\bar{\epsilon}, \bar{\phi})$, or even is smaller than $\bar{M}(\bar{\epsilon}, \bar{\phi})$. In this case, $\frac{\partial M}{\partial \phi}$ is either zero or a negative number. The computation of moment gradient with respect to strain fails which, in term, affects

the computation of ϵ and ϕ. A similar case occurs when $\frac{\partial M}{\partial \phi}$ is computed.

3.3. For convenience of description, let us first define the term 'main cycle' in Thiruvengadam's computation. This term represents the computation starting with assuming a set of trial forces for the inelastic panels and ends when the simultaneous equations are solved.

A good choice of $\bar{\epsilon}$ and $\bar{\phi}$ for the main cycles other than the first main cycle is difficult or even impossible in some cases, if solving the simultaneous equations is necessary, as in the method used by Thiruvengadam. In order to apply Aitkin's δ^2 correction method for the purpose of convergence, the force F assumed for ith panel for the second main cycle must take the computed values of force in the same panel from the first main cycle, and similarly, that for the third main cycle must take that computed value from the second main cycle. Because the computed value is farther away from the required value of force than the assumed value of the force, the assumed value in the second main cycle is worse than the assumed value in the first main cycle, and similarly the assumed value for the third main cycle is worst of all among the three main cycles. This divergent nature is shown in fig.(3.1).

Fig.(3.1) shows the divergence of the value of F for the successive three main cycles for the ith panel. A similar case occurs at each inelastic panel. After three successive main cycles, a much better assumed value is obtained for the fourth main cycle by

using the $\int^2$ correction method. However, at high loading stages, because of the severe divergence, corresponding to the divergent F and fixed M at ith panel, the computation of ϵ and ϕ frequently encounters the numerical difficulties stated in article 3.1. Since the values of $\bar{\epsilon}$ and $\bar{\phi}$, cannot be properly chosen, no matter what increments of $\bar{\epsilon}$ and $\bar{\phi}$ are used to find the gradients, the computation cannot give the required value of ϵ and ϕ for the given divergent F and fixed M at panel (i).

Other methods of choosing the assumed force for the next cycle, whether employing $\int^2$ correction technique or not, cannot provide convergence in the iterative process within a reasonable number of cycles.

Thiruvengadam's method 2 in article 2.7.7(D)b can avoid the difficulties caused by the divergence of F, but the difficulties stated in 3.1 and 3.2 still exist.

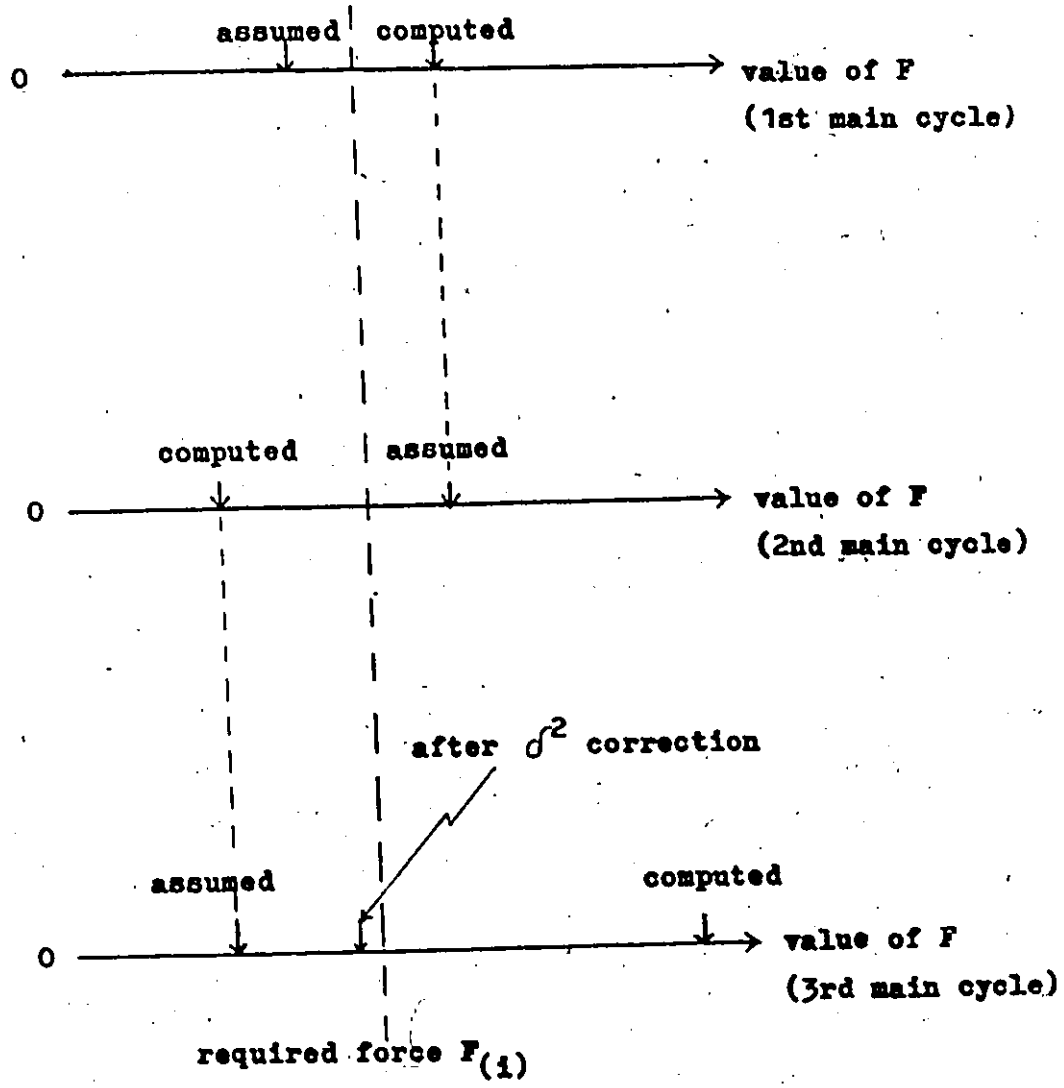


Fig. (3.1) Divergence of the Panel Force

CHAPTER 4

METHOD DEVELOPED TO OVERCOME THE NUMERICAL DIFFICULTIES

4.1 General

As stated in article 3.3, the divergence of the assumed force F at successive cycles causes serious numerical difficulties. To avoid these difficulties, a method without necessarily solving the simultaneous equations must be developed.

The method in this project was established after the study of all the difficulties mentioned above. In this method, a value is first assumed for the end slip, the slip at the first connector. The force $F_{(N+1)}$ at the other end beyond the last connector is then computed. If this force is sufficiently close to zero, a solution is obtained.

The detailed description is given in the subsequent paragraphs from which, it can be seen that the difficulties in article 3.3 will no longer exist. The method to remove the difficulties stated in articles 3.1 and 3.2 is given at the end of this chapter.

4.2 Computation of the Panel Force $F_{(i)}$ and the Connector Slip r_{i+1} from the Known Panel Force $F_{(i-1)}$ and Connector Slip r_i

If the slip at connector i is known, the computation of the slip at the i+1 th connector is accomplished by the method stated below

(see fig.(4.1)):

(1) The force at the connector i is found from the analytical expression of the load-slip curve for connector i, that is

$$Q_i = Q_i (r_i)$$

where $Q_i(r)$ is the expression of the load-slip curve for connector i.

(2) By using eq.(1-4), the force in the panel(i) is computed as:

$$F_{(i)} = F_{(i-1)} + Q_i \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1-4)$$

(3) The slip r_{i+1} is found in the following computation:

(A) If the panel is elastic, eq.(1-23) is used to find the slip at the i+1 th connector as

$$r_{i+1} = r_i + \alpha_i S_i F_{(i)} - \int_{s_{(i)}} \beta M \, dx \quad \ldots\ldots\ldots \quad (1-23)$$

The integral can be evaluated or replaced by equation (2-22) as:

$$A_{(i)} = - \int_{s_{(i)}} \beta M \, dx \doteq - \sum_{j=1}^{N_{(i)}} \beta_{(ij)} M_{(ij)} \Delta S_{(ij)} \quad \ldots \quad (2-22)$$

(B) If the panel is inelastic, eq.(1-7) must be used, thus,

$$r_{i+1} = r_i + \int_{s_{(i)}} (\epsilon_b - \dot{\epsilon}_s) ds \quad \ldots\ldots\ldots\ldots \quad (1-7)$$

The integral $\int_{s_{(i)}} (\epsilon_b - \epsilon_s) ds$ is found by the approximation expressed in eq.(2-23)

$$A_{(i)} = \int_{s_{(i)}} (\epsilon_b - \epsilon_s) ds \doteq \sum_{j=1}^{N_{(i)}} (\epsilon_b - \epsilon_s)_{(ij)} \Delta S_{(ij)} \quad \ldots (2-23)$$

Since $F_{(i)}$ is computed in step (2), the strains ϵ_b and ϵ_s can be found by using the method in article 2.7.5.

4.3 The Computational Procedure

(1) First determine the constants used in the analytical expression for the smooth load-slip curve for each connector.

(2) Compute the moment from the external loading.

(3) Assume a trial value of r_1 for the end connector. By knowing $F_{(0)}=0$, the force $F_{(1)}$ and the slip r_2 are found using the method in article 4.2 with the general index i set to 1.

In this step and the following steps, all the panels are first assumed to be elastic, unless they are found to be inelastic in the previous computation.

(4) After $F_{(1)}$ and r_2 are found, by setting i = 2, 3, ..., N $F_{(2)}$ and r_3, $F_{(3)}$ and r_4, ..., $F_{(N)}$ and r_{N+1} are found successively.

(5) Q_{N+1} is obtained from the load-slip curve for N+1 th connector, namely

$$Q_{N+1} = Q_{N+1}\,(r_{N+1})$$

and then $F_{(N+1)}$ is found as:

$$F_{(N+1)} = F_{(N)} + Q_{N+1} \qquad \text{..... (4-4)}$$

If the difference between zero and the value of $F_{(N+1)}$ is within the desired limit, the assumed value of r, is correct and the solution is obtained. For convenience, the computation from steps (3) to (5) are referred to as the main cycle for this method.

(6) If after a trial the solution is not obtained, a revised value of r_1 is used for the next cycle. The steps from (3) to (5) are repeated. This computation continues until the solution is obtained.

(7) Through the computation, the forces in the panels and at

the connectors are already obtained, the curvature and the strain for the elastic panels are found by employing eqs. (1-17), (2-24) and (2-25). The method is described in article 2.7.7 (C) (9). For the inelastic panels the curvatures and the strains are obtained through the computation (refer to 4.2.3(B)).

(8) The strains in the top fibre of the slab and the bottom fibre of the beam are checked to determine whether they have exceeded their respective yield limits. If any new panels have yielded, steps (3) to (7) must be repeated with these panels treated inelastically in the computation described in article 4.2.3.

For symmetric problems, only the half span needs to be considered. The computation is performed until the slip at the mid-span connector r_m is found. If r_m is sufficiently close to zero, the solution is obtained. If there is no connector at the mid-span in the composite beam, an imaginary connector having similar load-slip characteristics can be placed in the computation. Since for a real solution the connector at this point is subject to no force, this assumption will not affect the original solution.

From now on, the discussion is limited to the symmetric cases. The principle for unsymmetric cases is the same as that for the symmetric cases. Most methods and formulas applied to the symmetric case can also applied to the unsymmetric case. Only small modifications are required for a few formulas and some numerical values used in article 4.6.1.

Some techniques employed in the computation are given in the following articles.

4.4 Method of Choosing the First Trial Value of End Slip

It is difficult to choose a close trial value of end slip in general for the first main cycle for each loading, but relatively easy for the two initial loadings. It becomes much easier to do if the two initial loadings are small, well within the elastic limit, and also if some experience has been obtained from the previous computation.

The difficulties are further reduced if the finite difference equations are employed. Only the elastic finite difference equations

$$\frac{F_{(i+1)}}{k_{i+1}} - \left(\frac{1}{k_{i+1}} + \frac{1}{k_i} + \alpha_{(i)}S_{(i)}\right)F_{(i)} + \frac{F_{(i-1)}}{k_i} = -\int_{s_{(i)}}^{\beta} M ds \qquad \text{..... (1-19)}$$

are employed. After these are written for all the panels, a set of simultaneous equations are obtained. The constants k_i for each connector can be approximated by a single value $k = \frac{Q_0}{r_0}$ (see fig.(4.2)). r_0, a small value of slip, is chosen by estimation, and Q_0, the connector force corresponding to r_0, is computed from the load-slip expression. If eq.(2-1a) is used,

$$Q_0 = \frac{C}{r_0 - A} + B$$

After substituting the value k_i for each connector, the equations are solved. The value of $F_{(1)}$ is then obtained and the end slip r_1 is computed as:

$$r_1 = \frac{Q_1}{k} = \frac{F_{(1)} - F_{(0)}}{k} = \frac{F_{(1)}}{k} \qquad \text{..... (4-1)}$$

This value is then taken for the trial value of end slip in the first main cycle.

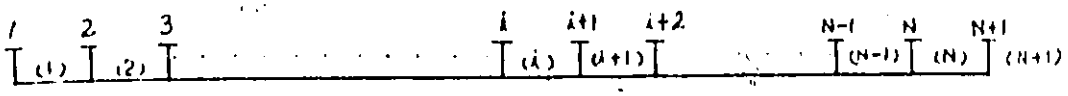


Fig. (4.1) Relative Position of the Panels and the Connectors

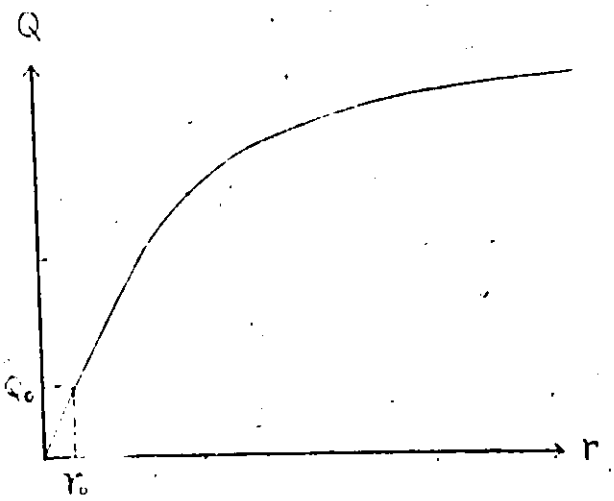


Fig. (4.2) Load-Slip Curve with an Almost Straight Initial Portion

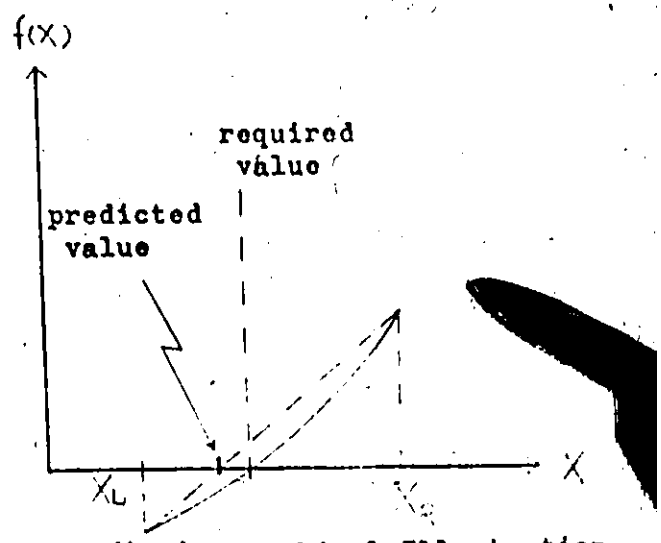


Fig. (4.3) Graphical Illustration of the Regula Falsi Method

Fig. (4.4) Negative Slips

Because the starting portion of any smooth load-slip curve is close to a straight line, the value k used for each connector is generally close to the average value of the slope of all connectors from a solution based on the smooth analytical curve. This method provides a good estimation for the assumed value for the first main cycle of the two initial loading stages.

After computing two initial loading stages, the trial value of r_1 for the first main cycle in the third loading can be found easily using

$$r_{1(3)} = r_{1(2)} + \frac{r_{1(2)} - r_{1(1)}}{M_{(2)} - M_{(1)}} \left(M_{(3)} - M_{(2)} \right) \quad \ldots\ldots \quad (4\text{-}2)$$

The number within parentheses in the subscript refers to the number of the loading stage. Eq.(4-2) is derived from the assumption that in the initial part of the moment - end slip curve, end slips are proportional to the external bending moments. This is a very good assumption for small loadings.

Whenever three consecutive loadings are computed, the trial value of the end slip for the first main cycle for the next loading stage is obtained from the Lagrangian Interpolation Formula.

4.5 Prediction of r_1, $\bar{\epsilon}$ and $\bar{\phi}$ from the Interpolation Formula

The Lagrangian Interpolation Formula plays an important role in the prediction of a trial value of end slip for the first main cycle for the fourth or subsequent loading stages. When any panel becomes inelastic, the starting values of strain and curvature , $\bar{\epsilon}_s$, $\bar{\epsilon}_b$ and $\bar{\phi}_b$, can also be obtained from the computed values in previous loading stages by employing the Lagrangian Interpolation Formula.

The formula is in the form of

$$y(x_{n+1}) = \sum_{i=1}^{n} \prod_{\substack{j=1 \\ j\neq i}}^{n} \left(\frac{x_{n+1}-x_j}{x_i-x_j} \right) y(x_i) \quad \ldots\ldots\ldots\ldots\ldots (4\text{-}3)$$

where x_1, $y(x_1)$; x_2, $y(x_2)$; ..... x_n, $y(x_n)$ are a set of known values. $y(x_{n+1})$ is the interpolated value corresponding to x_{n+1}.

To predict $y(x_{n+1})$, by using the formula an (n-1)th polynomial is constructed, which passes through the points $(x_1,y(x_1))$, $(x_2,y(x_2))$, ... $(x_n,y(x_n))$. For the present case, x_1, x_2, ... x_n are the previous consecutive loadings, and $y(x_1)$, $y(x_2)$, ... $y(x_n)$ represent the corresponding known quantities, r_1, ϵ_s , ϵ_b or ϕ_b, found in the previous computation. x_{n+1} is the loading for which the values are to be predicted while $y(x_{n+1})$ represents the values of r_1, $\bar{\epsilon}_s$, $\bar{\epsilon}_b$ or $\bar{\phi}_b$ from the interpolating formula.

4.6 Method of Finding a Revised Value of End Slip after the First Trial

4.6.1 Method of Using Multiplication Factors

This method stated here is that used to find a revised value of end slip for the next cycle before which all the assumed values of r_1 produce the mid-span slip r_m with the same sign. Whenever there is a change of sign of the mid-span slip, two better methods stated in the subsequent paragraphs are used.

As mentioned before, the method for symmetric problems is under discussion, for non-symmetric problems only slight modification is needed. For the symmetric problems, the slip at the mid-span should be zero, or sufficiently close to zero for practical purposes. If it is found to

be negative after a main cycle, the trial end slip is too small. If it is found to be positive, the assumed end slip is too large. To assume a trial value for the next cycle, a multiplication factor is applied to revise the trial value. An example is given here for the case where all the panels are elastic,

$$\text{if} \quad r_m \leq -.05 , \quad r^{(2)} = r^{(1)} \times 1.04$$

$$\text{if} \quad -.05 < r_m \leq -.001, \quad r^{(2)} = r^{(1)} \times 1.02$$

$$\text{if} \quad -.001 < r_m < 0 , \quad r^{(2)} = r^{(1)} \times 1.004$$

$$\text{if} \quad r_m \geq .05 , \quad r^{(2)} = r^{(1)} \times (2-1.04)$$

$$\text{if} \quad .001 \leq r_m < .05 , \quad r^{(2)} = r^{(1)} \times (2-1.02)$$

$$\text{if} \quad 0 < r_m < .001 , \quad r^{(2)} = r^{(1)} \times (2-1.004)$$

where $r^{(1)}$ is the end slip corresponding to the cycle just computed and $r^{(2)}$ is the assumed value of end slip for the next cycle. The numbers 1.04, 1.02, ... (2-1.004) , are all multiplication factors.

The choice of the ranges of r_m and the values of the multiplication factors is quite flexible. The only condition imposed is that the factors should not differ from unity too much such that the Regula Falsi method, stated in the later part of this article, can work effectively to accelerate the convergence. However, for the inelastic case, for which one or more panels yield, extra care should be taken in the choice of the values of the modification factors. If they are chosen to differ very slightly from unity, the required value of end slip cannot be obtained within a few cycles. However, when they differ significantly from unity, it may cause numerical difficulties in the computation of ϵ and ϕ for the next cycle, since $\bar{\epsilon}$ and $\bar{\phi}$ obtained from the cycle

just computed may not be in a small neighbourhood of ϵ and ϕ for the upcoming cycle.

After two or more cycles, the mid-span slip, r_m, may change sign. Whenever it does, it implies that the correct value of end slip is located between the last two trial values used. Then one of the following methods may be used to accelerate the process of convergence.

4.6.2 Bisection Method

Let X_L be the value of end slip producing a negative mid-span slip r_m. Let X_R be the slip which produces a positive r_m. Whenever a different sign for mid-span slip r_m appears, the values of end slip for the last two cycles are taken for the prediction of end slip for the next cycle.

The equation used for the new trial value X is

$$X = \frac{X_R + X_L}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4\text{-}4)$$

After the computation, if X produces a positive r_m, X will be taken for X_R in another prediction for the next cycle. Otherwise X is taken for X_L. The same procedure is repeated until r_m is sufficiently close to zero.

4.6.3 Regula Falsi Method

This method gives better prediction than the Bisection Method for a function close to linear.

Let the subscript n denote the cycle number. Let X be the value of the end slip. Then X_n is the assumed value of end slip at the nth cycle. $f(X_n)$ is used to represent the value of mid-span slip obtained

from assuming X_n at nth cycle.

Whenever mid-span slip changes sign, the Regula Falsi Method can be applied.

Assume that the mid-span slip r_m at nth cycle takes the sign opposite to that from n-1 th cycle. The two quantities, X_L and X_R, are defined in the same way as in the Bisection Method:

$$\text{if } f(x_n) < 0 \quad , \qquad X_L = x_n \quad , \quad X_R = x_{n-1}$$

$$\text{otherwise} \qquad\qquad X_L = x_{n-1}, \quad X_R = x_n$$

The predicted value is

$$X_{n+1} = \frac{X_R f(X_L) - X_L f(X_R)}{f(X_L) - f(X_R)} \qquad \text{..... (4-5)}$$

If the solution is still not obtained after using this new trial value, the procedure is repeated using x_{n+1} to renew one of the values, X_R or X_L , as follows:

$$\text{if} \quad f(x_{n+1}) < 0 \quad , \qquad X_L = x_{n+1} \quad , \quad X_R \text{ unchanged}$$

$$\text{if} \quad f(x_{n+1}) > 0 \quad , \qquad X_R = x_{n+1} \quad , \quad X_L \text{ unchanged}$$

Equation (4-5) is then used for a new trial value x_{n+2}. The Regula Falsi Method is a linear approximation. This becomes more clear when referring to fig.(4.3).

4.7 Techniques Used to Overcome Computational Difficulties

The difficulties stated in article 3.3 no longer exist when the method described in this chapter is adopted. The other difficulties are all related to the computation of ϵ_b and ϕ_b for a given F and M.

Those difficulties described in articles 3.1 and 3.2 can be considered to be the same type, but from different sources.

4.7.1 To overcome the difficulties outlined in article 3.1, the following techniques were used.

(A) The values of $\bar{\epsilon}_a$, $\bar{\epsilon}_b$ and $\bar{\phi}_b$ for the first main cycle at a new loading are obtained from the Lagrangian Interpolation Formula.

(B) At the same loading stage, the values of ϵ_a, ϵ_b and ϕ_b computed from one main cycle are taken for $\bar{\epsilon}_a$, $\bar{\epsilon}_b$ and $\bar{\phi}_b$ in the next cycle.

(C) If numerical difficulties appear in the computation of the gradients of F and M, or a desired accuracy is not obtained after an expected number of cycles, the computation is begun again with the same $\bar{\epsilon}_b$ and $\bar{\phi}_b$, but different increments of $\Delta\epsilon_b$ and $\Delta\phi_b$.

(D) Should difficulties still arise, the following modifications should be made.

(1) Reduce the increment of loading from the preceding loading if the difficulty occurs in the first main cycle.

(2) If difficulties are encountered in a main cycle other than the first main cycle, choose another set of multiplication factors which are closer to unity than the original set. This change reduces the difference between the two sets of values of ϵ_b and ϕ_b for the two consecutive main cycles. Hence the values of ϵ_b and ϕ_b obtained from the computation in the preceding main cycle can well serve as the $\bar{\epsilon}_b$ and $\bar{\phi}_b$ for the current cycle.

(3) For the very rare case in which many panels become inelastic and the computation happens to reach the portion on the load-

slip curve where a relatively acute rate of change of slope occurs, the methods listed above may not work well. For this case, the loading increments should be adjusted. The increment should be reduced, starting at least from the second loading stage counting back from the current loading stage. This method will give better prediction of r_1 from the interpolating polynomial. This change costs additional computer time, but always leads to solution of the problem.

4.7.2 To eliminate the difficulties described in article 3.2, the following method may be employed.

(A) If the stress-strain curve for the concrete and the cross-sectional shape of the slab are not too complicated to adopt the exact method of computation for the force and its location in the slab, the method described in 2.7.2, as one of the exact methods, can be used. This method not only can save computer time, but also eliminate the trouble due to coarse subdivision.

(B) If the method of subdivision is used, a sufficient number of divisions should be made so that after the increment of $\Delta\epsilon$ is made, $M(\bar{\epsilon}+\Delta\epsilon, \bar{\phi})$ obtained is larger than $M(\bar{\epsilon},\bar{\phi})$, and after the increment of ϕ is made, $M(\bar{\epsilon},\phi+\Delta\phi)$ is smaller than $M(\bar{\epsilon},\bar{\phi})$.

(C) The prescribed accuracy should be great enough so that the computed values of ϵ and ϕ are sufficiently close to the required ones.

4.7.3 If the techniques mentioned above are applied properly, all the difficulties in Thiruvengadam's method can be avoided. However, because of the nature of the method in this project, some minor computational difficulties are introduced unavoidably. Those difficulties are not severe and can be eliminated with relative ease. These

difficulties and their solutions are described in the following paragraphs.

(A) Referring to article 4.4, before some experience is gained, it may not be easy to make a good estimation of r_0 from which the assumed values of r_1 in the first main cycle of the first two initial loading stages are computed. Under this circumstance, a rough estimation of r_0 can be made for the first trial run of the computer program. Though a solution cannot be obtained from a quite poor estimation, the output of the computation does give some useful information. If the mid-span slip corresponding to every main cycle is positive, it indicates that the estimated value of r_0 and the assumed value of r_1 are larger than the respective required values. In order to get a smaller value for r_1 for the next computation, a smaller r_0 corresponding to a larger initial slope k is used. A similar treatment can be used for the case where all the mid-span slips turn out to be negative. If an object deck is used, a trial run of the computer program just takes a few seconds of computer time.

(B) Because of a poor estimation of r_0 and, hence, a poor assumed value of r_1, in the first main cycle of the first two initial loading stages, a negative connector force with extremely large magnitude may be produced in the proximity of midspan. When the magnitude of that force exceeds the allowable limit of the computer, the computation will stop. To prevent this, a negative number with a large magnitude should be chosen and set in the program as a limiting value of connector force. When any connector force is smaller than this value, the computation in the main cycle is switched to the initial

state with a larger revised value assumed for the end slip and the main cycle is renewed.

(C) As stated before, for the symmetric problems, the solution is based on a condition where a value of mid-span slip sufficiently close to zero is obtained. However, in some cases, this condition alone is not sufficient to obtain a solution.

Sometimes after the computation, a solution is obtained with several connectors having negative slip. The slip distribution may be in a fashion illustrated in fig.(4.4). This, of course, is unacceptable. To avoid this, either of the following methods can be employed.

(1) Reduce the loading stage. A new computation with better prediction of r_1 is started.

(2) For the computation of a main cycle, if a fairly large negative slip appears at a connector other than the mid-span connector, a multiplication factor with a value larger than unity is applied to enlarge the assumed value of end slip. The original computation is rejected and is replaced by a computation with the revised assumed value for end slip.

The difficulty just mentioned does not appear explicitly when some panels are yielded, because the difficulty in the computation of ϵ_b and ϕ_b will appear before any connector slip tends to have negative value. In this case, the method of reducing loading increment is usually effective for solving this problem.

All of the techniques stated above are well suited for computer programming.

After going through the above, it may be asked why the techniques described in articles 4.7.1 and 4.7.2 were not applied to Thiruvengadam's method 2 and this method retained without developing a new approach. These techniques can be applied to both of Thiruvengadam's methods to assist in reducing the computational difficulties. However they cannot completely eliminate the difficulties. As for Thiruvengadam's method 2, because of the necessity of solving the simultaneous equations, the load-slip curve has to be approximated by a series of straight lines. For convenience, call the left-most yielded panel the (i+1) th panel, located between the i+1 th and i+2 th connectors. At a new loading stage, if the i+1 th connector reaches the state where its load-slip characteristics switches from the initial straight line, corresponding to the preceding loading stage, to the next straight line on the load-slip curve, there will be an abrubpt change of the slope k_{i+1} at the i+1 th connector. After solving the simultaneous equations for panel (1) up to panel (i), the solution obtained will be affected by this change. Because of this sudden change, the force computed in each inelastic panel cannot be easily predicted, and difficulties in choosing $\bar{\epsilon}_b$ and $\bar{\Phi}_b$ will be encountered.

CHAPTER 5

MISCELLANEOUS COMPUTATIONAL TECHNIQUES

Most of the following techniques are used to save computing time and space or for the convenience of writing a computer program. Some are invented for higher accuracy of the results.

5.1 Pseudo Connectors or Pseudo Panels

The pseudo connector concept is introduced purely for the convenience of writing a program and saving computing space. It is only a mathematical transformation without possessing any physical meaning.

Earlier, when the spacing of a panel was not small, the integration of $(\epsilon_b - \epsilon_s)$ was replaced by the summation

$$A_{(i)} = \int_{s_{(i)}} (\epsilon_b - \epsilon_s)\, ds = \sum_{j=1}^{N_{(i)}} (\epsilon_b - \epsilon_s)_{(ij)} \Delta s_{(ij)} \quad \ldots\ldots\ldots \quad (2\text{-}23)$$

In order to evaluate the terms of this summation, the panel is subdivided into several sections. If the subdivision was actually made, considerable inconvenience would be caused in writing the computer program, since many one dimensional quantities would now require two dimensions. A better way of treating this problem is to introduce the concept of pseudo connectors. A pseudo connector is an imaginary connector possessing slip and without carrying any force. After introducing some pseudo connectors to a panel (i), the panels are divided into several sections called pseudo panels which are treated as ordinary panels. Because the spacing of these pseudo panels is small, subdivision is unnecessary when the term $A_{(i)}$ is evaluated. Then, the computation for the original panel with subdivision is replaced by the computation for the pseudo panels. In fig.(5.1) and the following description, each pseudo connector or panel is identified by placing a bar on the top.

Referring to fig. (5.1) and applying eq.(1-7) for each pseudo connector, because of small spacing, one may replace the integral by

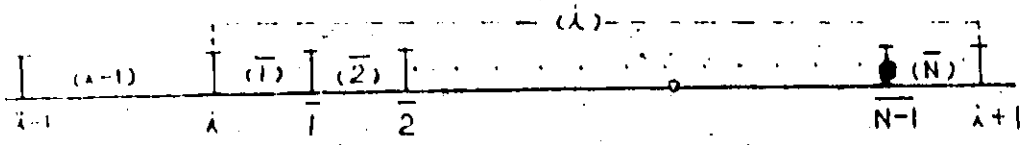


Fig. (5.1) Pseudo Connectors and Pseudo Panels

a multiplication as follow:

$$r_{\overline{1}} = r_i + (\epsilon_b - \epsilon_s)_{(\overline{1})} \Delta s_{(\overline{1})}$$

$$r_{\overline{2}} = r_{\overline{1}} + (\epsilon_b - \epsilon_s)_{(\overline{2})} \Delta s_{(\overline{2})}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$r_{i+1} = r_{\overline{N-1}} + (\epsilon_b - \epsilon_s)_{(\overline{N})} \Delta s_{(\overline{N})}$$

Summing up all the expressions above, one furthur obtains

$$r_{i+1} = r_i + \sum_{j=1}^{N_{(i)}} (\epsilon_b - \epsilon_s)_{(\overline{j})} \Delta s_{(\overline{j})} \quad \cdots\cdots\cdots\cdots \quad (5\text{-}1)$$

Eq.(5-1) is the same as eq.(1-7), it can be seen from the following:

(1) Equal spacing

$$\Delta s_{(\overline{j})} = \Delta s_{ij}$$

(2) The force at each pseudo panel is equal to the force at each subdivision, that is,

$$F_{(\overline{1})} = Q_i + F_{(i-1)} = F_{(i)},$$

$$F_{(\overline{2})} = F_{(\overline{3})} = \cdots\cdots = F_{(\overline{N})} = F_{(\overline{1})} = F_{(i)}$$

since $Q_{(\overline{I})} = 0$ for all pseudo connectors.

(3) From (2) above, for a fixed moment at the mid-point of each section and equal forces, the differential strain $(\epsilon_b - \epsilon_s)_{ij}$ is equal to $(\epsilon_b - \epsilon_s)_{(\overline{j})}$.

Therefore when N subdivisions are to be made in panel (i), the actual subdividing process can be conveniently replaced by putting N-1 pseudo connectors at the appropriate places in the panel.

By using a pseudo connector or subdivision, the discrete method is applicable for the composite beam with any kind of spacing, large or small, uniform or non-uniform. This is convenient for the theoretical study of a beam having very few connectors.

Because of the discrete nature of the method, the strains are found only at the mid-points of each panel or pseudo panel. If the strains at mid-span are required, a pseudo connector should be placed very close to mid-span to create a pseudo panel which has its mid-point very close to the mid-span of the beam.

5.2 The Desired Accuracy for the Solution in the Inelastic Loading Stage

When one or more panels become inelastic, the accuracy of the solution at each loading stage is better prescribed by two values. The first value is the value of mid-span slip, and the second value is the percentage difference of ϵ or ϕ at the most yielded panel between two consecutive main cycles. If the magnitude of the mid-span slip is smaller than the first value and the percentage deviation of the value of ϵ or ϕ at the most yielded panel from two consecutive cycles is smaller than the second value, the required solution is obtained.

5.3 The Desired Accuracy for the Computation of ϵ and ϕ for the Given F and M

As mentioned before, most of the numerical difficulties are encountered in this computation, especially when many panels become inelastic, or in the cases in which the interpolation formula cannot give a good prediction. When the required accuracy is not reached

after an expected number of cycles, it is not feasible to apply the techniques stated in article 4.7.1 immediately, since it requires additional computer time in order to institute a change. It is better to propose another value as a standard for lower accuracy. Through checking, if the force and moment corresponding to the computed ϵ and ϕ differ from the given force and moment within the limit not by this value, the computed ϵ and ϕ are conditionally accepted, and the computation in this main cycle is executed. Though the solution from this main cycle can not be accepted as a final solution of the loading, it does provide rough information regarding the correctness of the assumed value of end slip, whether it is too large or too small.

Because r_m obtained from this cycle is not accurate enough, the Regula Falsi or Bisection Method should not be used if one of the values of X_L or X_R, or both, is to be taken from the value of end slip assumed in this cycle. The method of finding a revised value of slip stated in article 4.6.1 should be used.

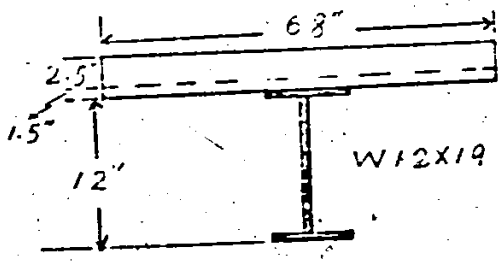
CHAPTER 6

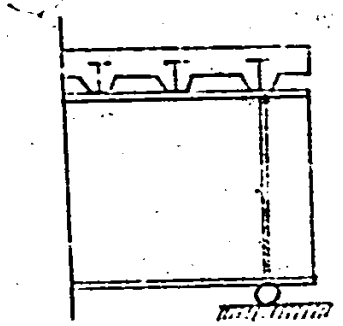
COMPUTATION FOR THE EXPERIMENTAL BEAMS

To verify the method and to check the computer program, several test beams were taken as examples for computation. The results agree with those for the experimental beams.

The four composite beams, tested by Robinson and Wallace (3), are designated B1, B2, B3 and B4. All the beams had the same geometry except the number of connectors and their spacing. The steel beam used was a W12x19. The slab was 68" wide and 4" in total depth, of which, 2.5" was the depth of the solid portion and 1.5" was the depth of the cellular floor. All the beams were unshored and simply supported. For each case, the span between the two supports was 252" and a single point load was applied at the mid-span. The dead load of each beam was 22 lb/in, producing a moment of 174.6 ksi at mid-span. 3" long and 3/4" diameter single studs were used. The stud connector layout for these beams is shown in fig.(6.1). Among the beams, B1 and B4 were identical except that B4 had one more connector placed at the mid-span.

The load versus slip curves from the push-out tests are shown in fig.(2,1). To study how the results would be affected when





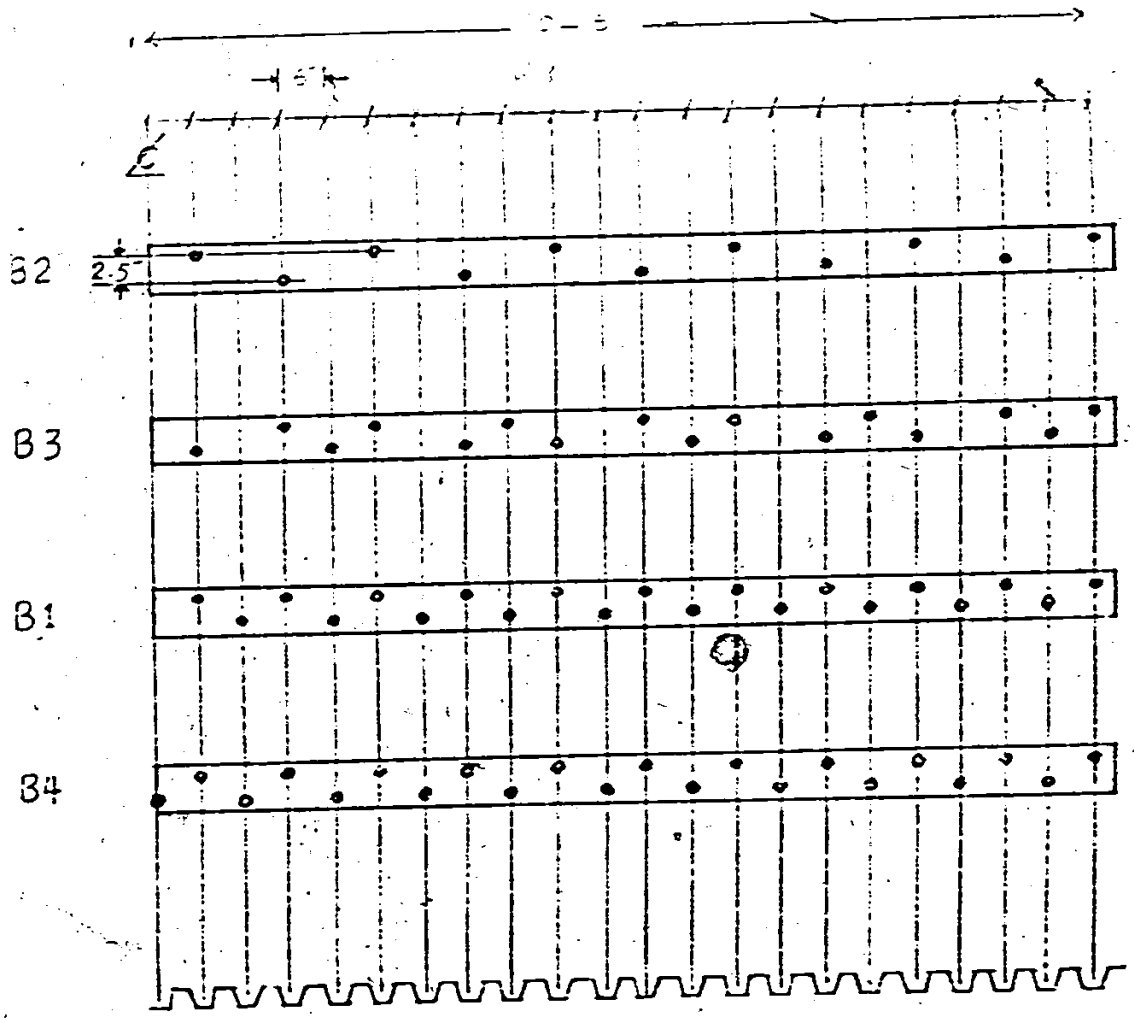


Fig. (6.1) Stud Connector Layout for the Experimental Beams

changing the load-slip curves, in the computation, two hyperbolas were used. Of these two, for the same value of slip, the one which gives smaller magnitude of force is called the lower hyperbola, the other is called the higher hyperbola. The lower hyperbola together with the experimental curves is first shown in fig.(2.1).

The two hyperbola and an exponential curve, $Q=A(1-e^{-Br})$, are shown in fig.(6.2) for comparision. Each curve is determined by two points which are to be passed through. The following are the curves and the set of points:

higher hyperbola (.01, 6), (.12, 11.3);

lower hyperbola (.02, 6.2), (.12,11.3);

exponential curve (.06, 1), (.12,11.3).

The first values in parentheses above represent the slips (inches), the second values represent the corresponding connector forces (kips). All the curves pass through the point (.12, 11.3), which is assumed to be the point of maximum connector force and maximum allowable slip. 11.3 kips was originally taken as the value of stud strength by Robinson and Wallace.

The steel stress-strain curve is shown in fig.(2.2). A value of 29×10^3 ksi is taken for E_b, and 44 ksi for the yield stress of the steel beam. The concrete stress versus strain curve is shown in fig. (2.3), with the stress yield value 3 ksi and E_s 3×10^3 ksi. The reason for using these practical design values for the computation is to check the degree of agreement between the computed and experimental results. If the agreement is found to be satisfactory other practical beams can be analyzed by a similar treatment.

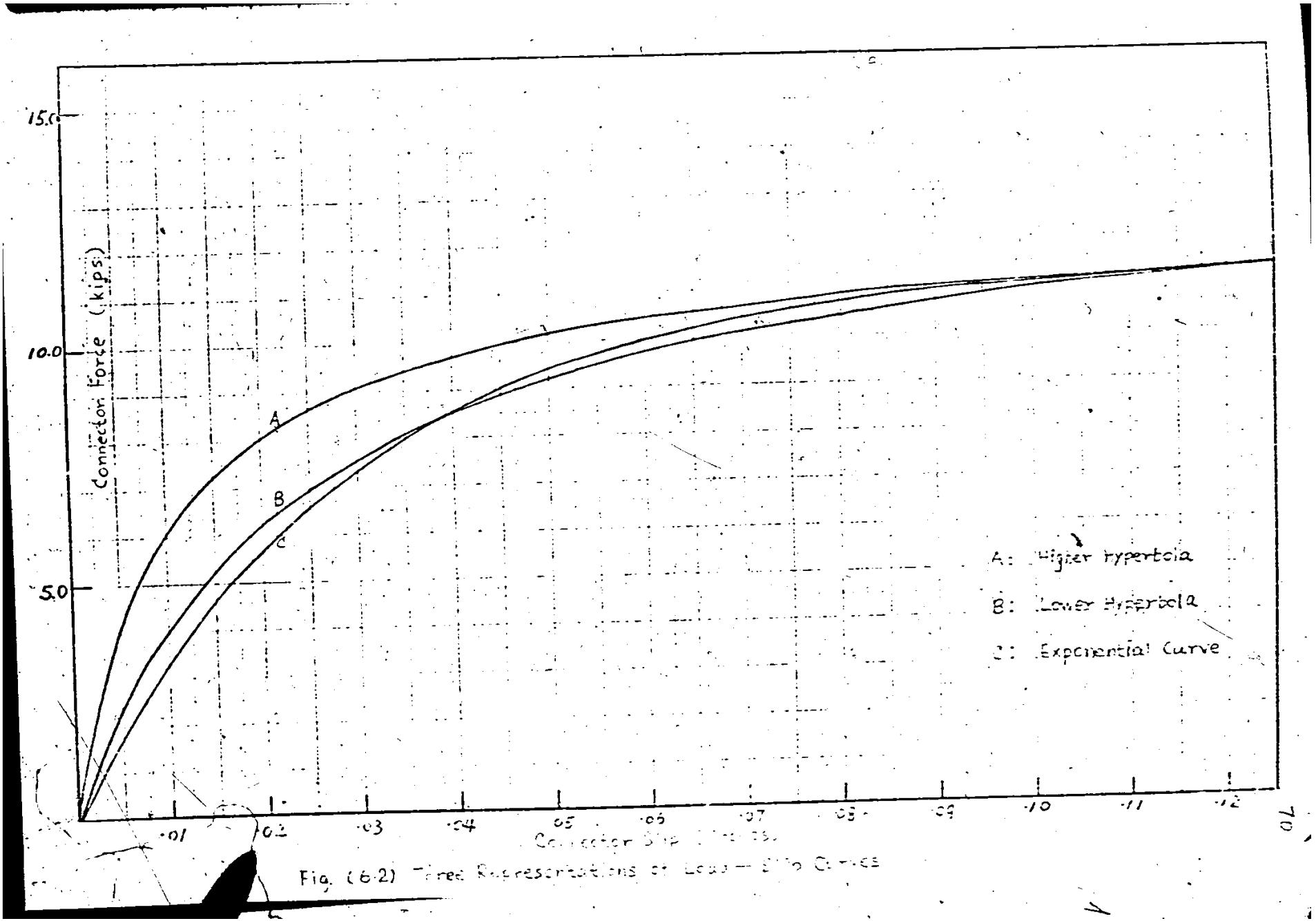


Fig. (6.2) Three Representations of Load — Slip Curves

The strain hardening factor α takes the value 0.004, starting at a point having the strain 0.015.

For further comparision, a computation of B1 and B4 is performed once more using the lower hyperbola load-slip curve with the yield stress, 41.2 ksi in the beam flange, and 46.4 ksi in the web, taken from the same source, reference (3). In the computation the concrete yield strain takes another higher value of 3.886 ksi which is the average value of 95% of the maximum strength listed in the reference. The results from all the computations are shown in fig.(6.3) to fig.(6.19). They are discussed in the following articles.

6.1 The Load Versus Deflection Curves (Figs.(6.3) to (6.6))

According to the nature of a shored beam and an unshored beam, the deflection for a shored beam is measured by taking the top level of the supports as the datum, while for an unshored beam, the deflection is measured by taking the deflected position of the steel beam due to dead load alone as the datum. The dead load deflection can be added to the deflection of the unshored beam just mentioned to give the deflection measured from the top level of the two supports. Because all the beams computed in this project are unshored beams, in all load versus deflection curves, the abscissa represents the deflection measured from the deflected position of the steel beam due to dead load alone, and the ordinate is the live loading or the moment due to live load. A short segment of a straight line is drawn down and to the left from the origin at each load-deflection curve. The horizontal projection of this segment provides the deflection of the steel beam due to dead load alone. The

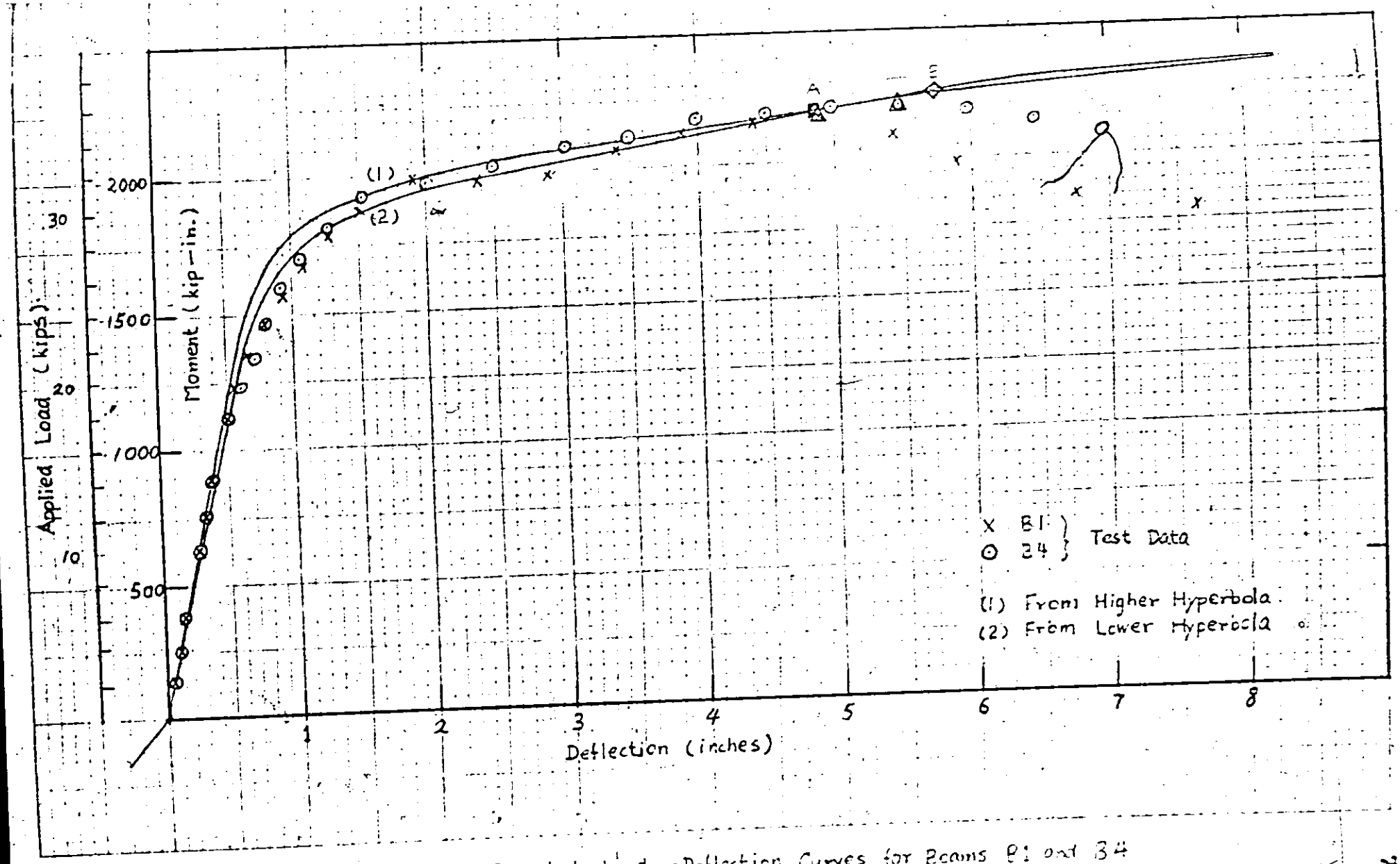


Fig. (6.3) Experimental and Computed Load—Deflection Curves for Beams B1 and B4

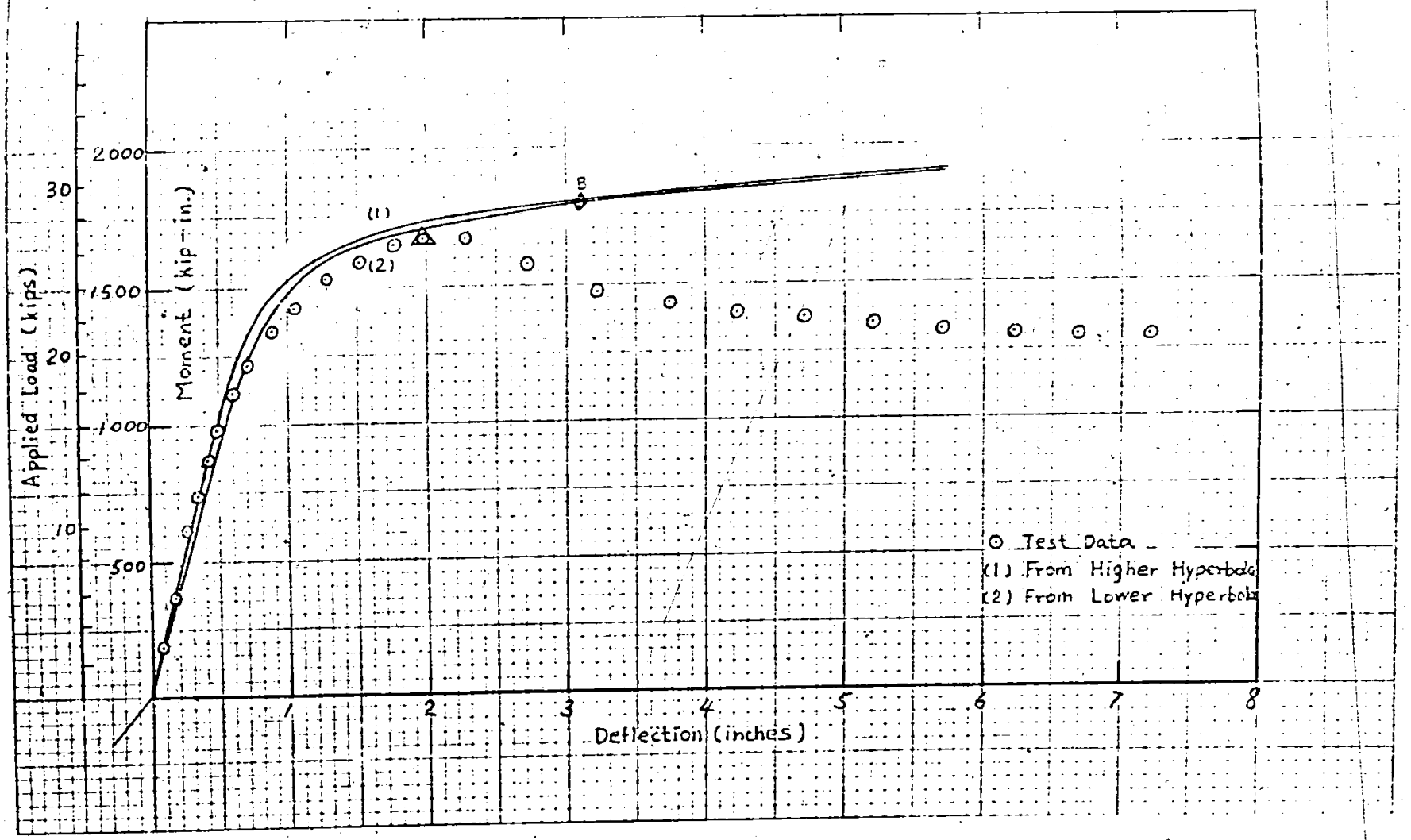


Fig. (6.4) Experimental and Computed Load—Deflection Curves for Beam B2

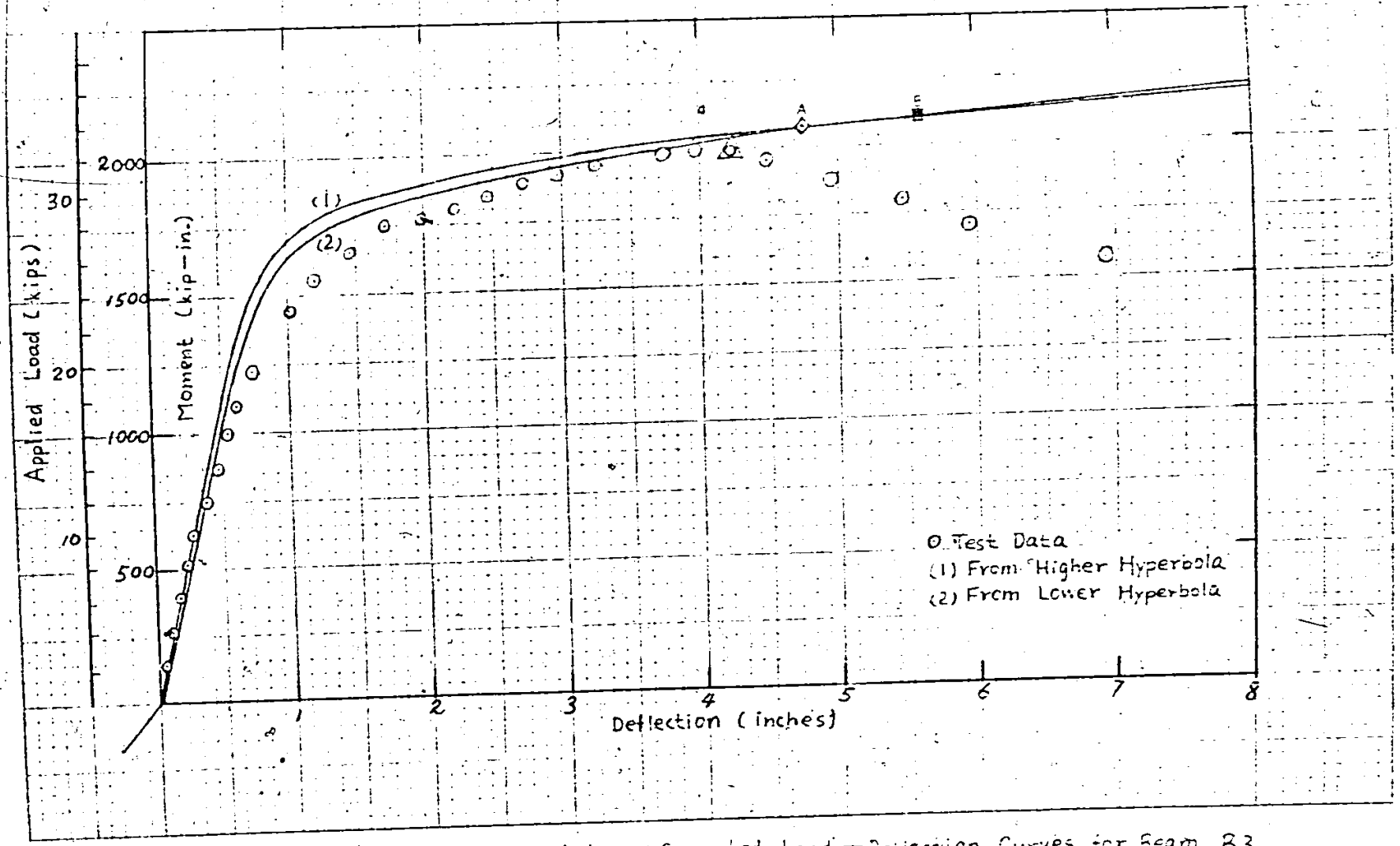


Fig. (6-5) Experimental and Computed Load—Deflection Curves for Beam B3

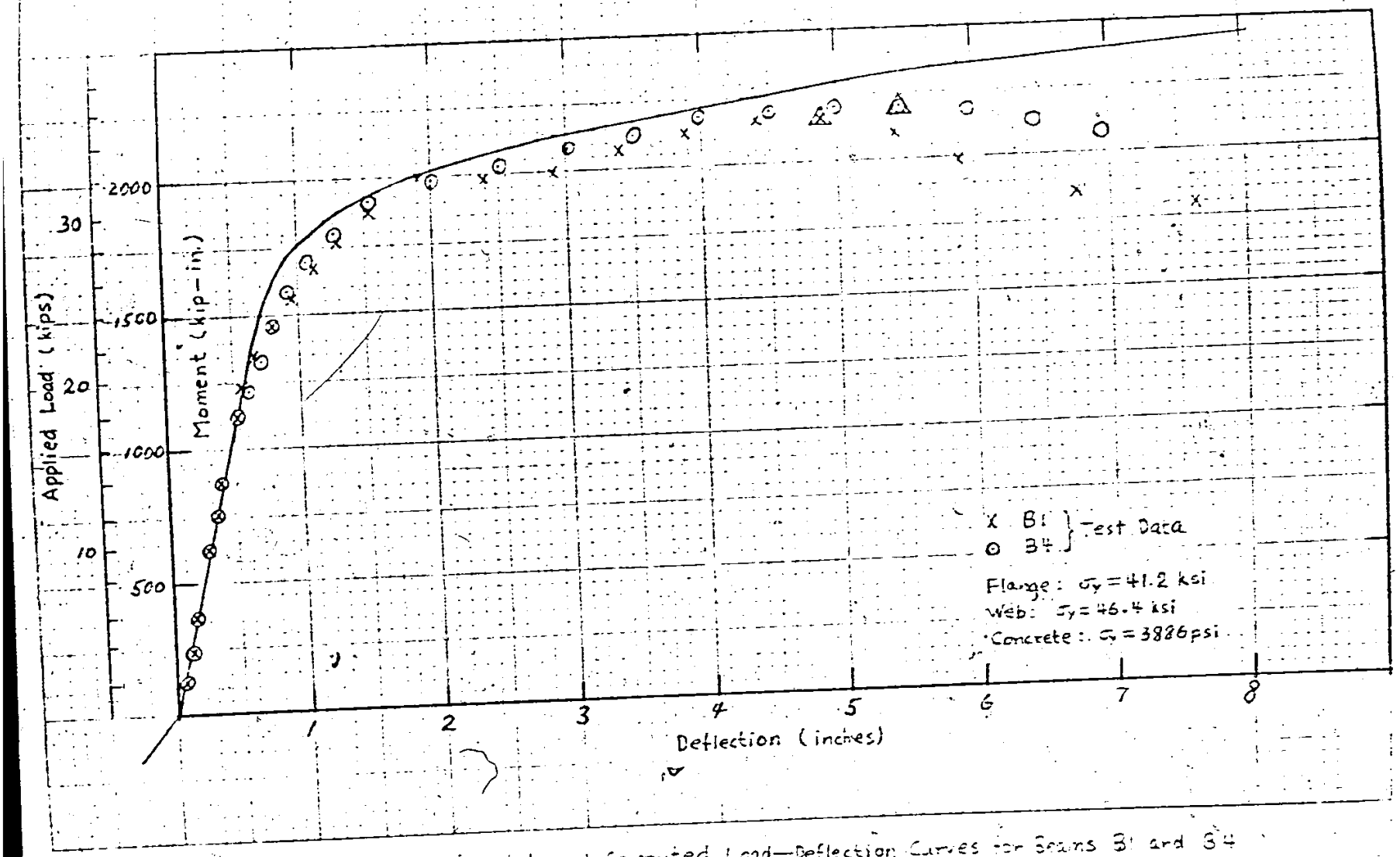


Fig. (6.5) Experimental and Computed Load—Deflection Curves for Beams B1 and B4

vertical projection of this segment provides the value of dead load or the moment due to dead load.

Observing the experimental curves, one may notice the triangular mark placed on each curve. This mark indicates the point of maximum loading, or the load-carrying capacity of the composite beam. At this loading, it is assumed that either the concrete crushes or one or more of the connectors have reached their maximum strength and started reducing their capacity.

Before the triangular marks, all the computed curves agree well with the experimental curves. Generally, using the lower hyperbola as the load-slip curve for the computation produces slightly better results. Therefore the lower hyperbola is employed for any future quantitative studies.

To determine the load-carrying capacity of the composite beam, a strain value of 0.006 was assumed as the concrete crushing strain and 11.3 kips was assumed as the maximum connector strength. A letter 'A' was placed beside each load-deflection curve together with the symbol ' □ ' to mark the point on the curve where the concrete strain reaches its crushing value. A letter 'B' was placed beside the curve together with the symbol ' ◇ ' to mark the point where the first connector reaches its maximum load. The load-carrying capacity of the composite beam is determined by either the load producing the concrete crushing strain or the connector failure load, whichever is reached first.

Table (6.1) shows the load-carrying capacity determined by the computation and the experiments. For beam B1 and B4, the approximately full connection case, (95%), crushing of the concrete governs the

Beams	B1	B4	B3	B2
percentage connection	95	95	76	53
load-carrying capacity from the experiments (kips)	34.5	35	31.75	27
concrete crushing load (kips)	34.75	34.75	33.75	*
connector failure load (kips)	35.75	35.75	33.05	28.5
computed load-carrying capacity (kips)	34.75	34.75	33.05	28.5
percentage difference	0.6	0.6	3.8	6.5
type of failure	concrete crushing	concrete crushing	connector failure	connector failure

(* the computation was terminated at the load of 31.25 kips which gives a concrete strain 0.0047)

Table (6.1) The Computed and Experimental Load-Carrying Capacity of the Experimental Beams

load-carrying capacity of the composite beam. For beams B3 and B2, failure of a connector governs the load-carrying capacity, as fewer connectors were used. The results in table(6.1) show that the computed load-carrying capacity is close to that from the experiments, especially for the beams with high percentage of connection.

6.2 The Load Versus Concrete Fibre Strain Curves (Figs.(6.7) to (6.10))

The agreement between the theoretical computed curves and the experimental ones is satisfactory. Some small disagreement is found. Over the range considered, experimental curves for beams B1, B4 and B3 are higher than the computed curves at the high loading stages. This can be interpreted by noticing that the yield stress used in the computation is 3 ksi, a design value, which is lower than the actual concrete strength. For beam B2 the agreement is good at high loadings, because the beam has lowest percentage of connection, and excessively large deflection appears before the occurence of large concrete strain. The strains computed have not reached the portion where the experimental curve is considerably higher than the curve used for the computation. This can be understood when referring to fig.(6.11).

Another diagreement is found in that the portion of the load versus top concrete fibre strain curves, corresponding to intermediate loading stages, the experimental curve provide larger values of strains than the computed curves. This also can be attributed to the difference of the assumed and actual concrete stress-strain curve shown in fig. (6.11). For the same stress, the actual stress-strain curve requires higher strain than the assumed curve at intermediate loading stages.

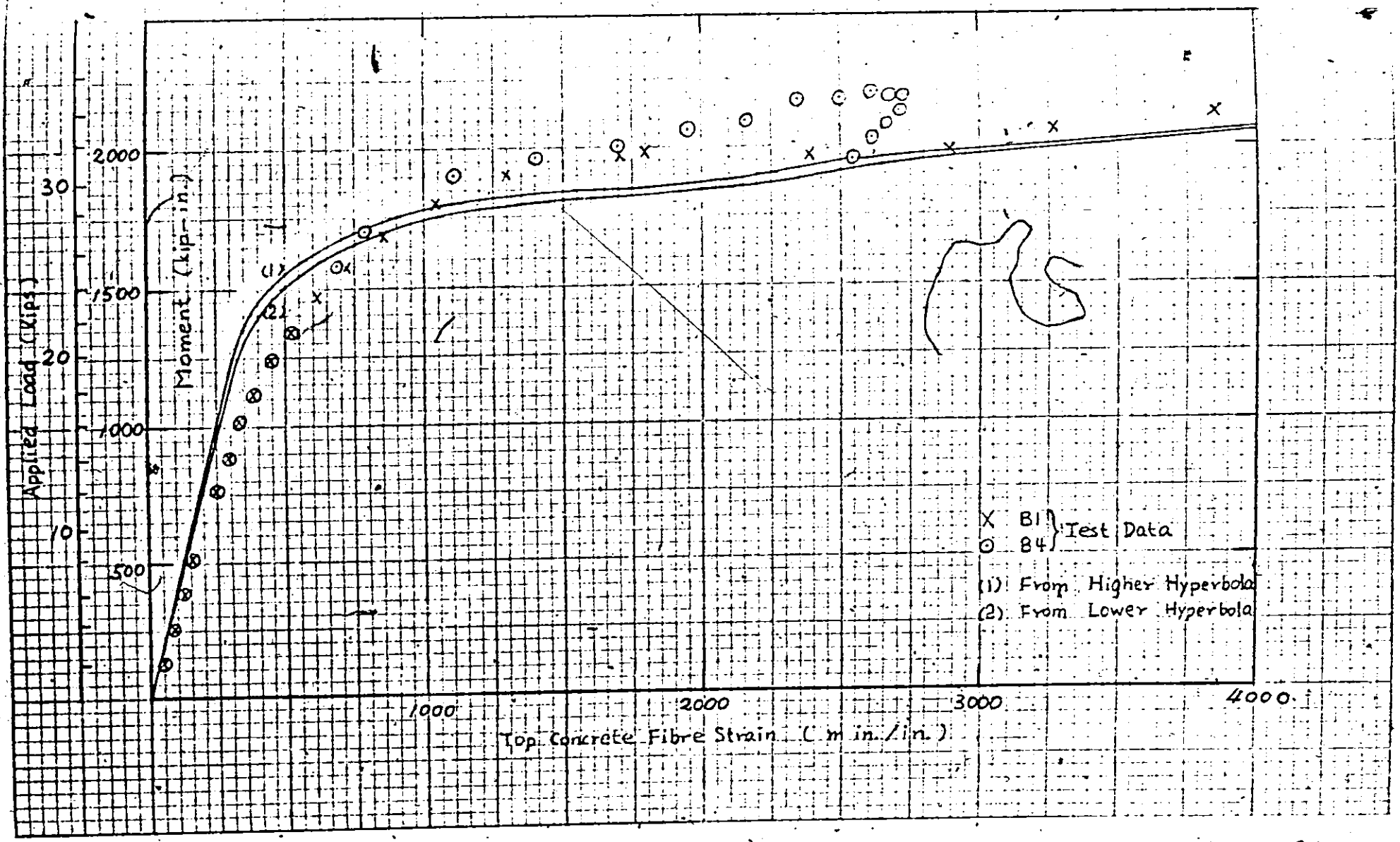


Fig. (6.7) Experimental and Computed Load—Top Concrete Fibre Strain Curves for Beams B1 and B4

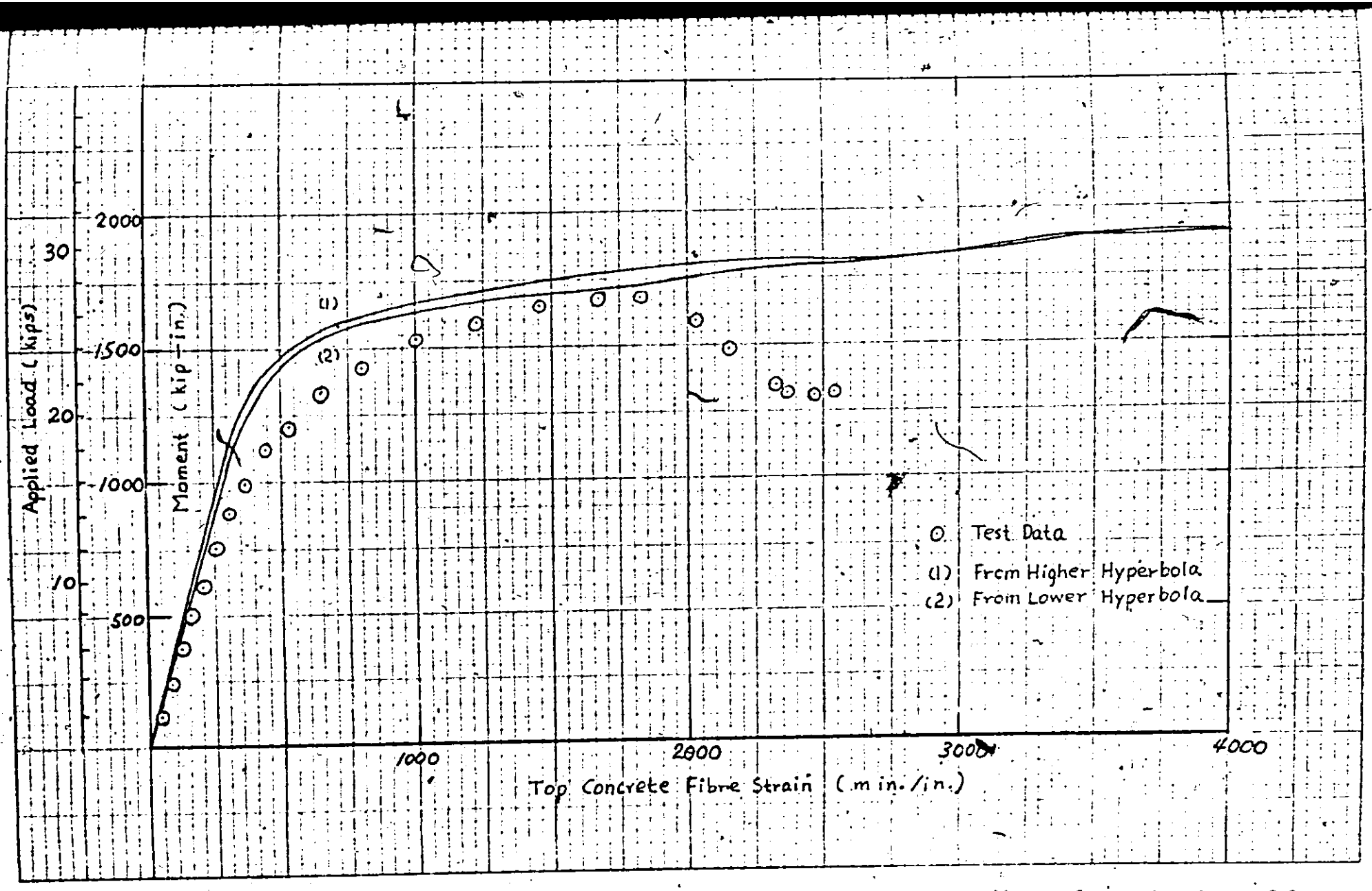


Fig. (6.8). Experimental and Computed Load — Top Concrete Fibre Strain Curves for Beam B 2

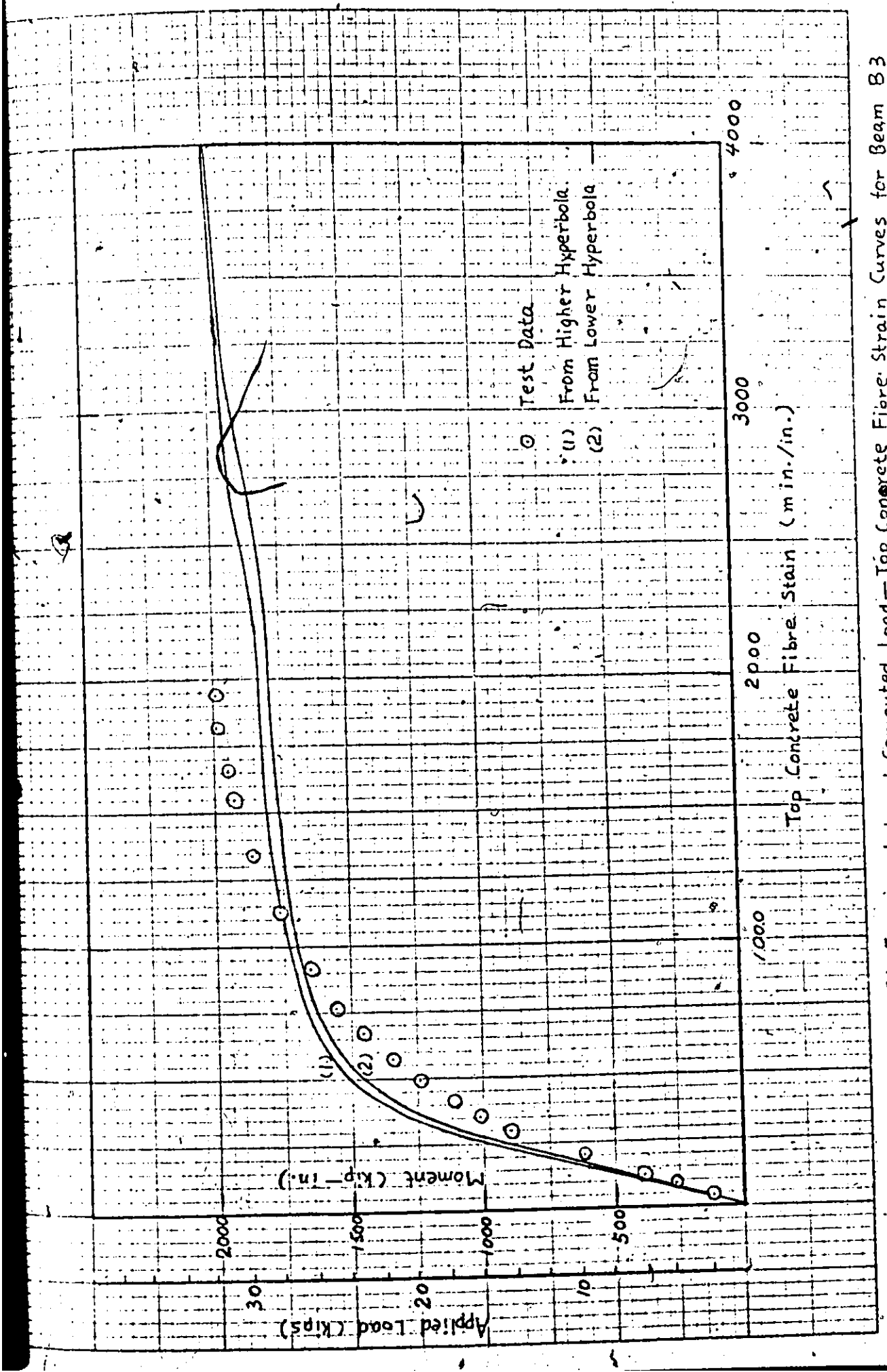


Fig. (6.9) Experimental and Computed Load—Top Concrete Fibre Strain Curves for Beam B3

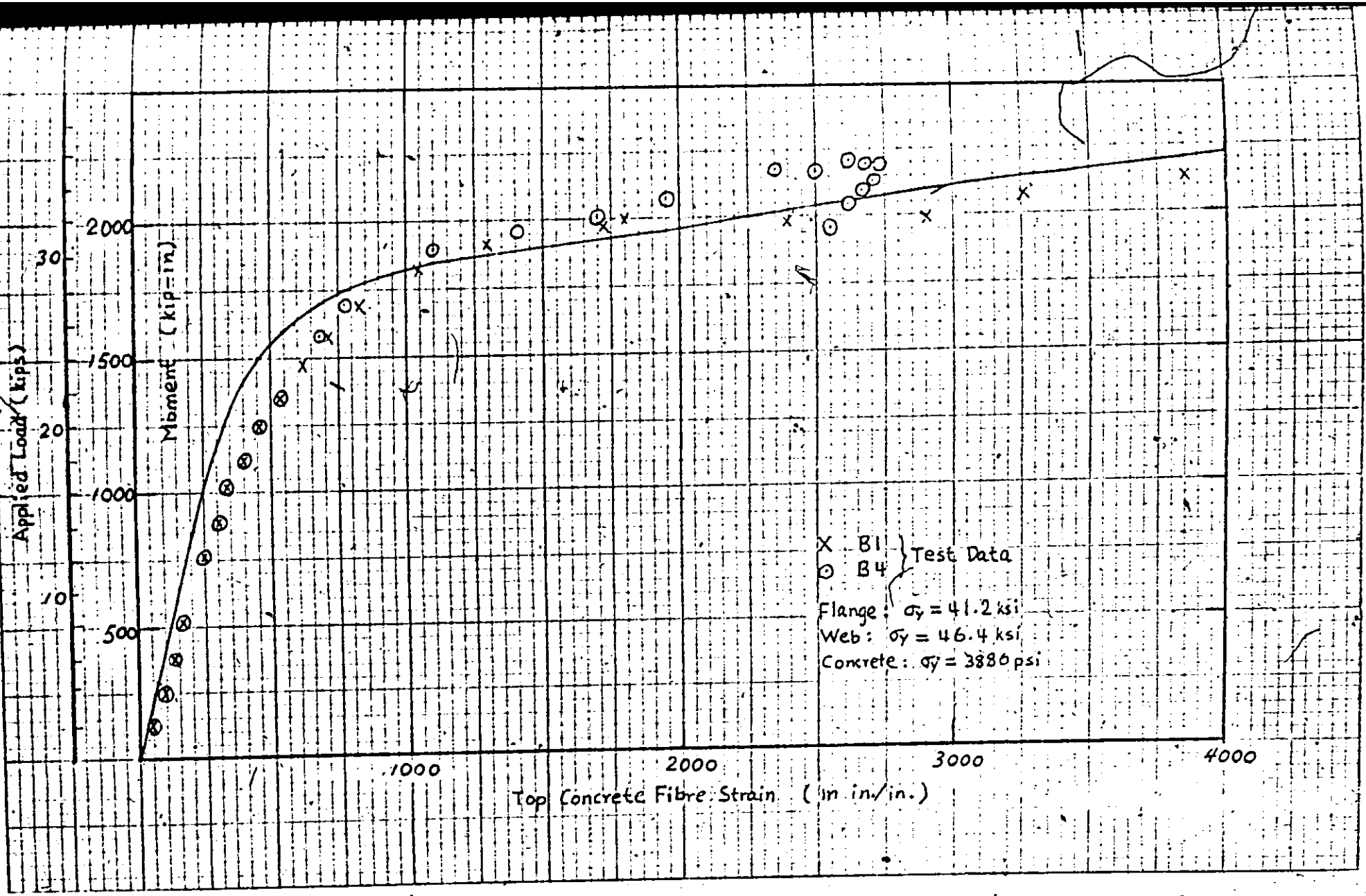
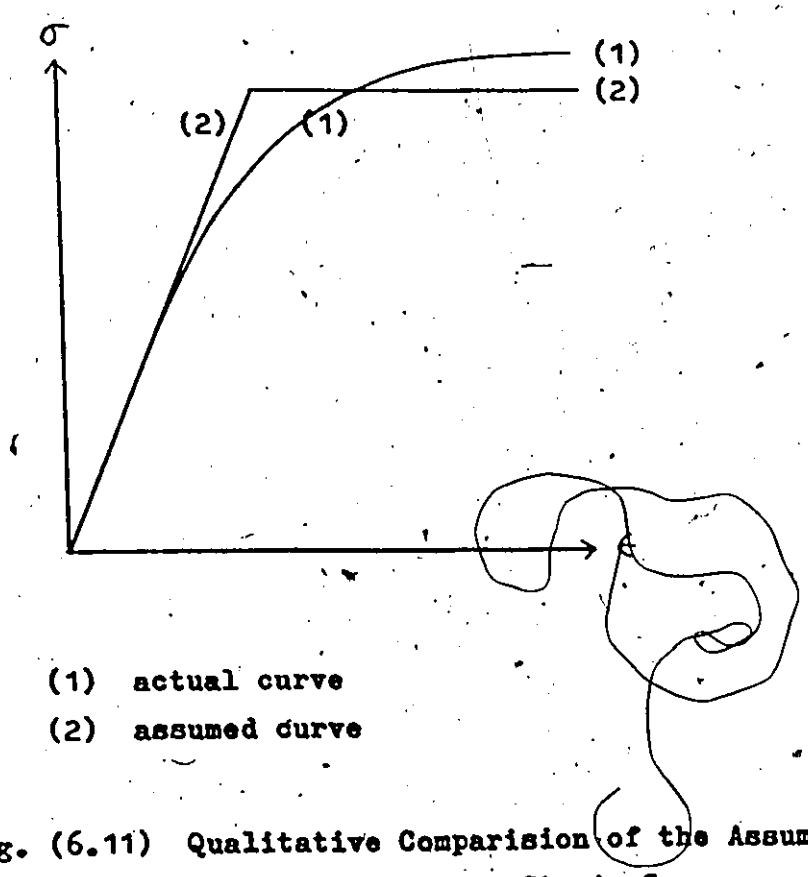


Fig. (6.10) Experimental and Computed Load – Top Concrete Fibre Strain Curves for Beams B1 and B4



(1) actual curve
(2) assumed curve

Fig. (6.11) Qualitative Comparision of the Assumed and Actual Concrete Stress-Strain Curve

Because the loading is directly related to the force of interaction, which is directly related to the stress, for the same loading, the experimental curves show higher strain than the computed curves.

6.3 The Load Versus End Slip Curves (Figs.(6.12) to (6.15)

Apparently the agreement is poor between the computed and tested curves at low loadings. The agreement is good at intermediate loading stage, and slightly worse at high loading stages.

The initial part of the experimental curves show smaller slips than those shown by the computation. Because the shear connection provided by the bond was not taken into account by the computation, the value of connector force given by the load versus slip curve used in the computation is lower than that from the actual load-slip relationship. Hence, the computed slips are larger than the measured ones. When the loading was sufficiently large, the bond was destroyed. The experimental load versus slip curves reflect this fact by showing a relatively high rate of change of curvature and then agree closely with the computed curves. At high loading stages, the experimental curves diverge from the computed curves showing higher values of end slip. This difference comes from the cracking of the slab, especially at the ribs, as stated in reference (3).

The curves computed by using the higher hyperbola give better agreement at low loading stages. In fact, the higher hyperbola has implicitly taken partially into account the contribution of shear force from the bond.

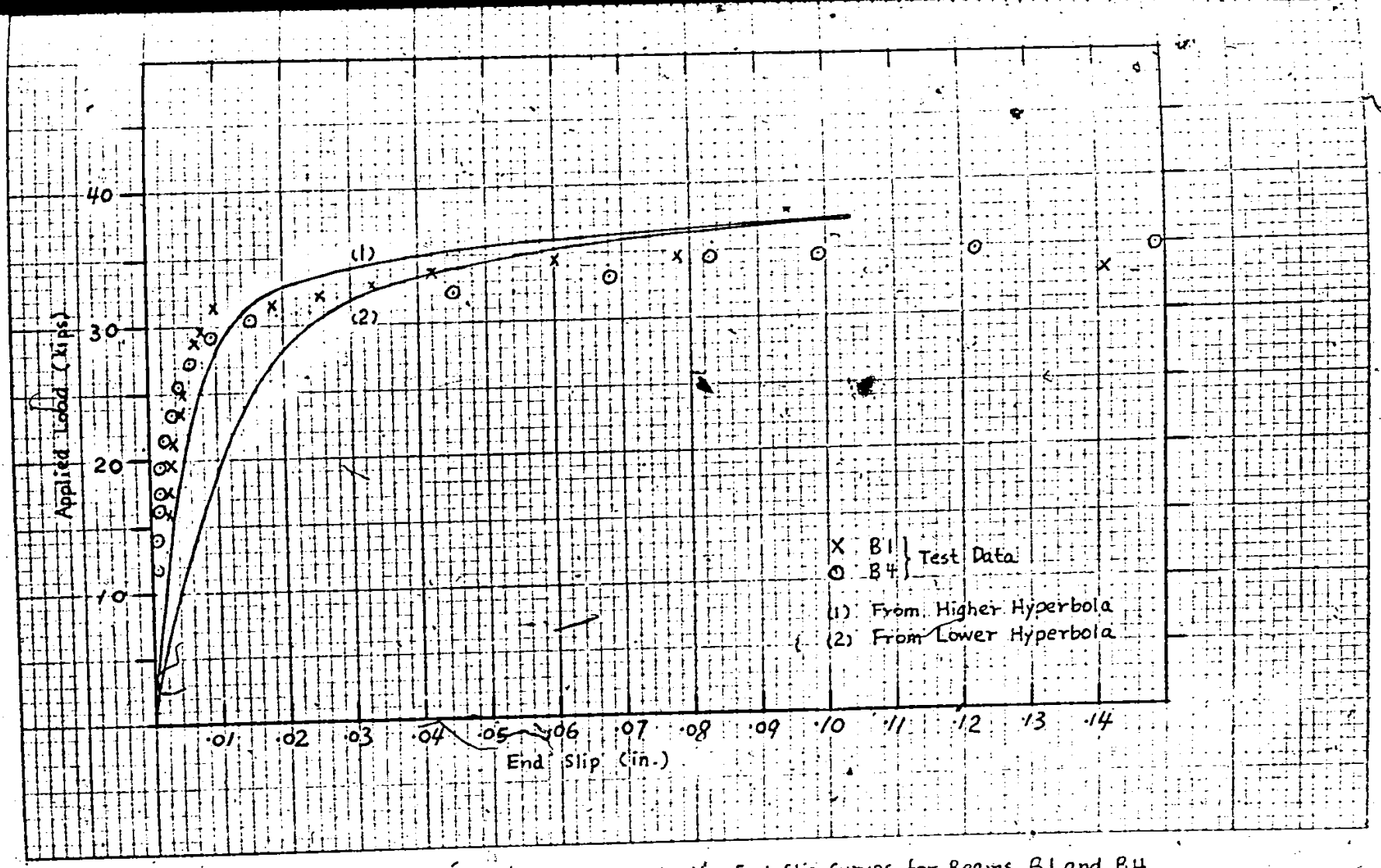


Fig. (6.12) Experimental and Computed Load—End Slip Curves for Beams B1 and B4

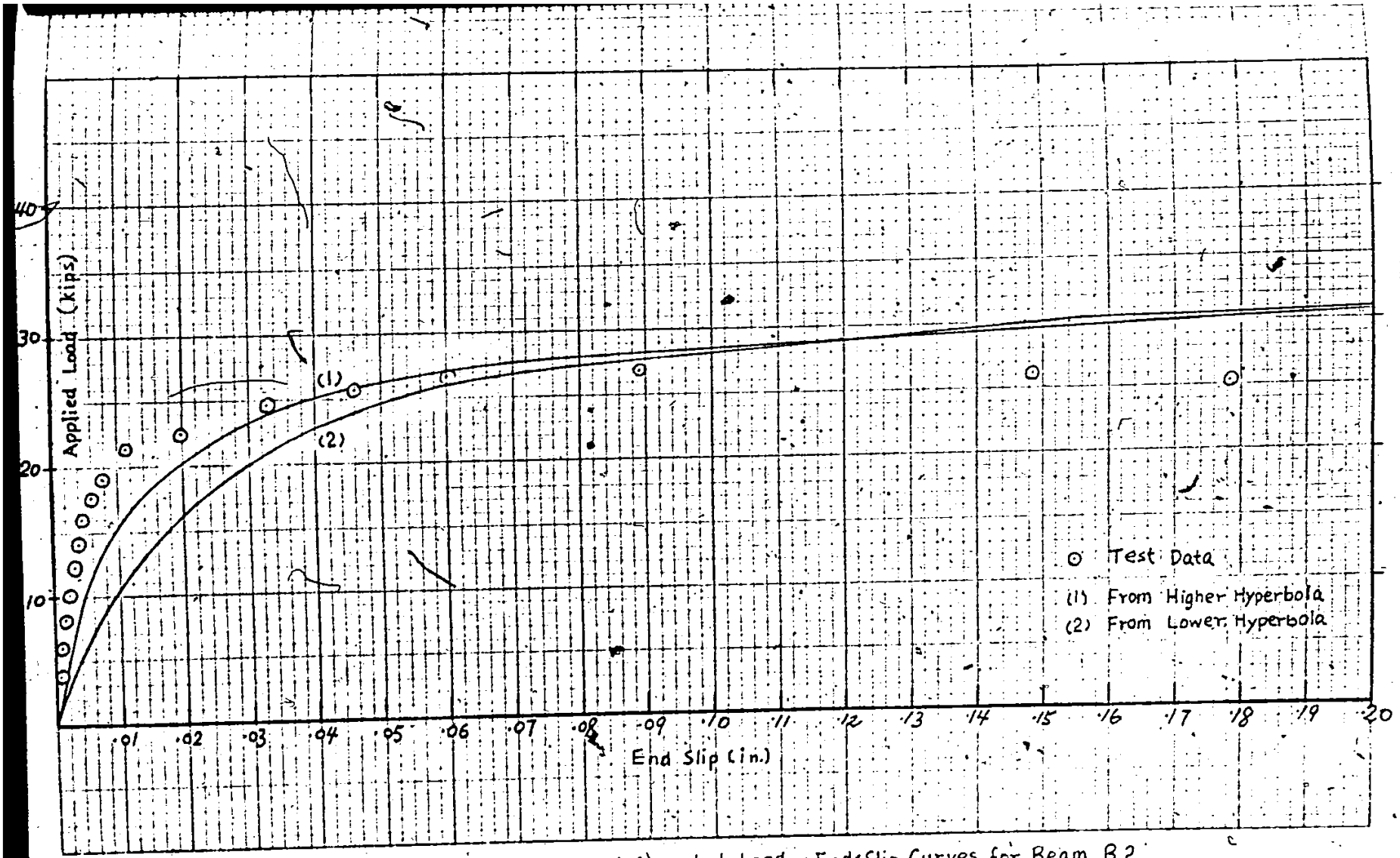


Fig. (6.13) Experimental and Computed Load—End-Slip Curves for Beam B2

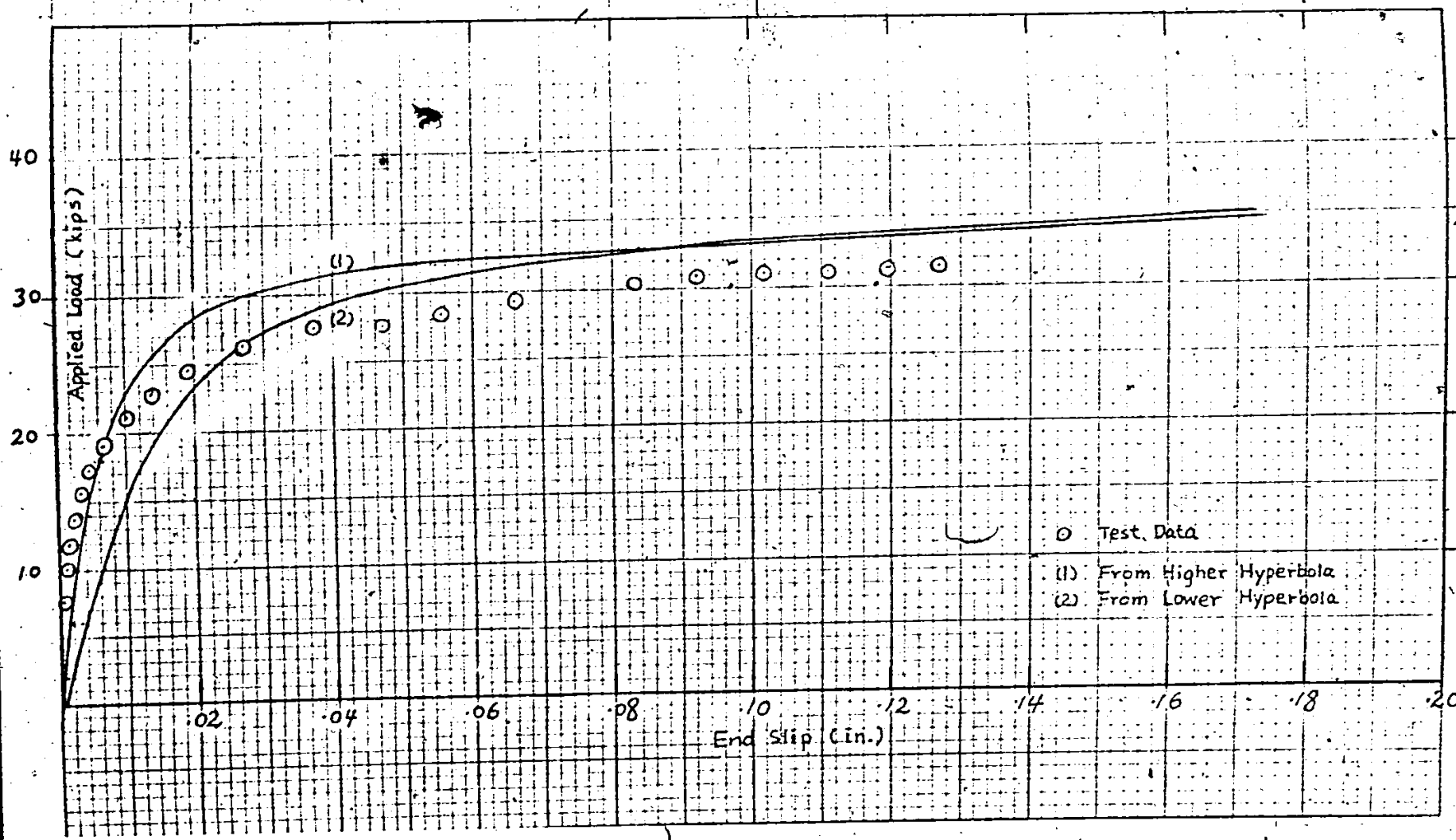


Fig. (6.14) Experimental and Computed Load—End Slip Curves for Beam B 3

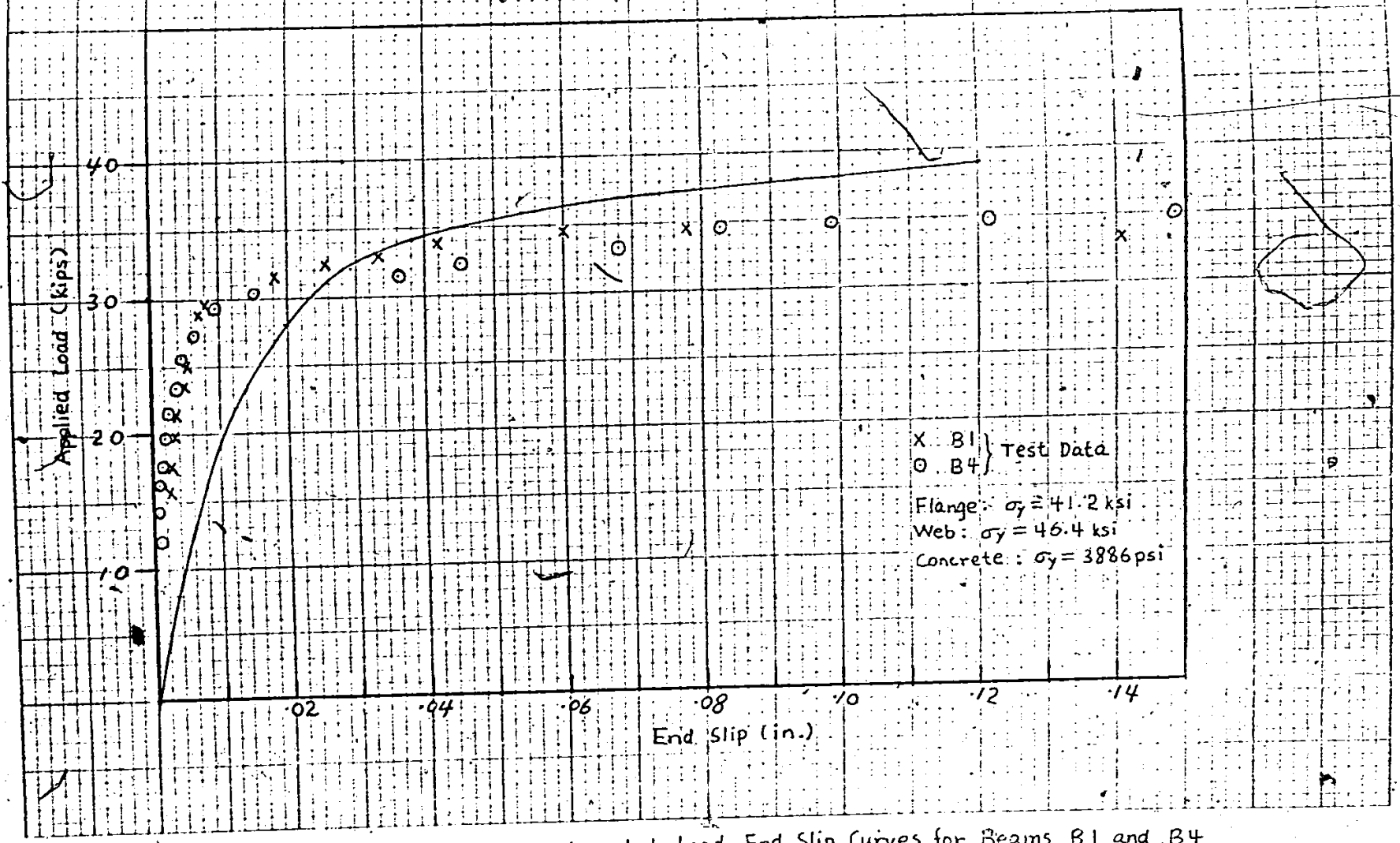


Fig. (6.15) Experimental and Computed Load—End Slip Curves for Beams B1 and B4

6.4 The Load Versus Bottom Steel Fibre Strain Curves (Figs.(6.16) to (6.19))

The agreement between the computed and experimental curves are not good for beams B4 and B1 at high loading, also for beams B2 and B3 at intermediate loading.

At the experiments, because the values of strains of beams B2 and B3 are excessively large at high loading, already out of the scale of the measuring instrument, only very few sets of the test values were obtained and plotted. If all sets of the strain values could be measured and plotted, it can be imagined that the experimental curves will agree well with the computed curves for B2 and B3. The reason that the computed strain curves of beams B1 and B4 are below the experimental ones can be explained by a similar argument as that made for the concrete strain curve. Referring to fig.(6.11), because the assumed concrete stress-strain curve is lower than the actual curve when strain value is high, the stiffness of the composite beam is underestimated and the computation leads to the curvatures being slightly larger than the actual curvatures. By drawing the strain and stress distribution diagram through a cross-section, one can judge that corresponding to the given force and moment a slight increase of the curvature will produce a relatively large increase of the bottom fibre steel strain. Further evidence is provided by the reduction of the difference between the two load versus strain curves shown in fig.(6.19), where the computation adopted a higher concrete yield stress value. Similar argument can be used to interprete the disagreement at intermediate loadings.

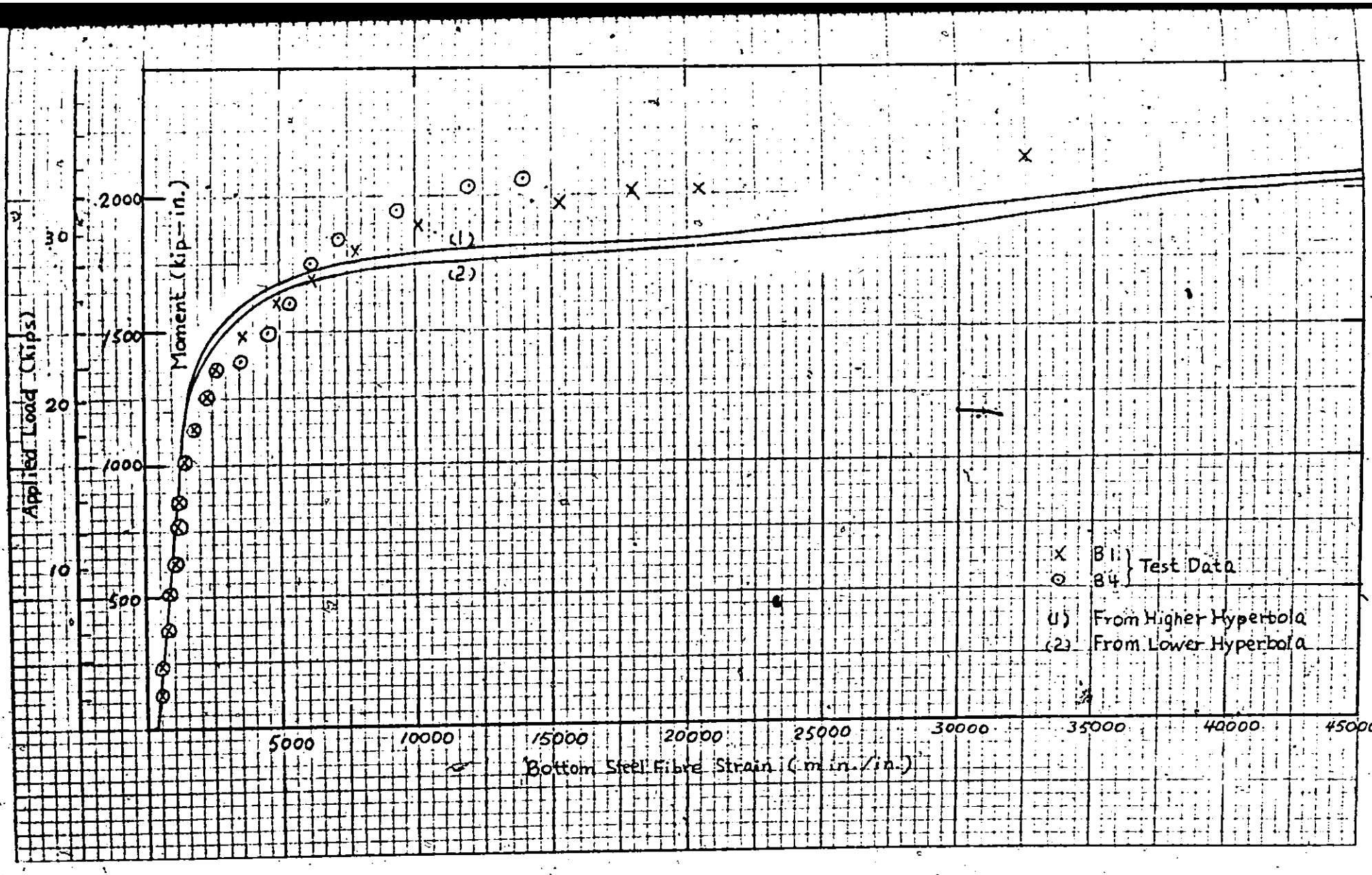


Fig. (6.16) Experimental and Computed Load—Bottom Steel Fibre Strain Curves for Beams B1 and B4

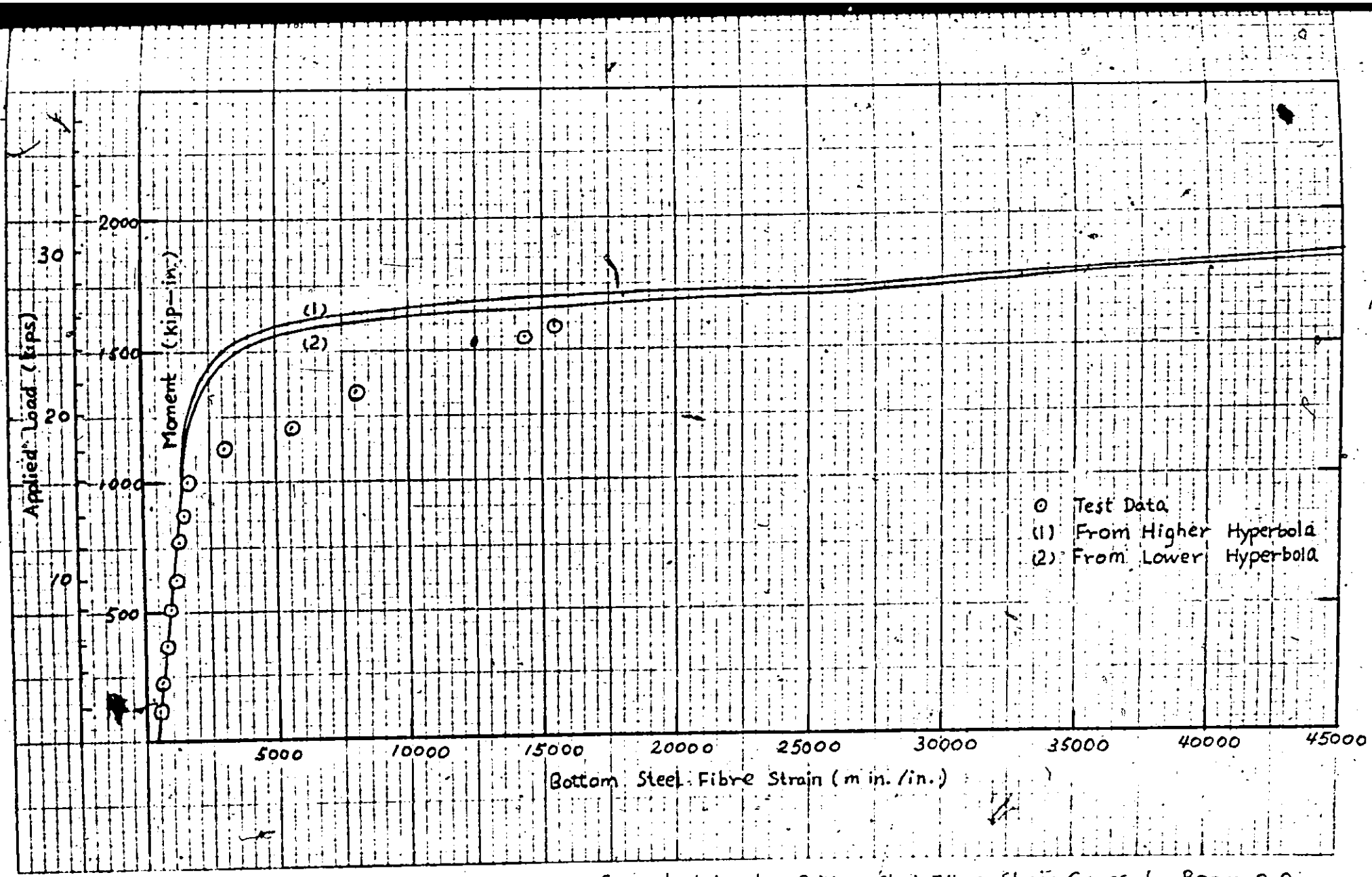


Fig. (6.17) Experimental and Computed Load—Bottom Steel Fibre Strain Curves for Beam B 2

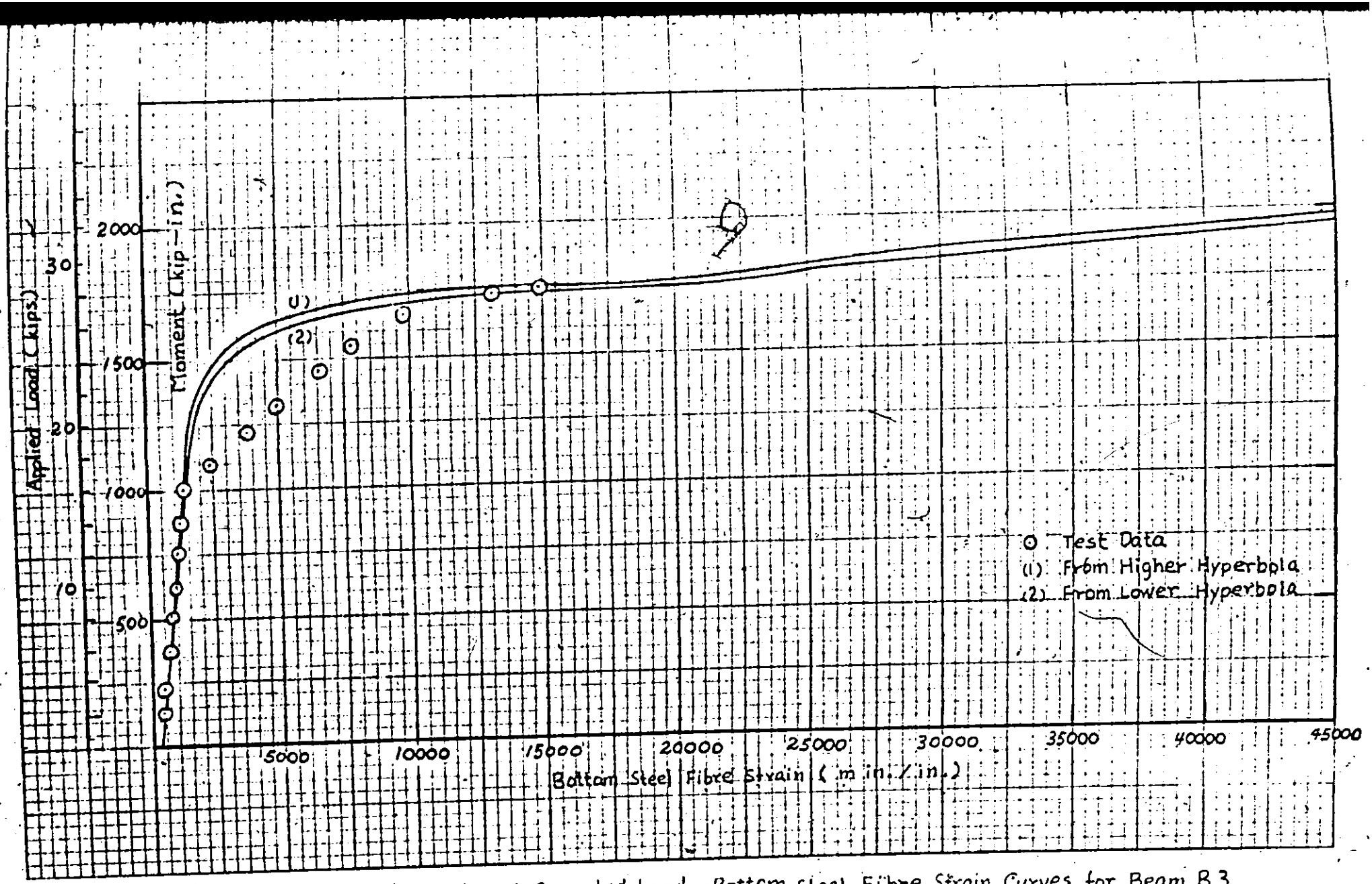


Fig. (6.18) Experimental and Computed Load—Bottom steel Fibre Strain Curves for Beam B3

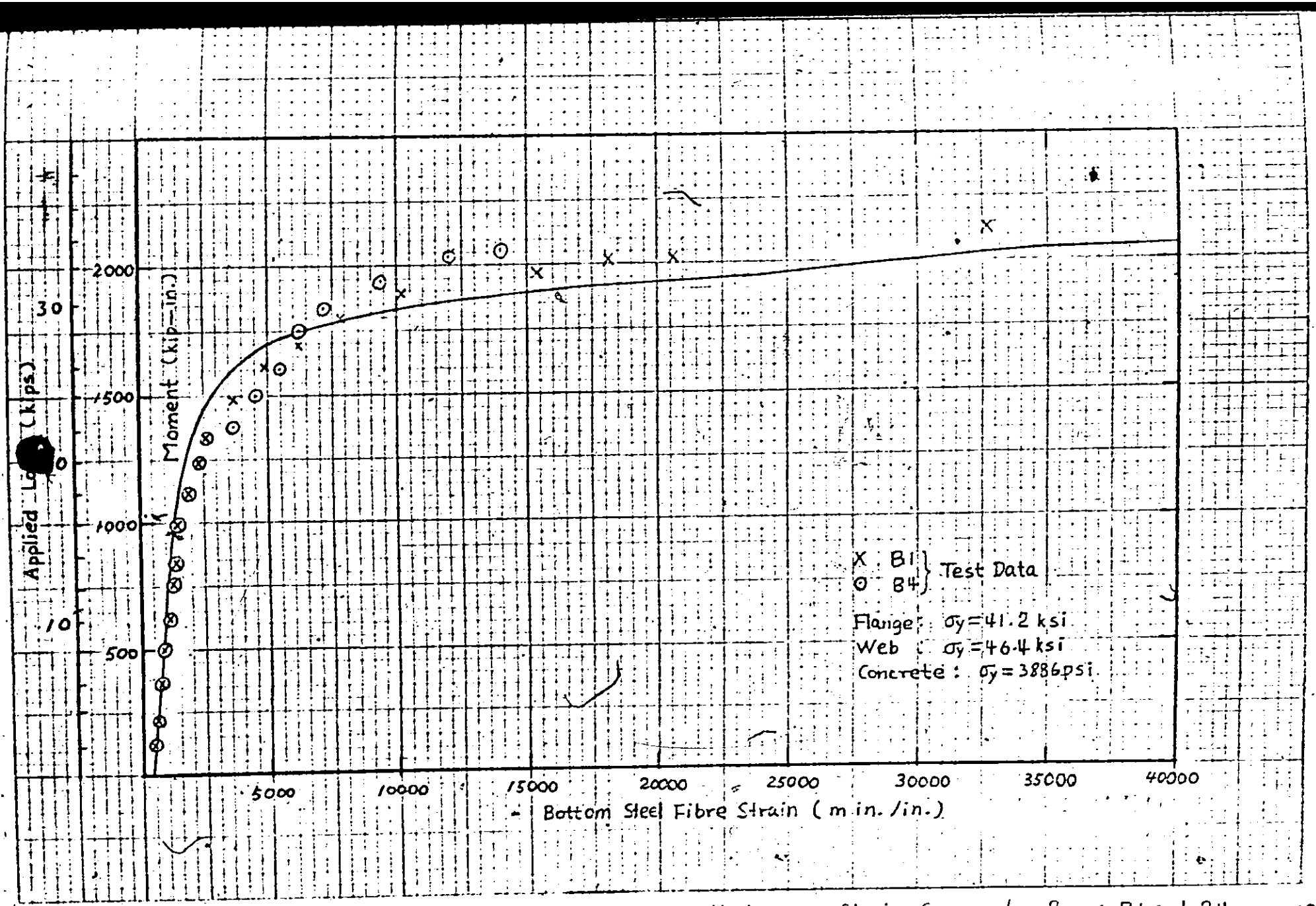


Fig. (6.19) Experimental and Computed Load—Bottom Steel Fibre Strain Curves for Beams B1 and B4

6.5 Summary

(A) By properly choosing the concrete crushing strain and the maximum connector strength, the computation provides a good prediction of the load-carrying capacity of the composite beam.

(B) Using a simplified concrete stress-strain curve does not affect the overall results very much. The noticeable difference produced by this simplification is the increase of the top concrete fibre strain and the bottom steel fibre strain at high loadings which may slightly reduce the concrete crushing load and the connector failure load and, hence, the load-carrying capacity.

For design purposes, the results are on the safe side.

If the reduction of the load-carrying capacity just mentioned above is to be eliminated, slightly higher values should be used for the concrete crushing strain and for the maximum connector strength.

(C) Through the comparision of the curves generated by using the different hyperbolas, a conclusion may be drawn.

The lower hyperbola generally gives better fitting of the experimental points, except the points representive the initial part of the load versus end slip relationship which has smaller values due to the contribution of bond. Referring to fig.(6.2), if an experimental curve is chosen for the load-slip curve and is to pass the two prescribed points, (0,0) and (.12,11.3), the results from the computation will be slightly worse than that previously shown. The exponential curve has a relatively low rate of change of curvature than a hyperbola between its initial portion and its middle portion, a portion

corresponding to the slips between the origin and the maximum slip. If the middle portion is properly chosen to give good fitting, the initial portion will be too low, lower than that of the lower hyperbola, and not suitable for fitting. If the initial portion is chosen appropriately, the middle portion will be too high to be used.

CHAPTER 7

DETERMINATION OF THE PRACTICAL LIMIT TO THE DEGREE OF CONNECTION FOR UNIFORMLY LOADED BEAMS

7.1 General

In order to determine the lower limit to the percentage of connection appropriate to practical design, a group of composite beams with various percentage of connection is taken as examples for the computation.

The composite beam with full connection designed for an interior bay of an office building is taken from reference (7). By reducing the number of connectors, different partial connection cases are obtained. The following values are common for all the cases;

Values for the slab:

f'_c = 3000 psi

E_s = 3×10^6 psi

working stress of the concrete = 1350 psi

total thickness of the slab, t_t = 4"

effective width of the slab, b' =69" •

• The effective width of the slab is determined from the three inequalities: $b' \leq L/4$, $b' \leq$ the distance between the center of two adjacent floor beams, and $b' \leq 2 \times 8\ t_t$ + width of the flange of the steel beam, for this case , the third inequality governs.

Values for the steel beam and cover plate:

F_y = 44 ksi

E_b = 29×10^6 psi

working stress of the steel = 29 ksi

The steel beam used is a W14x22

width of the cover plate = 3"

thickness of the cover plate = 0.5"

Values for the composite beam:

modular ratio = 9

span length = 360"

D. L. = 400 lb/ft

All the beams were simply supported and uniformly loaded. The cover plate extends through the whole span. Single stud connectors, 3" long and 3/4" in diameter, were used to provide shear connection. Since the types of slab and the connectors are the same as that used in beams series B in chapter 6, the same analytical representation for the load-slip characteristics was employed for the computation. From this representation, the maximum connector strength was chosen to be 11.3 kips. Then 31 connectors are required for the full connection case. The proposed percentages of connection for study are: 100%, 80%, 60%, 50%, 40%, 30%, 20%, and 10%. Because the way of construction limits the connector spacing such that the spacing can only take the values 6" or a multiple of 6", the above proposed percentages cannot be precisely obtained. An alternative set of percentages chosen for the study are: 97%, 80.6%, 61.3%, 51.6%, 42%, 32.2%, 22.6% and 9.7%.

If after a uniform placement of connectors there are extra connectors, it is better to place them filling the vacancies close to the supports where shear force is larger. The layout of connectors for all the beams is shown in fig.(7.1).

The stress versus strain curve for both the steel and the concrete were taken from chapter 6.

7.2 The Load Versus Deflection Curves

The load versus deflection curves are shown in fig.(7.2). While computing, no definite limitation was imposed on the termination of the computation, except one deflection condition. Since the loading causing a deflection of 12" is already above the actual load-carrying capacity for these beams, the computation was terminated at this large deflection.

For uniformly loaded beams, the top concrete fibre strain develop their magnitudes much more slowly than that for the single point load case. Taking the 97% case as an example, at the termination of loading the top fibre concrete strain has a small magnitude of .003. Before this loading connector failure occurs. Because fewer connectors are used in the other beams, they have smaller magnitudes of the top fibre concrete strains at the maximum load. Therefore connector failure governs the load-carrying capacity for all the cases.

A symbol 'X' is used to mark each curve in fig.(7.2) to indicate the loading causing first connector failure.

7.3 Moment Versus Maximum Connector Force Curves and the Connector

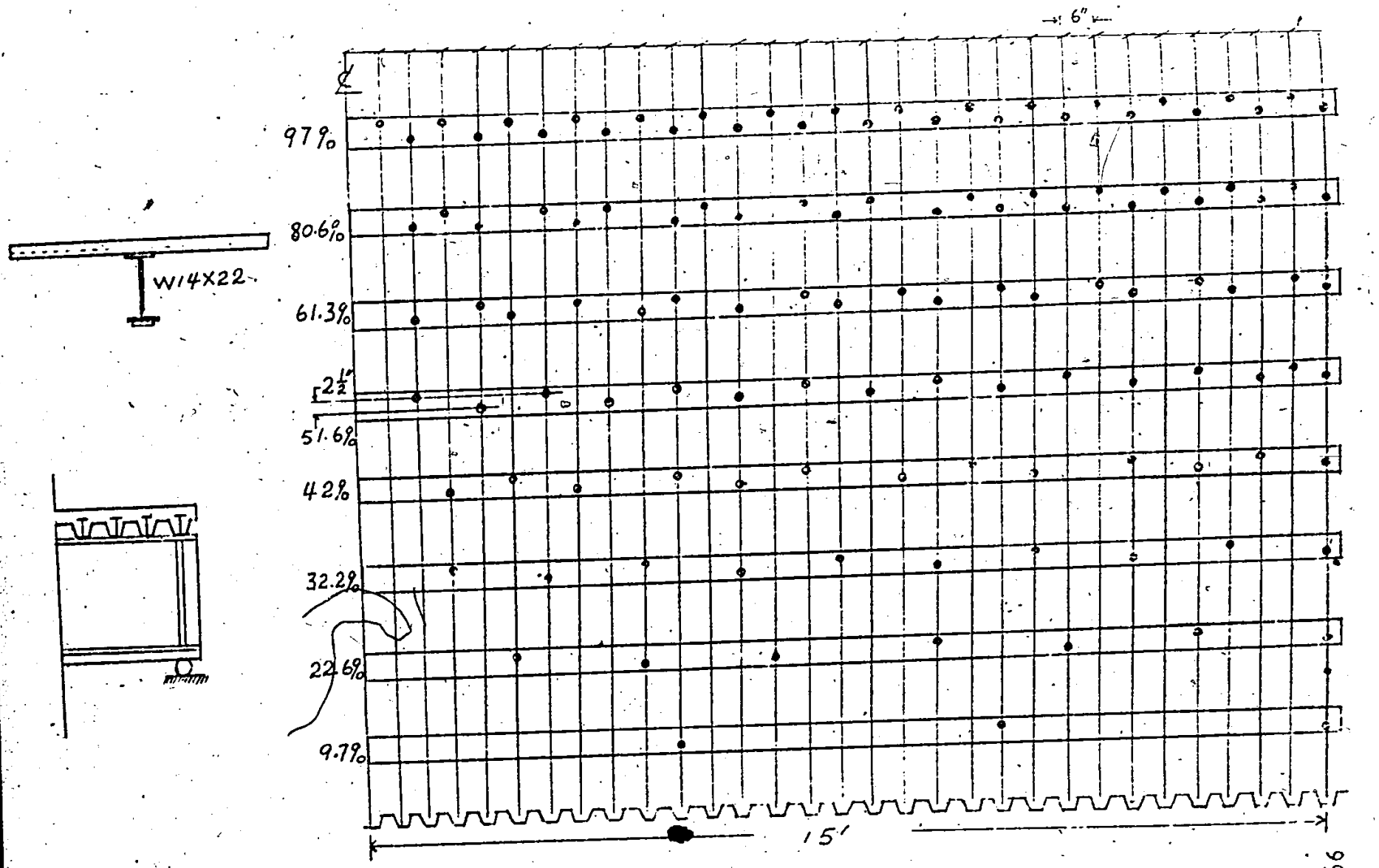


Fig. (7.1). Stud Connector Layout for Series of Beams with Various Percentage of Connection

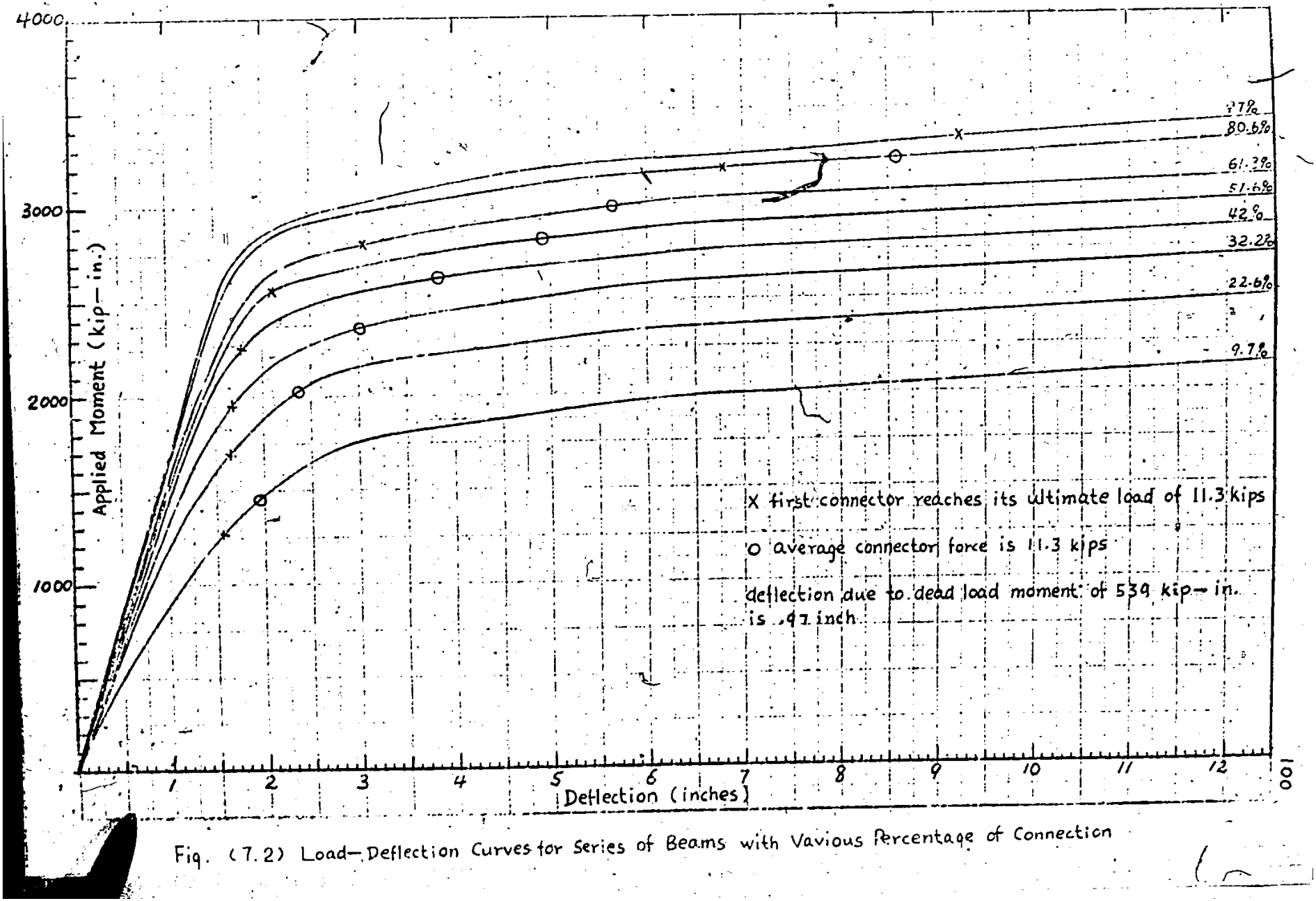


Fig. (7.2) Load—Deflection Curves for Series of Beams with Various Percentage of Connection

Force Distribution Curves (Figs.(7.3) and (7.4))

The variation of the connector forces in each beam with moment increase is shown in fig.(7.3). Corresponding to the same moment, the maximum connector force takes a larger value when percentage of connection is reduced. The increase in the maximum connector force from one beam to next beam having lower percentage of connection are approximately uniform except for the case from 97% connection to 80.6% connection. The beam with 80.6% connection has a relatively small connector force, slightly larger than the beam with 97% connection, for low and intermediate loadings. The reason for this can be found by observing the connector force distribution curve shown in fig.(7.4).

In fig.(7.4), the connector force distributions were plotted for the 97%, 80.6% and 51.6% beams at loadings close to the working loads and the ultimate loads for each beam respectively. Here the term 'ultimate load' means the load at which the first connector reaches its ultimate load of 11.3 kips. Curve (1) was plotted at 100% of the ultimate load and curve (4) was plotted at 98% of the working load for the beam with 97% connection. Curve (2) and curve (5) were plotted corresponding to 100% and 98.5% of the respective ultimate load and working load for the beam with 80.6% connection. Curve (3) and curve (6) are for the beam with 51.6% connection corresponding to 98% of its ultimate load and 99.5% of its working load respectively. The comparision of the magnitude of force at a connector from each curve does not have much meaning, since the curves were plotted at different loadings. It is more significant to compare the connector distributions among all the beams. The connector distributions are relatively more uniform for

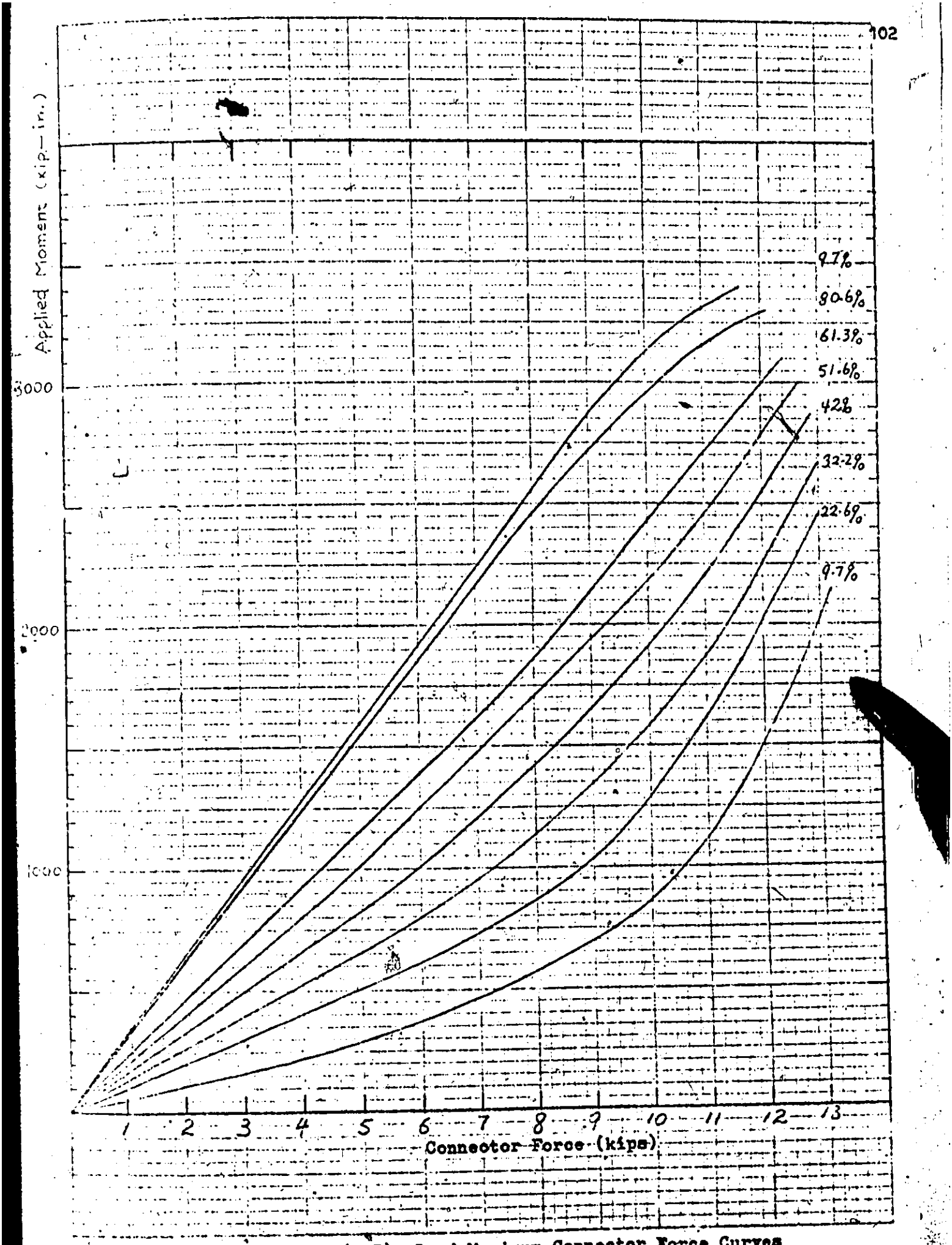


Fig.(7.3) Load-Maximum Connector Force Curves

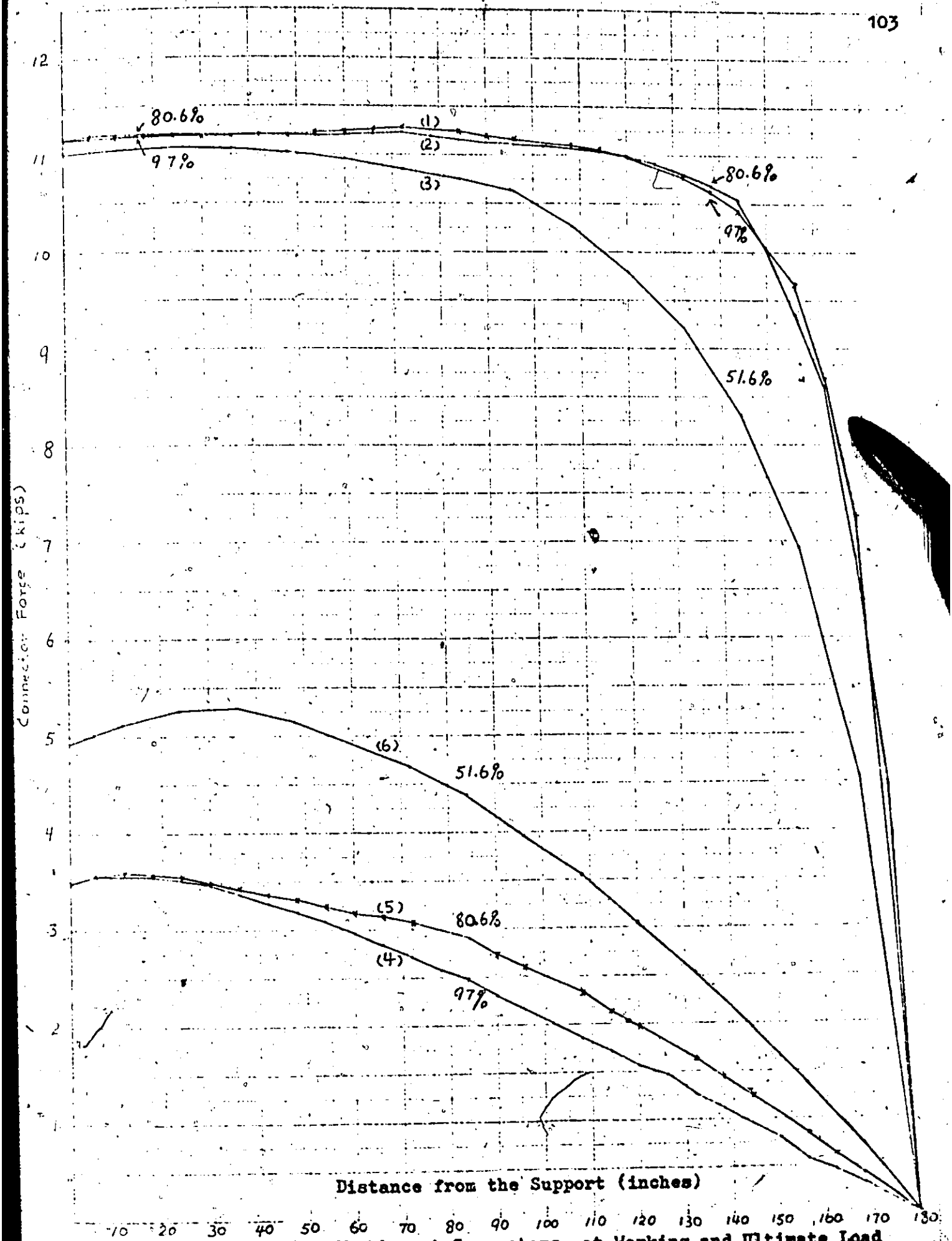


Fig.(7.4) Forces Distribution at Connectors, at Working and Ultimate Load

the beam with 80.6% connection. In other words, the force of interaction of the composite beam is relatively equally shared by the connectors.

7.4 Working Loads and Ultimate Loads

The working load moments and the ultimate load moments are tabulated in table (7.1). For the partial connection cases, the working load moments are found by reducing the effective width of the slab by multiplying by a factor, $\frac{n'}{n_o}$, with n_o being the number of connectors required for the 100% case and n' being the number of connector used in the partial connection case.

The working load is taken from the smaller of the following two loads; the load causing the steel working stress to be 29 ksi at the bottom steel fibre and the load causing the concrete working stress to be 1350 psi at the top concrete fibre.

Three types of ultimate moments are listed in table (7.1). The most significant one is the moment M_B, which causes the first connector to reach its ultimate load of 11.3 kips. This moment was taken as the load-carrying capacity of the composite beam. For comparision, another two moments, M_A and M_C are also listed in the table. M_A is the moment computed from the stress block and M_B is the moment at which the average of the connector forces is 11.3 kips. These two moments are for reference only, and not to be used to represent the load-carrying capacity of the composite beam.

Referring to fig.(7.5), it can be seen that when the percentage of connection is high, around 90%, M_C has a higher value than M_A. To explain this unexpected result, first examine the way of computing the

M_A — ultimate moment computed from stress block

M_B — moment at which first connector reaches 11.3 kips

M_C — moment at which average connector force is 11.3 kips

Q_{max} — the maximum connector force at working load

percentage connection	9.7	22.6	32.2	42	51.6	61.3	80.6	97
no. of connectors	3	7	10	13	16	19	25	30
M_A (k-in.)	2329	2749	3050	3300	3469	3580	3805	3918
M_B (k-in.)	1812	2225.5	2496	2803.4	3112.8	3348.3	3723	3889
M_C (k-in.)	1997	2572.5	2900	3164	3370	3538	3795	3960
working load moment (k-in.)	984	1412	1580	1602	1622	1631	1658	1670
Q_{max} (kips)	6.775	7.7	7.469	6.239	5.28	4.779	3.659	3.629
flexural load factor from M_A	2.37	1.95	1.93	2.06	2.14	2.195	2.29	2.345
flexural load factor from M_B	1.84	1.578	1.579	1.75	1.92	2.06	2.25	2.33
flexural load factor from M_C	2.03	1.82	1.84	1.98	2.08	2.17	2.29	2.37
connector load factor	1.67	1.47	1.51	1.81	2.14	2.36	3.09	3.12
stress which governs the working load	concrete		steel	steel	steel	steel	steel	steel

Table (7.1) The Value Of Ultimate Moments, Working Load Moment, and Load Factors for Each Beam

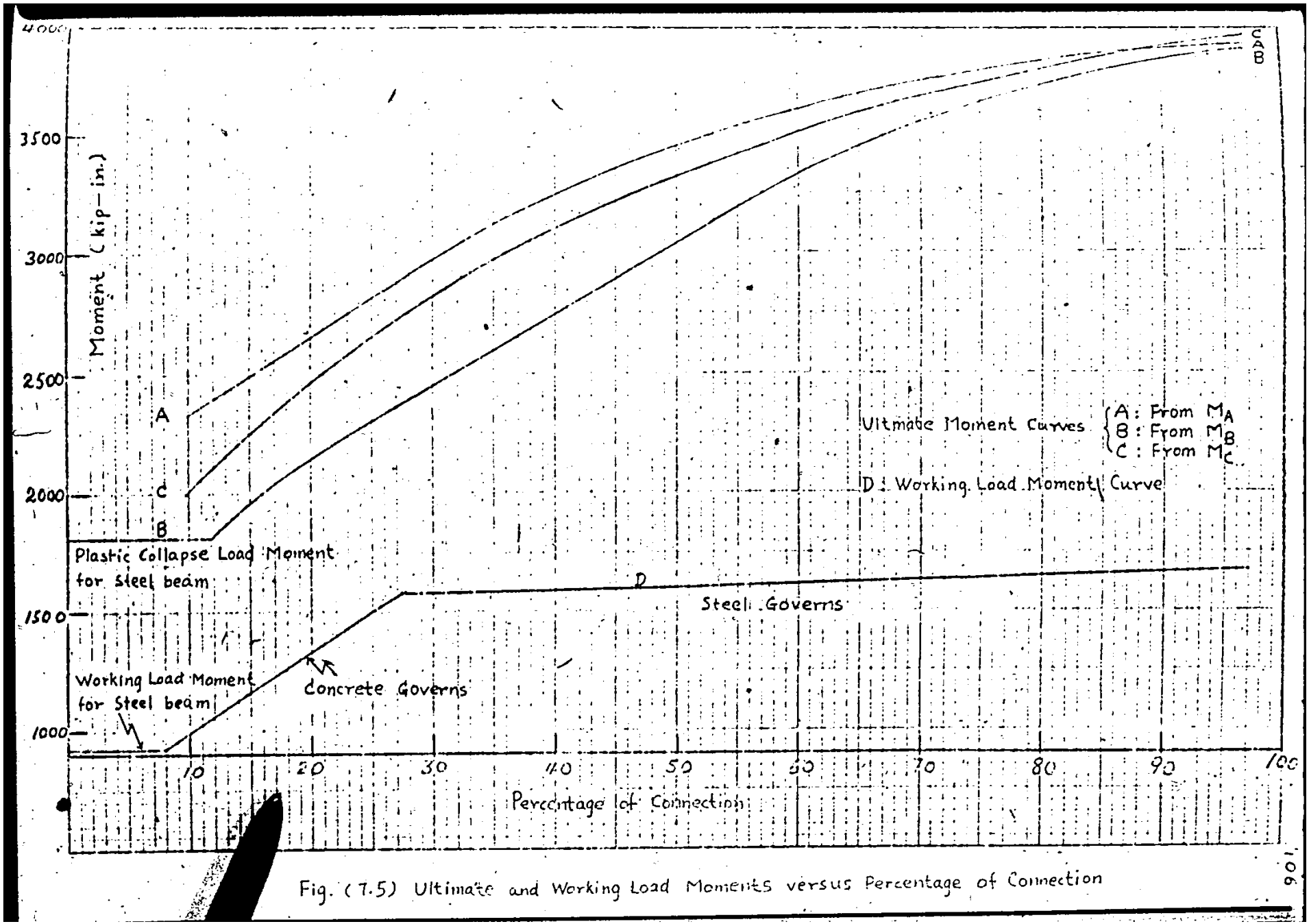


Fig. (7.5) Ultimate and Working Load Moments versus Percentage of Connection

maximum load by using the stress block. A factor 0.85 is multiplied by f'_c to create a rectangular stress block approximately replacing the original curvilinear stress distribution at the cross-section of the slab. Recalling that for the concrete stress-strain curve used in all the computations in this project, f'_c was directly taken as the maximum concrete stress without being multiplied by any factor. Therefore the concrete force in the slab computed in the latter case is larger than that in the former case. This leads to an M_C slightly greater than M_A. This occurs only when the percentage of connection of a beam is sufficiently high, because only for the beam with high percentage connection, at the loading corresponding to M_C, the stress distribution can be very close to that assumed in the stress block computation.

7.5 Load Factors

Two load factors are to be considered, the connector load factor and the flexural load factor. The connector load factor is defined as:

$$\text{connector load factor} = \frac{\text{ultimate strength of connector}}{\text{the maximum connector force at the working load}} \quad \ldots\ldots\ldots\ldots\ldots \quad (7\text{-}1)$$

where the ultimate strength of a connector is 11.3 kips. The flexural load factor is defined as:

$$\text{Flexural load factor} = \frac{\text{ultimate moment}}{\text{working load moment}} \quad \ldots\ldots\ldots \quad (7\text{-}2)$$

When one of the three types of ultimate moments, M_A, M_B and M_C, is

substituted in eq.(7-2), a flexural load factor is obtained. Taking M_B as the ultimate moment at which the first connector reaches its ultimate capacity and as the governing moment capacity, the most significant flexural load factor can be expressed as:

$$\text{Flexural load factor} = \frac{\text{connector failure moment}}{\text{working load moment}} = \frac{M_B}{\text{working load moment}}$$

All the load factors are also tabulated in table (7.1).

Fig. (7.6) shows a connector load factor curve and three flexural load factor curves derived from M_A, M_B and M_C. A common feathure exists in each curve: there is a rise at the portion close to the left end of each curve. This unexpected effect can be explained by referring to fig.(7.5). Observing the working load curve in fig.(7.5), the descending portion close to the left end of the curve corresponds to the low percentage of connection cases. This portion is computed by taking the load which first produces the concrete working stress at the top fibre of slab. Hence, the 9.7% connection case has a very low working load. This makes all the load factors related to this case becaome large and raises up the left end of the curves.

Considering the lowest degree of connection for design purposes and observing the variation of the load factor curves in fig. (7.6), it is found that 39% is the percentage of connection above which, no matter which curve is used, the flexural load factor is always large than 1.7 while 47.5% is the percentage above which the connector load factor is larger than 2. 1.7 and 2 are values generally suggested as

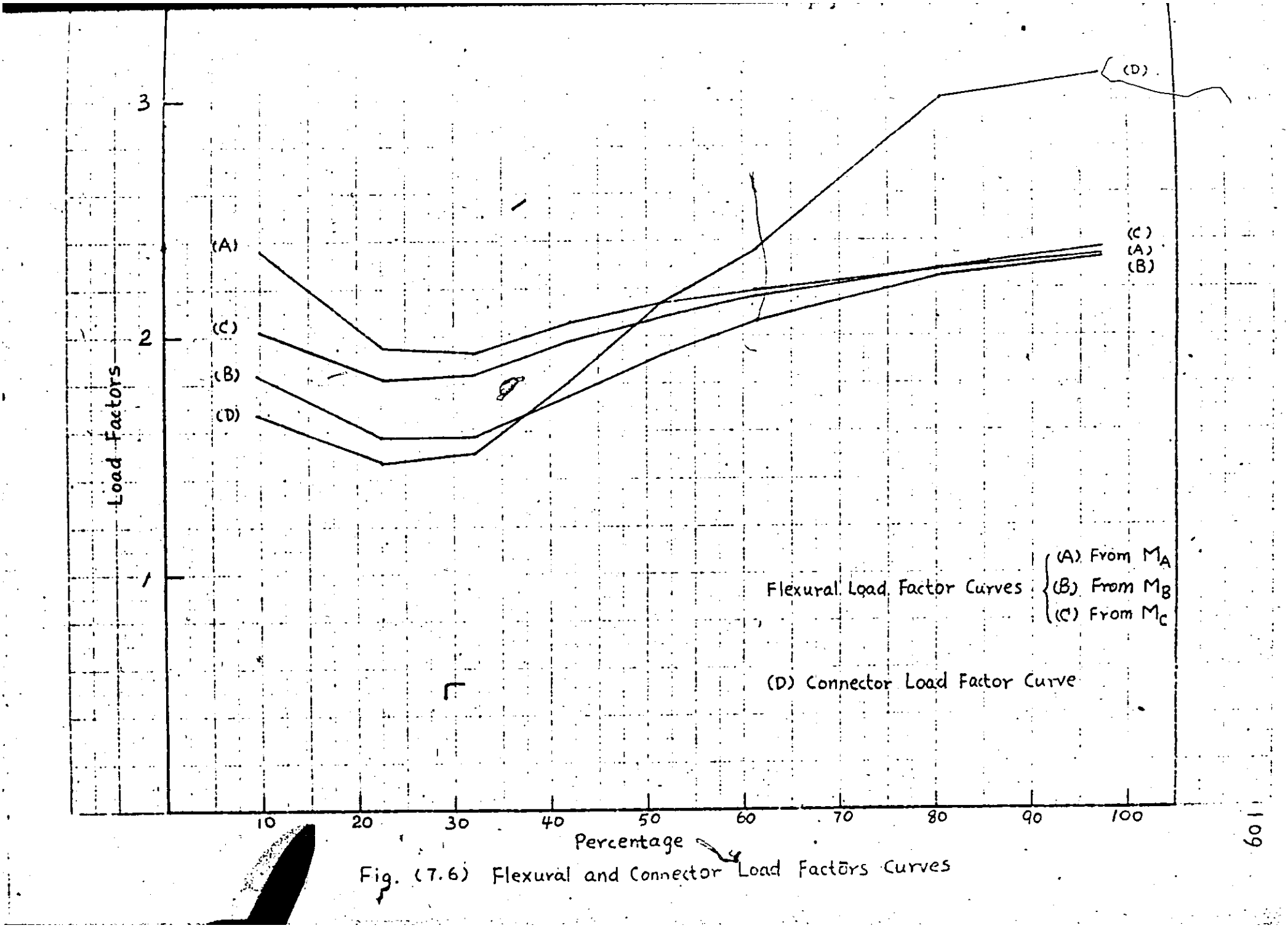


Fig. (7.6) Flexural and Connector Load Factors Curves

the respective flexural and connector load factors used for design. Therefore to meet both requirements, 47.5%, or approximately 50% should be taken as the lowest percentage allowed for partial connection in the design of single span composite beams. This agrees with the specification of the Canadian code (8) which sets 50% to be the minimum percentage of connection for loading.

However, the American Specification (9), clause 1.11.2.2 permits the use of partial connection ranging from 100 to zero percent by means of a formula which reduces the effective section modulus of a composite beam.

CHAPTER 8

SUMMARY AND CONCLUSION

8.1 Summary

A method for the analysis of a single span composite beam with discrete shear connections has been developed. The two basic assumptions on which the analysis is based are that the slab and beam deflect equally at all points along the span and the strain distribution in the slab and beam is linear.

The project consists of both the elastic and inelastic analysis of the composite beam. In all the inelastic cases, the shear connector, the concrete slab and the steel beam, are included in the consideration. Though the simplified stress-strain curve shown in fig.(2.3) was used in the computations in this project, the analysis is quite flexible, capable of employing various kinds of the stress-strain curves for the steel and concrete. For the connector load-slip characteristics, a smooth curve is adopted.

In spite of the nature of the analysis being essentially trial and error, by employing various numerical techniques and using the computer, the analysis leads to an efficient method which rapidly provides the solution for various quantities of general interest in the study of composite beam, such as strain distributions, connector forces and deflections.

The original objective of this project was to form a computer program based on the research result presented by Thiruvengadam (4). In this method, a set of simultaneous equations written for each panel was formulated and solved. Upon solving the equations, the solution obtained can be used to compute the quantities necessary for study, as mentioned in the proceding paragraph. However when his method was applied to the computation of some composite beams, very often, the computation could not proceed at high loadings because of numerical difficulties encountered. These difficulties were found to be directly related to the iterative technique used in the computation of strain and curvature of the beam. In order to overcome these difficulties, an entirely new approach was created and developed. This forms the method presented in this project. Starting with assuming trial values of end slip, by using the equilibrium and compatibility equations, a set of values for the connector slips and panel forces is derived, based on the assumed end slip. If the panel force at the other end of the composite beam is zero, the solution is obtained; otherwise the assumed value is revised. This method, not only eliminates the numerical difficulties of Thiruvengadam's method, but also reduces the number of cycles in the iterative procedure of computing the strain and curvature of the beam, and, hence, reduces the computer time.

The method and the computer program attached in the appendix was tested by comparing the computed results of some experimental beams with the test results. The computed results agree well with the experimental, even through a simplified form of the concrete stress- strain curve was used. Therefore the method and the computer program can be

successfully applied to analyze single span composite beams for the solution corresponding to the loading ranging from the very low stages to the ultimate loading stages.

Significant studies can be made by employing the computer program. One of them is the determination of the lower degree of connection which should be permitted for design, as done in chapter 7. In that chapter, a set of uniformly loaded composite beams with various percentages of connection was analyzed. Through the study of the variation of the flexural load factors and the connector load factors derived from the computational results, a practical limit to the degree of connection was proposed. Since the results are based on the study of a set of uniformly loaded simply supported composite beams with particular geometry and load-slip characteristics, they should not be taken as general values to guide the design. Nevertheless, it does demonstrate a method of approach. More representative values can undoubtedly be obtained through more extensive studies on various composite beams with various loading conditions and properties by using the same method.

All the composite beams computed in this project are simply supported with symmetric loadings. For other support conditions, or for non-symmetric loadings, or for both, the computer program requires only a slight modification.

The success of applying the numerical techniques in solving this problem is one of the many examples which indicate a possible way of solving relatively complicated nonlinear problems.

8.2 Conclusion

The study made in chapter 7 presents the value for the lower permissible limit to the degree of connection of practical beams. If the value 2 is taken as the connector load factor and the value 1.7 as the flexural load factor, the lower limit to the degree of connection should be approximately 50%.

If 1.7 is taken to be both the connector load factor and the flexural load factor, the lower permissible limit for partial connection will be approximately 40%.

APPENDIX

PROGRAM FOR THE ANALYSIS OF COMPOSITE BEAM

****

GENERAL DESCRIPTION

THIS PROGRAM IS DESIGNED TO COMPUTE THE SLIPS AND THE FORCES AT THE CONNECTORS, THE EXTREME FIBRE STRAINS IN THE SLAB AND THE STEEL BEAM, AND THE DEFLECTION AT THE MID-POINT OF THE PANELS IN THE COMPOSITE BEAM. THE COMPOSITE BEAM MUST BE SYMMETRICAL IN ITS GEOMETRY AND SUBJECTED TO A SYMMETRICAL LOADING ABOUT ITS MID-SPAN. THE STRESS-STRAIN RELATIONSHIPS FOR STEEL AND CONCRETE ARE ELASTIC -PLASTIC. STRAIN HARDENING OF THE STEEL CAN BE CONSIDERED. ALSO DIFFERENT VALUES OF YIELD STRESS CAN BE ASSIGNED TO THE WEB, THE FLANGE AND THE COVER PLATE.

THE PROGRAM IS DESIGNED FOR RECTANGULAR SLAB AND STEEL I-BEAM WITH OR WITHOUT COVER PLATE. FOR OTHER CROSS-SECTIONAL SHAPES AND OTHER STRESS-STRAIN RELATIONSHIPS, THE PROGRAM REQUIRES ONLY SLIGHT MODIFICATION.

THE PROGRAM IS WRITTEN IN FORTRAN IV LANGUAGE, AND TO BE RUN BY CDC 6400 SYSTEM.

LOADING

SYMMETRICAL LOADING MUST BE USED IN THIS PROGRAM. AT PRESENT THE PROGRAM IS DESIGNED FOR SINGLE POINT LOAD, TWO POINT LOADS AND UNIFORMLY DISTRIBUTED LOADS. IF SOME OTHER TYPE OF SYMMETRICAL LOADING IS DESIRED, THE CHANGES WILL BE NECESSARY ONLY IN THE PORTION OF BENDING MOMENT CALCULATION IN THE MAIN PROGRAM.

THE DEFLECTION FOR A SHORED BEAM AND AN UNSHORED BEAM

THE EFFECT OF DEAD LOAD HAS BEEN CONSIDERED AND THE PROGRAM CAN TAKE INTO ACCOUNT BOTH SHORED AND UNSHORED CASES. THE DEFLECTION COMPUTED IN THESE TWO CASES ARE EXPLAINED IN DETAIL IN ARTICLE 6.1 (P71).

NAME OF THE SUBROUTINES USED

THIS PROGRAM USES THE SUBROUTINES EXTRA, TERMS, REGULA, STRAIN, STAMS, FAMB AND FAMS. THE LIBRARY SUBROUTINE USED IS 'BNDSOL'.

SUBROUTINE EXTRA

SUBROUTINE EXTRA PREDICTS THE VALUE OF SLIP, CURVATURE AND THE EXTREME FIBRE STRAINS THROUGH EXTRAPOLATION BY EMPLOYING THE LAGRANGE INTERPOLATION FORMULA.

SUBROUTINE TERMS

SUBROUTINE TERMS COMPUTES THE TERMS WHICH MAY CHANGE THEIR VALUES WHEN THE CONCRETE SLAB CRACKS. THEREFORE THIS COMPUTATION SHOULD BE DONE AT EACH LOADING OR EACH MAIN PROGRAM CYCLE TO PROVIDE CURRENT VALUE OF THE QUANTITIES.

SUBROUTINE REGULA

SUBROUTINE REGULA FIRST TESTS THE CHANGE OF SIGN OF THE MID-SPAN SLIPS. IF THERE IS NO CHANGE OF SIGN, THE SUBROUTINE IS ESSENTIALLY NOT USED. IF THERE IS A CHANGE OF SIGN, THE REGULA FALSI METHOD OR THE BISECTION METHOD IS USED TO PREDICT THE VALUE OF END SLIP FOR THE NEXT CYCLE.

SUBROUTINE STRAIN

SUBROUTINE STRAIN COMPUTES THE CURVATURE AND STRAIN DISTRIBUTION IN A COMPOSITE BEAM FOR A GIVEN INTERACTIONAL FORCE AND MOMENT. THIS SUBROUTINE ONLY COMES INTO PLAY FOR INELASTIC PANELS.

SUBROUTINE STAMS

SUBROUTINE STAMS COMPUTES THE MOMENT AND EXTREME FIBRE STRAIN IN THE SLAB OF A COMPOSITE BEAM, FOR A GIVEN FORCE AND CURVATURE.

SUBROUTINE FAMB

SUBROUTINE FAMB COMPUTES FORCE AND MOMENT IN THE BEAM OF A COMPOSITE BEAM FOR A GIVEN STRAIN DISTRIBUTION. ANY SHAPE OF THE BEAM CAN BE ACCOMMODATED BY JUST MODIFYING THIS SUBROUTINE AND THE RELATED PART, 'DIVISION IN THE CRSSS-SECTION OF STEEL', IN THE MAIN PROGRAM.

SUBROUTINE FAMS

SUBROUTINE FAMS COMPUTES THE FORCE AND MOMENT IN THE SLAB OF A COMPOSITE BEAM FOR A GIVEN STRAIN DISTRIBUTION. THE STRESS-STRAIN RELATIONSHIP IS ELASTIC-PLASTIC. THE CROSS-SECTIONAL SHAPE OF THE SLAB IS RECTANGULAR. FOR THIS SIMPLE CASE, THE EXACT METHOD OF COMPUTATION IS USED. THE TENSION IN THE SLAB IS NEGLECTED. FOR COMPLICATED SHAPE OF SLAB, A MODIFICATION USEING THE METHOD OF SUBDIVISION SIMILAR TO THAT USED IN FAMB IS PREFERABLE.

NOTATION
****

AB--AREA OF BEAM WITHOUT COVER PLATE(IN*IN)

ACURA--PERCENTAGE DIFFERENCE OF THE CURVATURES FROM TWO CONSECUTIVE CYCLES , THIS IS ONE OF THE TWO KINDS OF ACCURACIES PRESCRIBED FOR THE SOLUTION

AREAP--AREA OF COVER PLATE (IN*IN) , IF THERE IS NO PLATE AREAP=0.

AS--AREA OF THE SLAB (IN*IN)

BF--WIDTH OF FLANGE (IN)

BI--MOMENT OF INERTIA OF STEEL BEAM WITHOUT COVER PLATE (IN**4)

BIT--MOMENT OF INERTIA OF STEEL BEAM WITH COVER PLATE (IN**4).

BM--EXTERNAL BENDING MIMENT ON THE COMPOSITE BEAM (LB-IN)

BMBE--BENDING MOMENT ON THE STEEL BEAM (LB-IN)

BMDL--BENDING MOMENT DUE TO DEAD LOAD (LB-IN)

BMSL--BENDING MOMENT ON THE SLAB (LB-IN)

BMTD--COMPUTED TOTAL INTERNAL MOMENT (LB-IN)

BS--WIDTH OF SLAB (IN)

CB--THE DISTANCE FROM THE FORCE CENTER OF THE BEAM TO THE TOP OF THE BEAM

CBD--THE DISTANCE FROM THE FORCE CENTER OF THE BEAM TO THE BOTTOM OF THE BEAM

CEST--THIS IS A FIXED VALUE OF STRAIN. WHEN THE STRAIN AT THE PANEL NEXT TO THE MID-SPAN IS LARGER THAN THIS VALUE, THE MULTIPLICATION FACTORS FAC ARE REPLACED BY FAC2

CS--HALF OF THE DEPTH OF THE UNCRACKED PORTION OF THE SLAB

CUR, CUR1--CURVATURE OF THE COMPOSITE BEAM

CURB, CURB1--CURVATRUE OF THE BEAM = CUR + CURD

CURD, CURD1--CURVATURE OF THE BEAM DUE TO D.L. =0. IF THE BEAM IS SHORED

CURS, CURS1--CURVATURE OF THE SLAB = CUR

DB--DEPTH OF THE STEEL BEAM WITHOUT COVER PLATE (IN)

DBT--TOTAL DEPTH OF THE STEEL BEAM WITH COVER PLATE (IN)

DC--DEPTH OF THE CELLULAR RIB (IN)

DEMAX--THE PRESCRIBED MAXIMUM VALUE OF DEFLECTION (IN) ,IF DEFLECTION AT ANY POINT EXCEEDS THIS VALUE THE COMPUTATION IS TERMINATED

DL--DEAD LOAD (LB/IN)

DN--THE DEPTH OF THE UNCRACKED PORTION OF THE SLAB

DS--DEPTH OF SLAB (IN)

DW1, DW2.....DW7--DW1 IS THE INITIAL LOADING INCREMENT, WHEN THE COMPOSITE BEAM YIELDS DW1 IS REPLACED BY DW2. IF THE LOADING .GE. WW3, THE INCREMENT IS DW3. IF THE LOADING .GE. WW4, THE INCREMENT IS DW4, ETC. ALL ARE IN UNIT OF POUNDS.

EAC--MULTIPLICATION FACTORS USED TO ENLARGE THE END SLIP ASSUMED FOR NEXT CYCLE IN ELASTIC COMPUTATION. (2.-EAC) ARE USED TO MULTIPLY THE END SLIP TO GET A SMALLER ASSUMED VALUE FOR THE NEXT CYCLE.

ER--MODULUS OF ELASTICITY OF THE BEAM (PSI)

ES--MODULUS OF ELASTIVCTY OF THE SLAB (PSI)

EACL--A MULTIPLICATION FACTOR SIMILAR TO EAC USED TO ENLARGE THE ASSUMED END SLIP

EACS--A MULTIPLICATION FACTOR USED TO REDUCE THE SLIP = 2.-EACL

ERR1,ERR2--THE LOWER ACCURACY PRESCRIBED IN 'STRAIN' SIMILAR TO ERR1 AND ERR2,WHEN THE DIFFERENCES BETWEEN THE COMPUTED FORCE AND MOMENT AND THE GIVEN ONES ARE GREATER THAN ERR1 AND ERR2,WHILE SMALLER THAN ERR1 AND ERR2 THE COMPUTED STRAIN AND CURVATURE ARE CONDITIONALY ACCEPTED

ERIN-- = ER12 FOR THE INITIAL STARTING THREE LOADING STAGES WHICH MAY BE ALLOWED TO HAVE LOWER ACCURACY IF THE TRIAL END SLIPS ARE DIFFICULT TO ESTIMATE

ERR1,ERR2--THE ACCURACY PRESCRIBED IN SUBROUTINE STRAIN,IN THE FORM OF MAXIMUM ALLOWABLE DIFFERENCE FOR FORCE AND MOMENT

ERST--THIS IS AN PERCENTAGE ACCURACY INITIALLY PRESCRIBED IN SUBROUTINE STRAIN AT INTERMEDIATE LOADING STAGES IN THE COMPARISION OF THE COMPUTED FORCE AND MOMENT WITH THE GIVEN ONES.

ER10--THIS IS A SMALL POSITIVE NUMBER USED TO PRESCRIBE THE DESIRED ACCURACY FOR INELASTIC COMPUTATION. IF THE MAGNITUDE OF MID-SPAN SLIP IS SMALLER THAN ER10 AND THE PERCENTAGE DIFFERENCE OF THE CURVATURES IN THE PANEL NEXT TO THE MID-SPAN AT THE TWO CONSECUTIVE CYCLES IS ALSO SMALLER THAN 'ACURA' THE SOLUTION IS OBTAINED.

ER12--THE PRESCRIBED ACCURACY IN ELASTIC COMPUTATION,SIMILAR TO ER10

FAC--SIMILAR TO EAC,USED FOR INELASTIC COMPUTATION

FRACT--A NUMNER USED TO REDUCE THE DIFFERENCE BETWEEN THE MULTIPLICATION FACTOR AND UNITY, WHEN NUMERICAL DIFFICULTY IS ENCOUNTERED IN THE SECOND OR THE SUBSEQUENT MAIN CYCLES

FAC2--SIMILAR TO FAC,USED FOR HIGHER LOADING

FALA--A MULTIPLICATION FACTOR USED TO ENLARGE THE ASSUMED END SLIP WHEN NEGATIVE CONNECTOR FORCES WITH LARGE MAGNITUDE APPEAR IN ESASTIC COMPUTATION

FACTOR--A NUMBER USED TO REDUCE THE INCREMENTS OF LOADINGS WHEN NUMERICAL DIFFICULTY IS ENCOUNTERED IN THE FIRST MAIN CYCLE

FANEG--A MULTIPLICATION FACTOR USED TO ENLARGE THE ASSUMED END SLIP WHENEVER A NEGATIVE VALUE OF PANEL FORCE APPEARS IN INELASTIC COMPUTATION

GC,GS,GST,GSL--THE VALUES OF THE CURVATURE OF THE BEAM , THE STRAINS OF THE BOTTOM STEEL FIBRE AND TOP CONCRETE FIBRE,AND THE END SLIPS AT THE

CONSECUTIVE LOADING STAGES RESPECTIVELY (IN THE EXTRAPOLATING COMPUTATION)

GSNH--AN EXTRAPOLATED VALUE OF STRAIN AT THE PANEL NEXT TO THE MID-SPAN

G2,Q2,G3,Q3--(IN),(LB),THE VALUES OF THE KNOWN SLIPS AND THE CORRESPONDING CONNECTOR FORCES WHICH SPECIFY THE TWO POINTS TO BE PASSED BY THE HYPERBOLIC Q-Y CURVE

HARST--THE STRAIN FROM WHICH ON STRAIN HARDENING IS INTRODUCED

INELA--REPRESENTS THE PANELS WHICH BECOME INELASTIC

JMK--NUMBER OF THE LOADING STAGES

K--REPRESENTS THE NUMNER OF INELASTIC PANELS

KER--INDICATES THE OCCURRENCE OF NUMERICAL DIFFICULTY

KKCC--THE NUMBER OF TIMES IN MAKING CHANGES TO VARY THE INCREMENT OF CURVATURE AND STRAIN IN COMPUTING F AND M GRADIENTS

KM--THE NUMBER OF TIMES IN MAKING THE CHANGES TO REDUCE THE LOADING INCREMENTS

KN--THE NUMBER OF TIMES IN MAKING THE CHANGES TO VARY THE VALUES OF MULTIPLICATION FACTORS

LABAL-THE NUMNERS CORRESPOND TO THOSE PSEUDO CONNECTORS WHEN ALL THE CONNECTORS ARE NUMBERED,AS 1,2,.....NH+1,STARTING FROM THE END CONNECTOR

LOAD--A VALUE TO INDICATE THE TYPE OF LOADINGS DESIRED , USE 1 FOR SINGLE POINT LOAD,USE 2 FOR TWO POINT LOADS AND 3 FOR UDL

MERM,MER1,MER2--THESE THREE NUMBERS ORGINALLY ARE SET ARBITRARILY SUCH THAT MER1 AND MER2 DIFFER FROM MERM. WHEN THE COMPUTED STRAIN AND CURVATURE ONLY SATISFY THE LOWER ACCURACY (FRE1 AND FRE2) MERM IS SET TO EQUAL MER1 OR MER2. THE END SLIP ASSUMED IN THIS CYCLE SHOULD NOT BE USED IN REGULA FALSI METHOD. BISECTION METHOD MAY BE USED WITH CARE. IN THE CASE THE END SLIPS FROM THE TWO CONSECUTIVE CYCLES DIFFER VERY SLIGHTLY AND THE SOLUTION IS STILL NOT OBTAINED,THE BISECTION METHOD SHOULD BE REPLACED BY THE METHOD USING MULTIPLICATION FACTORS

MMM--NUMBER OF MAIN CYCLES

N--NUMBER OF CONNECTORS IN THE WHOLE SPAN INCLUDING THE ACTUAL OR IMAGINARY CONNECTORS AT THE MID-SPAN

NCE, NC--NUMBER OF CYCLES ALLOWED FOR THE MAIN CYCLE COMPUTATION FOR THE ELASTIC AND THE INELASTIC CASES RESPECTIVELY,IF THE CORRESPONDING VALUE IS EXCEEDED THE COMPUTATION IS TERMINATED

NCHA--THE NUMBER OF TIMES ALLOWED FOR REDUCING THE LOADING INCREMENT,IF BEING EXCEEDED,THE COMPUTATION WILL BE TERMINATED

NFU,NW,NFD,NPL--NUMBER OF SUBDIVISION ASSIGNED TO THE UPPER FLANGE,THE WEB ,THE LOWER FLANGE AND THE COVER PLATE OF THE STEEL BEAM, IF THERE IS NO COVER PLATE,NPL=0

NLA-NUMBER OF PSEUDO CONNECTOR

NH-- THE NUMBER OF PANELS IN HALF SPAN.

NSHOR--ZERO FOR SHORED BEAM AND ONE FOR UNSHORED BEAM

PCUT--THE DISTANCE FROM THE SUPPORT TO THE CUT OFF POINT OF THE COVER PLATE, PCUT=0. IF THE PLATE EXTENDS TO THE WHOLE SPAN

Q--CONNECTOR FORCE

QIN--THE CONNECTOR FORCE COMPUTED FROM SLIN

RPF--STRAIN HARDENING FACTOR

SAS--ASSUMED END SLIPS FOR EACH MAIN CYCLE

SI--MOMENT OF INERTIA OF THE UNCRACKED PORTION OF THE SLAB

SLIN--THE VALUE OF SLIP WHICH GIVES THE SLOPE REQUIRED WHEN ESTABLISHING THE SIMULTANEOUS EQUATIONS IN THE INITIAL TWO LOADING STAGES

SD4--DISTANCE FROM THE SUPPORT TO THE FIRST CONNECTOR (IN)

SL--CONNECTOR SLIPS

SLMAX--THE ULTIMATE VALUE OF SLIP ,WHEN THE SLIP AT ANY CONNECTOR EXCEEDS THIS VALUE THE COMPUTATION WILL BE TERMINATED

SS--CONNECTOR SPASING

STBT,STBB--STRAIN AT THE TOP AND BOTTOM FIBRE OF BEAM

STST,STSB--STRAIN AT THE TOP AND BOTTOM FIBRE OF SLAB

STRSY,STRPY,STRWY,STRFY--YIELD STRESS OF CONCRETE,COVER PLATE,WEB AND FLANGE (PSI)

STSTF,STSBF,STBTF,STBBF--THE ULTIMATE VALUE (FAILURE VALUE) OF THE STRAINS AT THE TOP AND THE BOTTOM FIBRE OF SLAB AND BEAM RESPECTIVELY, WHEN THE STRAIN EXCEEDS THE CORRESPONDING VALUE,THE COMPUTATION IS TERMINATED

TF--THICKNESS OF FLANGE (IN)

TPL--THICKNESS OF COVER PLATE (IN)

TRYLD-- =0. IF THE LOADING INCREMENTS ARE SET BY THE INPUT, =1. WHEN THE INCREMENTS ARE AUTOMATICALLY SET BY THE PROGRAM

TW--THECKNESS OF WEB (IN)

U--DISTANCE OF THE FIRST LOAD FROM L.H.S. FOR THE CASE OF TWO POINT LOADS

W--INITIALLY IS THE STARTING LOADING,AFTER THE FIRST LOADING STAGE IT REPRESENTS THE LOADING AT ANY LOADING STAGE (LB/IN FOR UDL,OTHERWISE LB)

```
C     WW3,WW4,.....WW7--A SET OF LOADING INCREMENTS (SEE DW1,DW2.....DW7)
C
C     WW8--THE MAXIMUM VALUE OF W,WHEN BEING EXCEEDED, THE COMPUTATION WILL
C       BE TERMINATED
C
C     X--DISTANCE FROM THE L.H.S. TO CONNECTORS
C
C     XX--DISTANCE FROM THE L.H.S. TO THE CENTRE OF PANELS
C
C     ZL--TOTAL LENGTH OF THE COMPOSITE BEAM (IN)
C
C
C                              IMPORTANT NOTE
C                              **************
C     1. THE VALUES OF THE MULTIPLICATION FACTOR FAC AND FAC2 ARE FLEXIBLE,
C        BUT SHOULD NOT DIFFER MUCH FROM UNITY. THEY CAN BE AJUSTED TO AVOID
C        NUMERICAL DIFFICULTIES AND SAVE THE COMPUTER TIME. THE OTHER NUMERIC
C        FACTORS USUALLY NEED NOT BE CHANGED.
C     2. THE LOAD-SLIP CURVE SHOULD BE SMOOTH SUCH THAT EXTRAPOLATION CAN,
C        EFFECTIVELY PREDICT THE VALUES.
C     3. HOW TO INPUT THE PSEUDO CONNECTORS--AFTER THE INPUT OF THE SPACING O
C        THE ACTUAL CONNECTORS,INPUT THE TOTAL NUMBER OF CONNECTORS INCLUDING
C        THE PSEUDO ONES (N) AND THE NUMBER OF THE PSEUDO CONNECTORS (NLA).
C        THE NEXT INPUT WILL BE THE CONNECTOR SPACING AFTER INTRODUCING THE
C        PSEUDO CONNECTORS (SS(I)). FINALLY GIVE THE CONNECTOR NUMNER WHICH
C        CORRESPOND TO THE PSEUDO CONNECTORS (LABAL(I))
C        IF NO PSEUDO CONNECTOR IS ADDED,INPUT THE ORIGINAL NUMBER OF CONNECT
C        FOR N,0 FOR NLA AND THE ORIGINAL CONNECTOR SPACING FOR SS(I).
C        FOR LABAL(I), IT JUST NEEDS TO PUNCH AN '0'.
C     4. TO SAVE THE COMPUTER TIME, OBJECT (BINARY) DECK AND 'OPTIMIZATION'
C        IS SUGGESTED TO USE WHENEVER IT IS POSSIBLE.
C     5. BECAUSE THIS PROGRAM INVOLVES MANY VARIABLES. FOR BETTER CHECKING
C        A SPECIAL JOB CARD IS SUGGESTED TO USE, SUCH THAT THE LOCATIONS
C        OF ALL THE VARIABLES ARE PRINTED.
C     6. THE LOADING INCREMENT AT HIGH LOADING SHOULD BE SMALLER.
C
C
C     A DATA INPUT FOR A PRACTICAL BEAM IS GIVEN HERE FOR REFERENCE.
C     **************************************************************
C
      20 PER CENT OF CONNECTION, 7 CONNECTORS AT HALF SPAN
13.72       .23       .335      5.0       197.      2.5       69.       360.
0.0         0.0       1.5       33.3      .015      .004      1.5       10.
3000.       44000.    44000.    44000.    .18       .18       .9        .9
3000000.    29000000.
7.0         5.0       3.0       8.0       3.5       2.5       2.0
0.0         7.0       10.       120.      150.      170.      900.
  15    1     3  100    3     6    14
0.          0.5       0.
.02               .12             6200.           11300.          .0006
       50          50 .00001     .0001     .015      0.
1.009  1.008  1.008
1.009  1.008  1.008
1.04   1.02   1.004
.00002      .01       .9        1.045     .96       1.04                5
.0004       1.006
24.  24.  24.  30.  24.  24.  30.
  21    3
```

```
24.  24.  24.  30.  12.  12.  12.  12.  15.  15.
  6   8  10
C
C
C                              *****   PROGRAM BEGINS   *****
C                                      --------------
C
C
C
      DIMENSION TITLE(20)
      DIMENSION DEE(51),BMDL(50),Q(51),SL(51),CURD(51),AREA(51)
      DIMENSION GC(50,4),GS(50,4),GSL(50,4),GST(50,4)
      DIMENSION LABAL(50),TOTBM(50),FAC(6)
      DIMENSION ANTEG(50),HB(50),FAC(6),FAC2(6)
      DIMENSION INFLA(50),CURS(50),CURB(50),TRSL(3)
      DIMENSION XX1(51),ZK(51),X(51),A(3,50),B(50)
      DIMENSION STSB1(50),KOUT(50),TAREA(51)
      COMMON/BLOCK1/WM(4)
      COMMON/BLOCK2/ES(50),CS1
      COMMON/BLOCK3/BWM(50)
      COMMON/BLOCK4/BMS(50),BMB(50),BIT(50),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,FB,STFY,ES,EBP
      COMMON/BLOCK6/FB(50)
      COMMON/BLOCK7/CUR1(50)
      COMMON/BLOCK8/CURB1(50)
      COMMON/BLOCK9/CURS1(50)
      COMMON/BLOCK10/AS(50),SI(50),SE1(50),FAB(50),EIB(50),Z(50),CS(50)
      COMMON/BLOCK11/BET(50),ALP(50),BB(50),SS(50)
      COMMON/BLOCK12/DN(50)
      COMMON/BLOCK14/BM(50)
      COMMON/BLOCK13/DBT(50),CBD(50),CB(50)
      COMMON/BLOCK16/CURD1(50)
      COMMON/BLOCK17/STST(50),STBB(50)
      COMMON/BLOCK18/STSB(50)
      COMMON/BLOCK19/STBT(50)
      COMMON/BLOCK20/CUR(50)
      COMMON/BLOCK21/BH(165),DDB(165),DFA(165),HARST,NFU,NFD,NW,NPL,PCUT
      COMMON/BLOCK23/CURT(50),STBBT(50),STSTT(50)
      COMMON/BLOCK24/BMBF(50),BMSL(50),BMFD(50)
      COMMON/BLOCK25/MA,JM,NH,KER,KEM2
      COMMON/BLOCK26/STC(50),CUC(50)
      COMMON/BLOCK27/KKCC,MERM,MMM
      COMMON/BLOCK28/GSNH,ERST,SCAL1,SCAL2
      COMMON/BLOCK29/UU,VV,XL,XR,MER1,MER2,SAS(50),SLM(50),LAG
      COMMON/BLOCK30/XX(50)
C
C     THIS PRO. USES HYPERBOLIC Q-Y CURVE
C
C     DATA OF COMPOSITE BEAM
C
      READ(5,5300) TITLE
5300  FORMAT(20A4)
      WRITE(6,5301) TITLE
5301  FORMAT(1X,20A4,///)
      READ (5,49) DB,TW,TF,BF,BI,DS,BS,ZL
      WRITE(6,50) DB,TW,TF,BF,BI,DS,BS,ZL
      READ (5,49) U,SD4,DC,DL,HARST,RPE,SLMAX,DEMAX
      WRITE(6,50) U,SD4,DC,DL,HARST,RPE,SLMAX,DEMAX
      READ(5,49) STRSY,STRPY,STRWY,STRFY,STSTF,STSBF,STBTF,STBBF
```

```
      WRITE(6,50)STRSY,STRPY,STRWY,STRFY,STSTF,STSBF,STBTF,STBBF
      READ(5,2) ES,EB
      WRITE(6,2) ES,EB
      READ (5,31) DW1,DW2,DW3,DW4,DW5,DW6,DW7
      WRITE(6,85) DW1,DW2,DW3,DW4,DW5,DW6,DW7
      READ (5,31) W,WW3,WW4,WW5,WW6,WW7,WW8
      WRITE(6,85) W,WW3,WW4,WW5,WW6,WW7,WW8
      READ (5,1) N,NSHOR,LOAD,NW,NFU,NFD,NPL
      WRITE(6,1) N,NSHOR,LOAD,NW,NFU,NFD,NPL
      READ(5,9) WPL,TPL,PCUT
      WRITE(6,9)WPL,TPL,PCUT
40    FORMAT(8F10.5)
50    FORMAT(8F15.5)
2     FORMAT(2F10.0)
31    FORMAT(7F10.2)
85    FORMAT(7F15.2)
1     FORMAT(7I5)
9     FORMAT(3F10.5)
      READ(5,1000) G2,G3,Q2,Q3,SLIN
      WRITE(6,1000)G2,G3,Q2,Q3,SLIN
1000  FORMAT(5F15.4)
      READ (5,237) NCE,NC,ER12,ER10,ACURA,TRYLD
      WRITE(6,237) NCE,NC,ER12,ER10,ACURA,TRYLD
237   FORMAT(2I10,4F10.6)
      READ(5,238)(FAC(I),I=1,3)
      READ(5,238)(FAC2(I),I=1,3)
      READ(5,238) (EAC(I),I=1,3)
238   FORMAT(3F8.5)
      DO 7776 I=1,3
      FAC(I+3)=2.-FAC(I)
      FAC2(I+3)=2.-FAC2(I)
      EAC(I+3)=2.-EAC(I)
7776  WRITE(6,7755) I,FAC(I),FAC2(I),EAC(I)
7755  FORMAT(* NO*,I2,5X,*FAC*,F9.5,5X,*FAC2*,F9.5,5X,*EAC *,F9.5)
      READ(5,79) ERIN,CFST,FRACT,EACL,EACS,EALA,NCHA
      WRITE(6,79)ERIN,CFST,FRACT,EACL,EACS,EALA,NCHA
79    FORMAT(6F10.7,I10)
      READ(5,7) ERST,FANEG
      WRITE(6,7) ERST,FANEG
7     FORMAT(2F10.6)
C     INITIAL VALUE OF THE CONTROL FACTORS
      K=0
      MA=0
      JMK=0
      KKCC=0
      KER=0
      KSCH=0
      KM=0
      KN=0
      FACTOR=1.
      KMUL=0
      SCAL1=1.
      SCAL2=10.
      KEM2=0
      KNFG=0
C
      AB=2.*TF*BF+TW*(DB-2.*TF)
      STSY=STRSY/ES
      STRY=STRPY/EB
```

```
      STFY=STRFY/EB
      ERP=RPE*EB
C
C
C     DIVISION IN THE CROSS-SECTION OF STEEL
C
      NN=NFU+NW+NFD+NPL
      DO 131 KL=1,NN
      DDB(KL)=TF/FLOAT(NFU)
      IF(KL.GT.NFU) DDB(KL)=(DB-2.*TF)/FLOAT(NW)
      IF(KL.GT.(NFU+NW)) DDB(KL)=TF/FLOAT(NFD)
      IF(KL.GT.(NFU+NW+NFD)) DDB(KL)=TPL/FLOAT(NPL)
      BH(KL)=BF
      IF(KL.GT.NFU) BH(KL)=TW
      IF(KL.GT.(NFU+NW)) BH(KL)=BF
      IF(KL.GT.(NFU+NW+NFD))BH(KL)=WPL
  131 CONTINUE
      DEA(1)=DDB(1)
      DO 132 I=2,NN
  132 DEA(I)=DEA(I-1)+DDB(I)
      WRITE(6,140)((DDB(L),DEA(L),BH(L)),L=1,NN)
  140 FORMAT(2X,3E20.6)
C
C     LOAD-SLIP CHARACTERISTICS
C
      BV=(G2*Q2*Q3-G3*Q3*Q2)/(G2*Q3-G3*Q2)
      CV=(BV*G2*Q2-BV*BV*G2)/Q2
      AV=CV/BV
      DO 1100 I=1,30
      SL(I)=FLOAT(I)/200.
      Q(I)=(CV/(SL(I)-AV))+BV
 1100 WRITE(6,6005) SL(I),Q(I)
 6005 FORMAT(* SLIP AND FORCE*,F7.4,F13.2)
      NH=(N-1)/2
      NH1=NH+1
C
C      CONNECTOR SPACING AND THE COORDINATES OF THE CONNECTORS AND
C     PANELS
C
      READ(5,32) (SS(I),I=1,NH)
      WRITE(6,23) (SS(I),I=1,NH)
   32 FORMAT(16F5.1)
   23 FORMAT(* CONNECTOR SPACING*,16F6.2)
   63 IF(JMK.LT.3) GO TO 54
      READ(5,61) N,NLA
      WRITE(6,35)N,NLA
   35 FORMAT(* NO OF TOTAL CONN.AFTER INTRODUCING PSEUDO ONES*,I3,5X,
     1*NO OF PSEUDO CONN.*,I3)
      NH=(N-1)/2
      NH1=NH+1
      READ(5,32) (SS(I),I=1,NH)
      WRITE(6,64) (SS(I),I=1,NH)
   64 FORMAT(* NEW CONNECTOR SPACING*,16F7.2)
      READ(5,71) (LABAL(I),I=1,NLA)
      WRITE(6,75) (LABAL(I),I=1,NLA)
   75 FORMAT(* PSEUDO CONNECTOR*,40I3)
   71 FORMAT(40I3)
   61 FORMAT(2I5)
      W=WST
```

```
54    JM=0
      XX(1)=SD4+.5*SS(1)
      DO 3 I=2,NH
    3 XX(I)=XX(I-1)+.5*(SS(I-1)+SS(I))
      X(1)=SD4
      DO 36 I=2,NH1
   36 X(I)=X(I-1)+SS(I-1)
C
      QIN=(CV/(SLIN-AV))+BV
      DO 1002 I=1,NH1
1002  ZK(I)=QIN/SLIN
      DO 68 I=1,NH
      INFLA(I)=0
68    DN(I)=DS
C     TOTAL MOMENT OF INERTIA FROM THE COMBINATION OF BEAM & COVER PLATE
      AREAP=TPL*WPL
      YCM=(AB*(0.5*DB+TPL)+AREAP*0.5*TPL)/(AB+AREAP)
      BPI=BI+AB*((0.5*DB-(YCM-TPL))**2)+(WPL*(TPL**3))/12.
     1+AREAP*((YCM-0.5*TPL)**2)
      DO 360 I=1,NH
      BIT(I)=BI
      DBT(I)=DB
      CB(I)=0.5*DB
      CBD(I)=0.5*DB
      IF(XX(I).LE.PCUT) GO TO 360
      BIT(I)=BPI
      CBD(I)=YCM
      CB(I)=DB+TPL-YCM
      DBT(I)=DB+TPL
360   CONTINUE
C
C     COMPUTATION OF B.M. DUE TO D.L.
C
      TDL2=.5*DL*ZL
      DO 22 I=1,NH
   22 BMDL(I)=TDL2*XX(I)-DL*XX(I)*XX(I)*.5
      WRITE(6,99) ((XX(I),BMDL(I)),I=1,NH)
99    FORMAT( * XX*,F8.2,8X,*B.M.DUE TO D.L.*,F12.2)
      BMDLM=TDL2*(0.5*ZL)-DL*((ZL*0.5)**2)*0.5
      WRITE(6,7001) BMDLM
7001  FORMAT( * MID-SPAN B.M. DUE TO D.L. *,F12.2)
C
C     THE TREATMENT OF THE MOMENT AND THE CURVATURE DUE TO D.L. FOR THE
C     SHORED AND UNSHORED BEAM
C
      IF(NSHOR.GT.0) GO TO 43
C
C     CURVATURE DUE TO D.L. WILL BE ZERO IF THE BEAM IS SHORED
C
      DO 44 I=1,NH
   44 CURD(I)=0.
      GO TO 45
   43 DO 46 I=1,NH
46    CURD(I)=BMDL(I)/(EB*BIT(I))
      DO 48 I=1,NH
   48 BMDL(I)=0.
C
45    IF(JMK.EQ.3.AND.JM.EQ.0) GO TO 300
      GO TO 301
```

```
C
C       THIS BRANCH IS FOR THE COMPUTATION OF THE D.L. EFFECT FOR UNSHORED
C       BEAM
300     IF(NSHOR-1) 301,302,302
302     W=0.
        BMLL=0.
        BMMSP=BMDLM
        DO 304 I=1,NH
        CUR(I)=CURD(I)
        CURB(I)=CURD(I)
        STBT(I)=-CURB(I)*CB(I)
304     STBB(I)=CURB(I)*CBD(I)
        WRITE(6,303)
303     FORMAT(/////,* AT THIS LOADING THE D.L. EFFECT IS COMPUTED*)
        WRITE(6,311)
311     FORMAT(1H0,/,6X,*X*,8X,*STBT*,10X,*STBB*,/)
        WRITE(6,312) ((XX(I),STBT(I),STBB(I)),I=1,NH)
312     FORMAT(1H0,F8.2,2F16.8)
        GO TO 305
301     CONTINUE
C
C       LOADING INCREMENT AND THE INITIAL VALUE OF OTHER CONTROL FACTORS
C       THE VALUE OF MER1 & MER2 ARE SET ARBITRARILY SO AS TO DIFFER FROM
C       MERM
        MMM=1
        WST=W
        DELW=DW1
 18     CONTINUE
        MERM=0
        MER1=100
        MER2=100
        CURP=0.
        LAG=0
        JM=JM+1
        JMK=JMK+1
        IF(TRYLD.EQ.1.) GO TO 8000
        IF(K.NE.0)   DELW=DW2
        IF(W.GE.WW3) DELW=DW3
        IF(W.GE.WW4) DELW=DW4
        IF(W.GE.WW5) DELW=DW5
        IF(W.GE.WW6) DELW=DW6
        IF(W.GE.WW7) DELW=DW7
        GO TO 77
8000    DELW=1500.
        IF(K.NE.0) DELW=1500.
        IF(JMK.LE.6) GO TO 900
        IF(STBB(NH).GT.0.008)DELW=500.
        IF(STBB(NH).GT.0.02) DELW=250.
        IF(LOAD.EQ.3)DELW=(2./ZL)*DELW
        GO TO 77
900     IF(LOAD.EQ.3)DELW=(2./ZL)*DELW
77      W=W+DELW*FACTOR
        IF(JM.EQ.1.AND.NSHOR.EQ.0)  W=0.
        IF(JM-4) 6100,6100,6101
6100    WM(JM)=W
        GO TO 6102
6101    DO 6103 I=1,3
6103    WM(I)=WM(I+1)
204      WM(4)=W
```

```
6102  WRITE(6,34) W
34    FORMAT(1H0,///,4H W =,F11.3)
C
C     B.M. CALCULATION
C
      REACL=W*0.5
      IF(LOAD.EQ.2) GO TO 102
      IF(LOAD.EQ.3) GO TO 1991
      DO 4 I=1,NH
      BM(I)=REACL*XX(I)+BMDL(I)
    4 CONTINUE
      BMLL=REACL*(ZL*0.5)
      GO TO 103
102   DO 108 I=1,NH
      IF(XX(I).GT.U.AND.XX(I).LE.(ZL/2.)) BM(I)=BMDL(I)+REACL*U
108   IF(XX(I).LE.U)  BM(I)=BMDL(I)+REACL*XX(I)
      BMLL=REACL*U
      GO TO 103
1991  REACL=W*ZL*.5
      DO 1992 I=1,NH
      BM(I)=BMDL(I)+REACL*XX(I)-.5*W*XX(I)*XX(I)
1992  CONTINUE
      BMLL=REACL*(.5*ZL)-.5*W*((.5*ZL)**2)
103   DO 5 I=1,NH
      IF(NSHOR.EQ.0) TOTBM(I)=BM(I)
      IF(NSHOR.EQ.1) TOTBM(I)=BM(I)  +(TDL2*XX(I)-DL*XX(I)*XX(I)*0.5)
5     CONTINUE
      BMMSP=BMDLM+BMLL
C
C     USE THE LIBRARY FUNCTION TO SOLVE THE FINITE DIFFERENCE EQU.FOR
C     THE FIRST 2 TRIAL VALUES OF SLIP
C
      IF(JMK.GT.6) GO TO 8
      IF(JMK.GT.3) GO TO 2001
      DO 5101 I=1,NH
      CALL TERMS(I,EB,ES)
92    FORMAT(F40.8)
      B(I)=BB(I)
      A(2,I)=-(1./ZK(I)+1./ZK(I+1)+ALP(I)*SS(I))
      A(3,I)=1./ZK(I+1)
5101  A(1,I)=1./ZK(I)
      A(1,1)=0.
      A(3,NH)=0.
      A(2,NH)=-(1./ZK(NH)+ALP(NH)*SS(NH))
      CALL BNDSOL(A,B,3,1,NH)
      Q(1)=B(1)
      DO 5105 I=2,NH
5105  Q(I)=B(I)-B(I-1)
      Q(NH+1)=0.
      DO 5665 I=1,NH
5665  SL(I)=Q(I)/ZK(I)
      SL(NH+1)=0.
      TRSL(JMK)=SL(1)
      IF(JMK.LT.3) GO TO 18
      IF(JMK.EQ.3) GO TO 63
C
C     ST. BAR,CUR. BAR AND STSTT BAR ARE FOUND HERE BY EXTRAPOLATION
8     CALL EXTRA(1,GSL,SUML)
      WRITE(6,92) SUML
```

```
      GSL(1,4)=SUML
      DO 902 I=1,NH
      CALL EXTRA(I,GC,SUML)
      GC(I,4)=SUML
      CALL EXTRA(I,GS,SUML)
      GS(I,4)=SUML
      CALL EXTRA(I,GST,SUML)
      GST(I,4)=SUML
  902 CONTINUE
      GSNH=GS(NH,4)
      IF(GSNH.GE.CFST) GO TO 9250
      GO TO 851
 9250 DO 9253 I=1,6
 9253 FAC(I)=FAC2(I)
  851 SAS(MMM)=GSL(1,4)
      WRITE(6,2858) SAS(MMM)
 2858 FORMAT(//,* SLIP ASSUMED*,F15.6)
C
C     GREEN CARD-THE BEGINNING OF ELASTIC COMPUTATION*****************
C
      IF(K.NE.0) GO TO 70
 7001 IF(JMK.EQ.4) SAS(MMM)=TRSL(1)
      IF(JMK.EQ.5) SAS(MMM)=TRSL(2)
      IF(JMK.EQ.6) SAS(MMM)=GSL(1,2)+((GSL(1,2)-GSL(1,1))/(WM(2)-WM(1)))
     1*(WM(3)-WM(2))
 1624 SL(1)=SAS(MMM)
      I=1
      NLAB=1
  565 Q(I)=(CV/(SL(I)-AV))+BV
      IF(I.EQ.LABAL(NLAB)) GO TO 15
      GO TO 16
   15 NLAB=NLAB+1
      Q(I)=0.
   16 CALL TERMS(I,EB,ES)
      IF(I-1) 800,800,801
  801 B(I)=Q(I)+B(I-1)
      GO TO 802
  800 B(I)=Q(I)
  802 HB(I)=B(I)*ALP(I)*SS(I)+BB(I)
      SL(I+1)=SL(I)+HB(I)
      IF(B(I).LT.(-200000.)) GO TO 289
      IF(Q(I).LT.(-200000.)) GO TO 289
      GO TO 290
  289 SAS(MMM)=SAS(MMM)*FALA
      KSCH=KSCH+1
      WRITE(6,291)
  291 FORMAT(* ORIGINAL SLIP ASSUMED IS TOO SMALL*)
      IF(KSCH.GE.80) GO TO 26
      GO TO 1624
  290 I=I+1
      IF(I.LT.NH1) GO TO 565
      SLM(MMM)=SL(NH1)
      IF(JM.LE.3.AND.ABS(SLM(MMM)).LE.ERIN) GO TO 57
      IF(ABS(SLM(MMM))-ER12) 57,57,2323
 2323 WRITE(6,1239) MMM,SAS(MMM),SLM(MMM)
      MMM=MMM+1
      IF(MMM.GT.NCE) GO TO 26
C     PREDICT THE VALUE OF SLIP FOR NEXT CYCLE
      IF(SLM(MMM-1).LE.(-.03 )) SAS(MMM)=SAS(MMM-1)*EAC(1)
```

```
      IF(SLM(MMM-1).GT.(-.03 ).AND.SLM(MMM-1).LE.(-.001 ))
     1SAS(MMM)=SAS(MMM-1)*EAC(2)
      IF(SLM(MMM-1).GT.(-.001 ).AND.SLM(MMM-1).LE.0.) SAS(MMM)=
     1SAS(MMM-1)*EAC(3)
      IF(SLM(MMM-1).GE.(0.03 )) SAS(MMM)=SAS(MMM-1)*EAC(4)
      IF(SLM(MMM-1).LT.(0.03 ).AND.SLM(MMM-1).GE.(.001 ))
     1SAS(MMM)=SAS(MMM-1)*EAC(5)
      IF(SLM(MMM-1).LT..001 .AND.SLM(MMM-1).GE.0.) SAS(MMM)=
     1SAS(MMM-1)*EAC(6)
      IF(SLM(MMM-1).LE.(-.1)) SAS(MMM)=SAS(MMM-1)*EACL
      IF(SLM(MMM-1).GE.1.) SAS(MMM)=SAS(MMM-1)*EACS
      IF(MMM.LE.2) GO TO 1624
      CALL REGULA (MMM,MERM)
      IF(ABS(SAS(MMM-1)-SAS(MMM-2)).LE.0.0000003) SAS(MMM)=(XL+XR)*0.5
      GO TO 1624
67    WRITE(6,1239)MMM,SAS(MMM),SLM(MMM)
      WRITE(6,1248)
      SREC=SAS(MMM)
      MMB=MMM-1
      MMM=1
      KSCH=0
      LAG=0
      IF(SL(NH1-2).LE.0.) GO TO 470
      GO TO 471
470   KMUL=KMUL+1
      W=W-(DELW/FLOAT(KMUL+1))
      IF(KMUL.EQ.5) GO TO 26
      IF(JM.GT.4) GO TO 204
      WM(JM)=W
      GO TO 6102
471   KMUL=0
C
C     GREEN CARD-THE END OF ELASTIC COMPUTATION**************************
C
C
      DO 14 I=1,NH
      CUR(I)=(BM(I)-B(I)*Z(I))/SEI(I)
      CURS(I)=CUR(I)
      CURB(I)=CUR(I)+CURD(I)
      STST(I)=-B(I)/(ES*AS(I))-CS(I)*CURS(I)
      STSB(I)=CURS(I)*DS+STST(I)
      STBT(I)=B(I)/(EB*AB)-CURB(I)*CB(I)
      STBB(I)=B(I)/(EB*AB)+CURB(I)*CBD(I)
      DN(I)=-STST(I)/CUR(I)
      IF(DN(I).GT.DS.OR.DN(I).LT.0.) DN(I)=DS
14    CONTINUE
      WRITE(6,330) (STBB(I),I=1,NH)
330   FORMAT(  * STBB*,9E15.5)
C
C     CHECK FOR THE YIELDING OF THE PANELS
C
      DO 52 I=1,NH
      IF(ABS(STST(I)).LE.STSY.AND.STBB(I).LE.STBY) GO TO 52
      IF(K.EQ.0) GO TO 53
      DO 84 LL=1,K
      IF(INELA(LL).EQ.I) GO TO 52
84    CONTINUE
53    K=K+1
      INELA(K)=I
```

```
   52  CONTINUE
       IF(K.EQ.0) GO TO 55
       SAS(MMM)=SREC
   70  L=1
C
C      THE BEGINNING OF INELASTIC COMPUTATION----------------------------
C
       KOUT(L)=K
       WRITE(6,72) (FAC(I),I=1,6)
   72  FORMAT(/,* FAC*,8F10.6,/)
       NE=NH-K
       NE1=NE+1
       KREPT=0
   73  SL(1)=SAS(MMM)
       WRITE(6,3151)MMM,SAS(MMM)
 3151  FORMAT(1X,*     * CYCLE NO *,I5,9X,*SLIP ASSUMED*,F20.8)
       NLAB=1
C      COMPUTATION OF THE ELASTIC PANELS
       I=1
  765  Q(I)=(CV/(SL(I)-AV))+BV
       IF(I.EQ.LABAL(NLAB)) GO TO 56
       GO TO 67
   56  NLAB=NLAB+1
       Q(I)=0.
   67  CALL TERMS(I,EB,ES)
       IF(I-1) 700,700,701
  701  B(I)=Q(I)+B(I-1)
       GO TO 702
  700  B(I)=Q(I)
  702  HB(I)=B(I)*ALP(I)*SS(I)+BB(I)
       SL(I+1)=SL(I)+HB(I)
       I=I+1
       IF(I.LT.NE1) GO TO 765
       DO 60 I=1,NE
       CUR(I)=(BM(I)-B(I)*Z(I))/SEI(I)
       CURS(I)=CUR(I)
       CURB(I)=CUR(I)+CURD(I)
       STST(I)=-B(I)/(ES*AS(I))-CS(I)*CURS(I)
       STSB(I)=CURS(I)*DS+STST(I)
       STBT(I)=B(I)/(EB*AB)-CURB(I)*CB(I)
       STBB(I)=B(I)/(EB*AB)+CURB(I)*CBD(I)
       DN(I)=-STST(I)/CUR(I)
       IF(DN(I).GT.DS.OR.DN(I).LT.0.) DN(I)=DS
   60  CONTINUE
       II=NE1
       Q(NE1)=(CV/(SL(NE1)-AV))+BV
       IF(NE1.EQ.LABAL(NLAB)) GO TO 113
       GO TO 200
  113  NLAB=NLAB+1
       Q(NE1)=0.
  200  B(II)=B(NE)+Q(NE1)
       WRITE(6,78) B(NE),Q(NE1),SL(NE1)
   78  FORMAT(3F30.2)
C
C      COMPUTATION OF THE INELASTIC PANNELS
 1235  STSTT(II)=GST(II,4)
       STBBT(II)=GS(II,4)
       CURT(II)=GC(II,4)
       CURD1(II)=CURD(II)
```

```
      BMREP=BM(II)
      IF(MERM.EQ.MER1.OR.MERM.EQ.MER2) GO TO 11
      GO TO 68
11    IF(ABS(SAS(MMM-1)-SAS(MMM-2)).GT.0.0000001) GO TO 68
      SCAL2=SCAL1
      SCAL1=SCAL1*0.5
68    CALL STRAIN(B,BM,CURD1,II)
      BM(II)=BMREP
1247  FORMAT (*$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$*,/)
      IF(KKCC.GT.4.AND.MMM.EQ.1) GO TO 201
      IF(KKCC.GT.4.AND.MMM.NE.1) GO TO 202
      GO TO 6106
201   IF(KM.EQ.6) GO TO 26
      W=(WM(3)+WM(4))*0.5
      IF(KREPT.EQ.1) GO TO 6
      IF(KM.EQ.1) GO TO 6
      FACTOR=0.75
6     KM=KM+1
      WRITE(6,6112)
6112  FORMAT(* THE LOADING INCREMENT IS CHANGED TO A SMALL VALUE--------
     1-------------------------*,//)
      GO TO 204
202   IF(KN.EQ.NCHA) GO TO 26
      SAS(MMM)=SAS(MMM-1)+(SAS(MMM)-SAS(MMM-1))/5.
      KN=KN+1
      IF(KN.EQ.1) GO TO 4444
      DO 6111 I=1,6
      IF(I.LE.3) FAC(I)=(FAC(I)-1.)*FRACT+1.
      IF(I.GE.4) FAC(I)=2.-FAC(I-3)
      FAC2(I)=FAC(I)
6111  CONTINUE
4444  WRITE(6,6110)
6110  FORMAT(* THE VALUE OF SLIP IS REASSUMED----------------------
     1-------------------------*,//)
      GO TO 73
6106  CUR(II)=CUR1(II)
      CURS(II)=CURS1(II)
      CURB(II)=CURB1(II)
      DN(II)=-STST(II)/CURS(II)
      IF(DN(II).GT.DS.OR.DN(II).LT.0.) DN(II)=DS
      STSB1(II)=STSB(II)+CUR1(II)*DC
      ANTEG(II)=(STBT(II)-STSB1(II))*SS(II)
      SL(II+1)=SL(II)+ANTEG(II)
      Q(II+1)=(CV/(SL(II+1)-AV))+BV
      IF((II+1).EQ.LABAL(NLAB)) GO TO 104
      GO TO 12
104   NLAB=NLAB+1
      Q(II+1)=0.
12    IF(II-NH) 1234,1236,1236
1234  II=II+1
      B(II)=B(II-1)+Q(II)
      IF(B(II).GT.0.) GO TO 1235
      KNEG=KNEG+1
      SAS(MMM)=SAS(MMM)*FANEG
      IF(KNEG.EQ.10) GO TO 26
      GO TO 73
1236  ER11=0.
      IF(ABS(CURB(NH)-CURP).LE.(ACURA*CURB(NH))) ER11=1.
      CURP=CURB(NH)
```

```
      SLM(MMM)=SL(NH+1)
      IF(ABS(SLM(MMM)).LE.ER10.AND.ER11.EQ.1) GO TO 59
      WRITE(6,1239)MMM,SAS(MMM),SL(NH+1)
 1239 FORMAT(* CYCLE*,I3,5X,*END SLIP ASSUMED*,F12.8,6X,*MID-SPAN SLIP*
     1,F12.8)
      WRITE (6,4333)
 4333 FORMAT(*-------------------------------------------------*,
     1*-------------------------------------------------*,/)
      MMM=MMM+1
      KNEG=0
      IF(MMM.GT.NC) GO TO 26
C     PREDIC THE VALUE OF SLIP FOR NEXT CYCLE
      IF(SLM(MMM-1).LE.(-.025)) SAS(MMM)=SAS(MMM-1)*FAC(1)
      IF(SLM(MMM-1).GT.(-.025).AND.SLM(MMM-1).LE.(-.002))
     1SAS(MMM)=SAS(MMM-1)*FAC(2)
      IF(SLM(MMM-1).GT.(-.002).AND.SLM(MMM-1).LE.0.) SAS(MMM)=
     1SAS(MMM-1)*FAC(3)
      IF(SLM(MMM-1).GE.(0.025)) SAS(MMM)=SAS(MMM-1)*FAC(4)
      IF(SLM(MMM-1).LT.(0.025).AND.SLM(MMM-1).GE.(.002))
     1SAS(MMM)=SAS(MMM-1)*FAC(5)
      IF(SLM(MMM-1).LT..002.AND.SLM(MMM-1).GE.0.) SAS(MMM)=
     1SAS(MMM-1)*FAC(6)
      SASRE=SAS(MMM)
      IF(MMM.LE.2) GO TO 73
      CALL REGULA (MMM,MERM)
      IF(ABS(SAS(MMM-1)-SAS(MMM-2)).LE.0.0000003) SAS(MMM)=SASRE
      GO TO 73
   59 WRITE(6,1239) MMM,SAS(MMM),SL(NH+1)
      WRITE (6,1248)
 1248 FORMAT (*$$$$$$$$$$ THIS IS LAST CYCLE$$$$$$$$$$*)
      SREC=SAS(MMM)
      MMB=MMM-1
      MMM=1
      LAG=0
      CURP=0.
      MER2=100
      MERM=0
      MER1=100
      SCAL1=1.
      SCAL2=10.
C
C     THE END OF INELASTIC COMPUTATION----------RED-------
C
      DO 433 I=1,NH
      CURT(I)=CURB(I)
      STBBT(I)=STBB(I)
      STSTT(I)=STST(I)
  433 CONTINUE
C     CHECK FOR YIELDING OF PANELS. IF SOME NEW PANELS BECOME INELASTIC
C     TREAT THEM INELASTICALLY
C
      KO=K
      DO 62 I=1,NH
      IF(ABS(STST(I)).LE.STSY.AND.STBB(I).LE.STBY) GO TO 62
      DO 66 LL=1,K
      IF(INELA(LL).EQ.I) GO TO 62
   66 CONTINUE
      K=K+1
      INELA(K)=I
```

```
  62  CONTINUE
      L=L+1
      KOUT(L)=K
      KNEW=KOUT(L)-KO
      WRITE(6,2352)KNEW
2352  FORMAT (1X,* NO OF NEW YIELDED PANELS  IS *,I4)
      WRITE(6,1247)
      NE=NH-K
      NF1=NF+1
      MMM=1
      IF(KNEW.EQ.0) GO TO 55
      SAS(MMM)=SREC
      KREPT=1
      DO 74 I=1,NH
      GST(I,4)=STST(I)
      GC(I,4)=CURB(I)
74    GS(I,4)=STBB(I)
      GO TO 73
C
55    CONTINUE
C
      DO 76 I=1,NH
      CURT(I)=CURB(I)
      STBBT(I)=STBB(I)
      STSTT(I)=STST(I)
76    CONTINUE
      DO 1110 I=1,MMB
      DSAS=SAS(I)-SREC
      RATS=SREC/SAS(I)
      WRITE(6,1117) I,DSAS,RATS,SAS(I),SLM(I)
1117  FORMAT( * NO*,I3,5X,*SLIP DIFF*,F9.7,5X,*RATIO*,F10.8,5X,*SAS*,
     1F11.8,5X,*MID-SPAN SLIP*,F10.7)
1110  CONTINUE
      MMM=1
      Q(NH1)=(CV/(SL(NH1)-AV))+BV
C
C
      NF1=NH+1-K
      DO 432 I=1,NH
      BMBE(I)=CURB(I) * EB * BIT(I)
      BMSL(I)=CURS(I) * ES * SI(I)
      BMFD(I)=BMBE(I)+BMSL(I)+B(I)*Z(I)
 432  CONTINUE
      DO 141 I=1,K
      IN1=INFLA(I)
      BMBE(IN1)=BMB(IN1)
      BMSL(IN1)=BMS(IN1)
      BMFD(IN1)=BMM(IN1)
 141  CONTINUE
C
      DO 9951 I=1,NH
      IF(DN(I)-DS) 9950,9951,9950
9950  WRITE(6,9952) I,DN(I)
9952  FORMAT(  *  PANEL NO *,I3,5X,*DN*,F10.3)
9951  CONTINUE
C     COMPUTATION OF DEFLECTION
C
305   XX1(1)=.66666667*SD4
      AREA(1)=CUR(1)*.5*SD4
```

```
      TAREA(1)=AREA(1)
      DO 37 I=2,NH1
      XX1(I)=XX(I-1)
      AREA(I)=CUR(I-1)*SS(I-1)
   37 TAREA(I)=TAREA(I-1)+AREA(I)
      REACT=TAREA(NH1)
      DO 38 J=1,NH1
      DDIO=REACT*X(J)
      SUM=0.
      DO 39 I=1,NH1
      IF(XX1(I).GT.X(J)) GO TO 40
      SUM=SUM+AREA(I)*(X(J)-XX1(I))
   39 CONTINUE
   40 DEF(J)=DDIO-SUM
   38 CONTINUE
      WRITE(6,3001) W,BMLL
 3001 FORMAT(1X,///,* LIVE LOAD FORCE*,F11.3,8X,*LIVE LOAD MOMENT AT MID
     1-SPAN*,F13.0)
      WRITE(6,7008) BMMSP
 7008 FORMAT(* TOTAL B.M. AT MID-SPAN*,F14.2)
      WRITE(6,41) (DEF(I),I=1,NH1)
   41 FORMAT(1HO,*DEF*,13F9.5)
      IF(JM.EQ.0.AND.NSHOR.EQ.1) GO TO 301
C
      WRITE(6,412)
  412 FORMAT(1HO,/,5X,*X*,8X,*Q*,10X,*SL*,/)
      WRITE(6,411)  (( X(I),Q(I),SL(I)),I=1,NH1)
  411 FORMAT(1HO,F8.2, F12.0,F12.7)
      WRITE(6,144)
  144 FORMAT(1HO,5X,*XX*,13X,*STST*,12X,*STSB*,12X,*STBT*,12X,*STBB*,12X
     1,*CUR*,13X,*CURB*,12X,*F*,/)
      WRITE(6,33)((XX(I),STST(I),STSB(I),STBT(I),STBB(I),CUR(I),CURB(I),
     1B(I)),I=1,NH)
   33 FORMAT(1HO,F8.2,7E16.8)
      WRITE(6,1502)
 1502 FORMAT(1HO,6X,*XX*,12X,*BMSL*,12X,*BMBF*,12X,*BMFD*,12X,*TOTBM*)
      WRITE(6,1501)((XX(I),BMSL(I),BMBF(I),BMFD(I),TOTBM(I)),I=1,NH)
 1501 FORMAT(1X, F8.2,4F16.8)
      IF(K.EQ.0) GO TO 145
      WRITE(6,143) K,(INELA(L),L=1,K)
  143 FORMAT(1HO,*NO. AND INELASTIC PANELS *,I6,18I4/)
C
C     CHECK AGAINST EXCESSIVE YIELDING OF THE PANELS AND CONNECTOR FAILURE
C     THAT IS EXCESSIVE SLIP IN THE CONNECTOR. IF IT IS SO STOP OTHERWISE
C     INCREASE THE LOAD
C
  145 DO 93 I=1,NH
      IF(STBB(I).GE.STBBF .OR.ABS(STBT(I)).GE.STBTF ) GO TO 94
      IF(ABS(STST(I)).GE.STSTF.OR.STSB(I).GE.STSBF) GO TO 95
   93 CONTINUE
      DO 90 I=1,NH1
      IF(ABS(SL(I)).GE.SLMAX)GO TO 91
      IF(DEF(I).GT.DFMAX) GO TO 5000
   90 CONTINUE
      GO TO 100
   91 WRITE(6,96)
   96 FORMAT(*FAILURE OF CONNECTOR(S)*)
      GO TO 26
   94 WRITE(6,97) I,STBB(I),STBT(I)
```

```
 97   FORMAT(1H0,32HEXCESSIVE YIELDING IN PANEL NO. ,I4,15H   STRAINS AR
     1E ,2F15.4)
      GO TO 26
 95   WRITE(6,98) I,STST(I),STSB(I)
 98   FORMAT(1H0,26HCONC CRUSHES IN PANEL NO. ,I4,15H   STRAINS ARE ,2E1
     25.4)
      GO TO 26
5000  WRITE(6,3000)
3000  FORMAT(* DEFLECTION EXCEEDS THE ALLOWABLE LIMIT*)
      GO TO 26
100   IF(JM.GE.4) GO TO 6000
C     RECORD  THE VALUES USED IN EXTRAPOLATION IN NEXT CYCLE
      DO 901 I=1,NH
      GS(I,JM)=STBRT(I)
      GC(I,JM)=CURT(I)
      GSL(I,JM)=SL(I)
901   GST(I,JM)=STSTT(I)
      GO TO 6104
6000  DO 6001 I=1,NH
      DO 6001 MJ=1,2
      GS(I,MJ)=GS(I,MJ+1)
      GC(I,MJ)=GC(I,MJ+1)
      GST(I,MJ)=GST(I,MJ+1)
6001  GSL(I,MJ)=GSL(I,MJ+1)
      DO 6003 I=1,NH
      GS(I,3)=STBRT(I)
      GC(I,3)=CURT(I)
      GSL(I,3)=SL(I)
6003  GST(I,3)=STSTT(I)
6104  IF (W.LE.WW8) GO TO 18
  26  STOP
      END
C
C
      SUBROUTINE EXTRA(I,GYY,SUML)
C     ----------------------------
C
      DIMENSION GYY(50,4)
      COMMON/BLOCK1/WM(4)
      SUML=0.0
      DO 7273 J=1,3
      PROL=GYY(I,J)
      DO 7283 L=1,3
      IF(J-L) 7293,7283,7293
7293  PROL=PROL*((WM(4)-WM(L))/(WM(J)-WM(L)))
7283  CONTINUE
7273  SUML=SUML+PROL
      RETURN
      END
C
      SUBROUTINE TERMS(I,EB,ES)
C     -------------------------
C
      COMMON/BLOCK4/BMS(50),BMB(50),BIT(50),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK10/AS(50),SI(50),SEI(50),FAB(50),EIB(50),Z(50),CS(50)
      COMMON/BLOCK11/BET(50),ALP(50),BB(50),SS(50)
      COMMON/BLOCK12/DN(50)
      COMMON/BLOCK13/DBT(50),CBD(50),CB(50)
      COMMON/BLOCK14/BM(50)
```

```
      AS(I)=DN(I)*BS
      CS(I)=.5*DN(I)
      Z(I)=(DS-CS(I))+CB(I)+DC
      SI(I)=BS*DN(I)**3./12.
      SEI(I)=EB*BIT(I)+ES*SI(I)
      EAB(I)=EB*AB*ES*AS(I)/(EB*AB+ES*AS(I))
      EIB(I)=SEI(I)+EAB(I)*Z(I)**2
      BET(I)=Z(I)/SEI(I)
      ALP(I)=EIB(I)/(EAB(I)*SEI(I))
      BB(I)=-BET(I)*BM(I)*SS(I)
      RETURN
      END
C
      SUBROUTINE REGULA(MMM,MERM)
C     ---------------------------------
C
      COMMON/BLOCK29/UU,VV,XL,XR,MER1,MER2,SAS(50),SLM(50),LAG
      IF(LAG.LT.1) GO TO 2007
      IF(SLM(MMM-1)*VV) 2009,2009,2010
2010  XL=SAS(MMM-1)
      VV=SLM(MMM-1)
      MER1=MMM-1
      GO TO 2005
2009  XR=SAS(MMM-1)
      UU=SLM(MMM-1)
      MER2=MMM-1
      GO TO 2005
2007  IF(SLM(MMM-1)*SLM(MMM-2)) 2002,2002,25
2002  LAG=LAG+1
      IF(SLM(MMM-1)) 2003,2003,2004
2003  VV=SLM(MMM-1)
      UU=SLM(MMM-2)
      XL=SAS(MMM-1)
      XR=SAS(MMM-2)
      MER1=MMM-1
      MER2=MMM-2
      GO TO 2005
2004  UU=SLM(MMM-1)
      VV=SLM(MMM-2)
      XR=SAS(MMM-1)
      XL=SAS(MMM-2)
      MER2=MMM-1
      MER1=MMM-1
2005  SAS(MMM)=(XL*UU-XR*VV)/(UU-VV)
      IF(MERM.EQ.MER1.OR.MERM.EQ.MER2)SAS(MMM)=(XL+XR)*0.5
25    RETURN
      END
C
      SUBROUTINE STRAIN(FST,BMST,CURD1,I)
C     ---------------------------------------
C
      DIMENSION FST(50),BMST(50),CURD1(50)
      COMMON/BLOCK3/BMM(50)
      COMMON/BLOCK4/BMS(50),BMB(50),BIT(50),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,FB,STFY,ES,EBP
      COMMON/BLOCK6/FB(50)
      COMMON/BLOCK7/CUR1(50)
      COMMON/BLOCK8/CURB1(50)
      COMMON/BLOCK9/CURS1(50)
```

```
      COMMON/BLOCK13/DBT(50),CBD(50),CB(50)
      COMMON/BLOCK17/STST(50),STBB(50)
      COMMON/BLOCK18/STSB(50)
      COMMON/BLOCK19/STBT(50)
      COMMON/BLOCK20/CUR(50)
      COMMON/BLOCK23/CURT(50),STBBT(50),STSTT(50)
      COMMON/BLOCK25/MA,JM,NH,KER,KEM2
      COMMON/BLOCK26/STC(50),CUC(50)
      COMMON/BLOCK27/KKCC,MERM,MMM
      COMMON/BLOCK28/GSNH,ERST,SCAL1,SCAL2
      KER=0
      KKCC=0
      KK2=0
      CUR1(I)=CURT(I)
      STBB(I)=STBBT(I)
      BMST(I)=BMST(I)+CURD1(I)*EB*BIT(I)
      CURB1(I)=CUR1(I)
      WRITE(6,604) I,CUR1(I),STBB(I)
  604 FORMAT( *  INELASTIC PANEL  NO*,I2,5X,*CURVATURE BAR*,E13.6,5X,*ST
     1RAIN BAR*,E14.6)
      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CURD1(I)
      CALL STAMS (FB,CURS1,I)
      FB2=FB(I)
      BMB2=BMM(I)
      FB4=FB(I)
      BMB4=BMM(I)
      CUC(I)=0.1E-07
      STC(I)=0.1E-07
 8012 NLK=0
      NLM=0
      NLL=0
      GO TO 2
 7051 CUC(I)=0.1E-05
      STC(I)=0.1E-05
      NLK=1
      GO TO 2
 7052 CUC(I)=0.1E-06
      STC(I)=0.1E-08
      NLL=1
      GO TO 2
 7053 CUC(I)=0.1E-08
      STC(I)=0.1E-06
      NLM=1
    2 IF(KK2.GE.2) GO TO 58
      GO TO 59
   58 CUC(I)=0.5E-08
      STC(I)=0.5E-08
   59 CURB1(I)=CUR1(I)+CUC(I)
      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CURD1(I)
      IF (FB(I).LE.0.) GO TO 8019
      MA=1
      CALL STAMS (FB,CURS1,I)
      MA=0
      IF(KEM2.EQ.1) GO TO 8019
      FBC=FB(I)
      BMBC=BMM(I)
      CURB1(I)=CUR1(I)
```

```
      IF(KKCC.EQ.0) GO TO 505
      WRITE(6,606) CUC(I),FBC,BMBC
606   FORMAT (* DEL CUR*,E16.8,4X,*F DEL CUR*,F15.6,4X,*M DELT CUR*,F16.
     18)
505   STBB(I)=STBB(I)+STC(I)
      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CURD1(I)
      CALL STAMS (FB,CURS1,I)
      IF(KEM2.EQ.1) GO TO 8019
      FBST=FB(I)
      BMBST=BMM(I)
      IF(KKCC.EQ.0) GO TO 506
      WRITE(6,607)STC(I),STBB(I),FBST,BMBST
 607  FORMAT(1H0,*DELT ST*,1E12.5,4X,*STRAIN P*,1E13.5,5X,*F DELT ST*,F
     112.1,6X,*M DEL ST*,1E11.4)
506   DFDC=(FBC-FB2)/CUC(I)
      DMDC=(BMBC-BMB2)/CUC(I)
      DFDST=(FBST-FB2)/STC(I)
      DMDST=(BMBST-BMB2)/STC(I)
      IF(DFDST.LE.0..OR.DMDST.LE.0.) GO TO 3
      IF(DFDC.GE.0..OR.DMDC.GE.0.) GO TO 3
      DENO=DFDST*DMDC-DMDST*DFDC
      IF(DENO.NE.0..AND.DFDC.NE.0.) GO TO 7060
3     KER=1
8019  KKCC=KKCC+1
      KK2=1
      WRITE(6,8010) KKCC
8010  FORMAT(/////,*0000000000000000000000000000000000000KKCC IS *,I3,//)
      IF(KKCC.GT.4) GO TO 5
      CUR1(I)=CURT(I)
      STBB(I)=STBBT(I)
      FB2=FB4
      BMB2=BMB4
      IF(NLK.EQ.0) GO TO 7051
      IF(NLL.EQ.0) GO TO 7052
      IF(NLM.EQ.0) GO TO 7053
7060  STBB2N=(DMDC*(FST(I)-FB2)-DFDC*(BMST(I)-BMB2))/(DFDST*DMDC-D
     1FDC*DMDST)
      CURIN2=(FST(I)-FB2-DFDST*STBB2N)/DFDC
      IF(KKCC.EQ.0) GO TO 10
      WRITE(6,989)
989   FORMAT(*          DFDC          *,*          DMDC          *,*
     1 DFDST          DMDST          DELT ST          *,
     2*DELT CUR*)
      WRITE(6,7) DFDC,DMDC,DFDST,DMDST,STBB2N,CURIN2
7     FORMAT(2X,6E20.8)
10    STBB(I)=STBB(I)-STC(I)+STBB2N
      CUR1(I)=CUR1(I)+CURIN2
      IF(KKCC.EQ.0) GO TO 122
      WRITE(6,960)STBB(I),CUR1(I)
960   FORMAT(*  STR  PRIME AND CURVATURE PRIME*,2E14.5)
122   CONTINUE
      IF(STBB(I).GT.1..OR.CUR1(I).GT.1.) GO TO 8019
      IF(STBB(I).GT.0..AND.CUR1(I).GT.0.) GO TO 8
      GO TO 8019
8     CURB1(I)=CUR1(I)
      CALL FAMB(STBB,CURB1,I)
      CURS1(I)=CURB1(I)-CURD1(I)
      CALL STAMS (FB,CURS1,I)
```

```
      IF(KEM2.EQ.1) GO TO 8019
      FB2=FB(I)
      BMB2=BMM(I)
      KK2=KK2+1
      ERFAC=ERST
      IF(GSNH.GE.0.008) ERFAC=ERST/4.
      IF(GSNH.GE.0.05) ERFAC=ERST/25.
      ERR1=ABS(FST(I))*ERFAC*SCAL1
      ERR2=ABS(BMST(I))*ERFAC*SCAL1
      IF(KKCC.EQ.0) GO TO 121
      WRITE(6,632) FST(I),FB2,BMST(I),BMB2
  632 FORMAT(*  FORCE ASSUMED IN THE PANNEL*,F12.1,5X,*F SOLVED
     1*,F15.4,5X,*MOMT ASSUMED*,F13.0,5X,*MSOL*,F15.4)
  121 CONTINUE
      IF(ABS(FST(I)-FB2).LE.ERR1.AND.ABS(BMST(I)-BMB2).LE.ERR2)
     1GO TO 4
      IF(KK2.GE.4) GO TO 4
      GO TO 2
    4 CONTINUE
      IF(KK2.EQ.4) MERM=MMM
      ERE1=ABS(FST(I))*ERFAC*SCAL2
      ERE2=ABS(BMST(I))*ERFAC*SCAL2
      IF(ABS(FST(I)-FB2).GT.ERE1.OR.ABS(BMST(I)-BMB2).GT.ERE2)
     1GO TO 8019
      STBBT(I)=STBB(I)
      STSTT(I)=STST(I)
      CURT(I)=CUR1(I)
      CUR1(I)=CURS1(I)
      WRITE(6,6) KK2,I
    6 FORMAT(2X,*((((((((((    KK2= *,I2,*    ))))))))))*,*PANNEL NO*,
     1I2,/)
    5 RETURN
      END
C
      SUBROUTINE STAMS(FB,CURS1,I)
C     ----------------------------
C
      DIMENSION FB(50),CURS1(50),FB1(10)
      COMMON/BLOCK2/FS(50),CS1
      COMMON/BLOCK3/BMM(50)
      COMMON/BLOCK4/BMS(50),BMB(50),BIT(50),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,FB,STFY,ES,EBP
      COMMON/BLOCK13/DBT(50),CBD(50),CB(50)
      COMMON/BLOCK17/STST(50),STBB(50)
      COMMON/BLOCK18/STSB(50)
      COMMON/BLOCK23/CURT(50),STBBT(50),STSTT(50)
      COMMON/BLOCK25/MA,JM,NH,KER,KEM2
      COMMON/BLOCK28/GSNH,ERST,SCAL1,SCAL2
      KEM2=0
      KC=0
      NNA=0
      NNB=0
      STST(I)=STSTT(I)
       CALL FAMS(STST,CURS1,I)
      FB1(1)=FS(I)
      DSTST=-5.0E-6
    6 CONTINUE
      DO 2 L=1,6
      IF(MA.EQ.1) DSTST=DSTST*(-1.0)
```

```
      STST(I)=STST(I) + DSTST
       CALL FAMS(STST,CURS1,I)
      FD1=FS(I)
      DFDF=(FD1-FB1(L))/DSTST
      IF(DFDF.NE.0.) GO TO 5
      STST(I)=STSTT(I)
      IF(NNA.EQ.1) GO TO 7
      DSTST=-1.0E-04
      NNA=1
      GO TO 6
    7 IF(NNB.EQ.1) GO TO 9
      DSTST=-5.0E-03
      NNB=1
      GO TO 6
    9 KFM2=1
      GO TO 4
    5 DSTST1=(FB(I)-ABS(FB1(L)))/DFDF
      STST(I)=STST(I)-DSTST1-DSTST
       CALL FAMS(STST,CURS1,I)
      FRCOF=0.0001
      IF(GSNH.GE.0.015) FRCOE=0.00001
      IF(GSNH.GE.0.045) FRCOE=0.000002
      ERR3=ABS(FB(I))*ERCOE
      IF(KFR.GE.1) FRR3=FRR3*0.0001
      IF(MA.EQ.1) DSTST=DSTST*(-1.0)
      IF(ABS(FB(I)-ABS(FS(I))).LE.ERR3)  GO TO 3
      FB1(L+1)=FS(I)
    2 CONTINUE
    3 Z1=CS1+CB(I)
      BMM(I)=BMB(I)+BMS(I)+FB(I)*Z1
    4 RETURN
      END
C
      SUBROUTINE FAMB(STBB,CURB1,I)
C     ----------------------------
C
      DIMENSION STBB(50),CURB1(50),FBB(165),BMBB(165),TFBB(165),
     $TBMBB(165),STBD(165),STRBD(165)
      COMMON/BLOCK4/BMS(50),BMB(50),BIT(50),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,FB,STFY,ES,EBP
      COMMON/BLOCK6/FB(50)
      COMMON/BLOCK13/DBT(50),CBD(50),CB(50)
      COMMON/BLOCK19/STBT(50)
      COMMON/BLOCK21/BH(165),DDB(165),DFA(165),HARST,NFU,NFD,NW,NPL,PCUT
      COMMON/BLOCK30/XX(50)
      STBT(I)=STBB(I)-CURB1(I)*DBT(I)
      STBD(1)=STBT(I)
      STRBD(1)=STBD(1)*FB
      IF(STBD(1).GE.STFY.AND.STBD(1).LE.HARST) STRBD(1)=STRFY
      IF(STBD(1).GT.HARST)STRBD(1)=STRFY+(STBD(1)-HARST)*EBP
      IF(XX(I).LE.PCUT) NN=NFU+NFD+NW
      IF(XX(I).GT.PCUT) NN=NFU+NW+NFD+NPL
      DO 4 KL=1,NN
      STRY=STRFY
      IF((KL+1).GT.NFU) STRY=STRWY
      IF((KL+1).GT.(NFU+NW)) STRY=STRFY
      IF((KL+1).GT.(NFU+NW+NFD)) STRY=STRPY
      STBY=STRY/EB
      STBD(KL+1)=STBD(KL) + CURB1(I)*DDB(KL)
```

```
      STRBD(KL+1)=STBD(KL+1)*EB
      IF(ABS(STBD(KL+1)).GE.STRY.AND.ABS(STBD(KL+1)).LE.HARST)
     1STRBD(KL+1)=STRY*STBD(KL+1)/ABS(STBD(KL+1))
      IF(ABS(STBD(KL+1)).GT.HARST) STRBD(KL+1)=(STRY+(ABS(STBD(KL+1))-
     1HARST)*EBP)*STBD(KL+1)/ABS(STBD(KL+1))
      FBB(KL)=(STRBD(KL+1)+STRBD(KL))*.5*DDB(KL)*BH(KL)
      BMBB(KL)=(DFA(KL)-DDB(KL)*.5)*FBB(KL)
      IF(KL.GT.1)  GO TO 8
      TFBB(KL)=FBB(KL)
      TBMBB(KL)=BMBB(KL)
      GO TO 4
    8 TFBB(KL)=TFBB(KL-1) + FBB(KL)
      TBMBB(KL)=TBMBB(KL-1) + BMBB(KL)
    4 CONTINUE
      FB(I)=TFBB(NN)
      YYY=DBT(I)-(TBMBB(NN)/FB(I))
      BMB(I)=ABS(CBD(I)-YYY)*FB(I)
      RETURN
      END
C
C
      SUBROUTINE FAMS(STST,CURS1,I)
C     -----------------------------
C
      DIMENSION STST(50),CURS1(50)
      COMMON/BLOCK2/FS(50),CS1
      COMMON/BLOCK4/BMS(50),BMB(50),BIT(50),TW,TF,BF,DS,BS,AB,DC
      COMMON/BLOCK5/STSY,STRSY,STRPY,STRWY,STRFY,FB,STFY,ES,EBP
      COMMON/BLOCK18/STSB(50)
      STSB(I)=STST(I)+DS*CURS1(I)
      DNAS=ABS(STST(I))/CURS1(I)
      IF(DNAS.GE.DS)  DNAS=DS
      CS1=DS-.5*DNAS+DC
      STBO=STST(I)+DNAS*CURS1(I)
      ZMABO=STBO*ES*(-1.)
      ZMAT=STST(I)*ES*(-1.)
      IF(ABS(STST(I)).GE.STSY) ZMAT=STRSY
      DEZMA=ZMAT-ZMABO
      H=(STSY-ABS(STBO))/CURS1(I)
      IF(H.GT.DNAS) H=DNAS
      F1=ZMAT*DNAS*BS
      F2=DEZMA*0.5*H*BS
      FS(I)=F2-F1
      YBA=(F1*DNAS*0.5-(F2*H)/3.)/(F1-F2)
      BMS(I)=(0.5*DNAS-YBA)*FS(I)
      RETURN
      END
        6400 END OF RECORD
      REMA B2, USE THE FINAL PROGRAM
 12.16        .24         .349        4.01        130.        2.5         68.         252.
0.0           C.0         1.5         22.0        .015        .004        .30         8.0
3000.         44000.      44000.      44000.      .18         .18         .9          .9
3000000.      29000000.
 500.         2000.       250.        2000.       1000.       500.        250.
0.0             500.      1000.       25000.      27000.      29000.      34000.
   23     1     1  100     3     6     0
0.          0.          127.
.02                .12                6200.              11300.              .0008
        50          40 .00001     .0001     .015       0.0
```

```
1.008    1.006    1.005
1.005    1.004    1.004
1.04     1.009    1.004
.00002     .01        .9          1.045       .96         1.04                5
.0004      1.006
12.  12.  12.  12.  12.  12.  12.  12.  12.  12.  6.
   25     1
12.  12.  12.  12.  12.  12.  12.  12.  12.  12.  5.9  0.1
 12
          END OF FILE
```

CD TOT 1190