

# The Extended Innovation Kalman-Sliding Filter for Nonlinear Estimation

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**ABSTRACT**—Predicting and planning a path and extracting the current location are important aspects in the fields of navigation, localization, and autonomous vehicles. This brief paper belongs to these applications with measurement signals that are obtained from linear sensors. The kinematic states of a vehicle, and the maneuvering angle, are extracted by a filter from a noisy environment. Filters are considered to be either accurate or robust, and typically not both (a trade-off exists). In this paper, we introduce a method that combines accuracy with robustness. The well-known extended Kalman filter (EKF) is combined with the relatively new sliding innovation filter (SIF). The proposed algorithm makes use of the EKF gain and structure while utilizing the robustness of the SIF switching-based gain in an effort to provide a good estimate of the states. The result is a suboptimal nonlinear estimation strategy that resists uncertainties and disturbances. The proposed filter is demonstrated on a vehicle in the Cartesian coordinate while maneuvering and performing turns. The results are compared to the classical EKF and SIF.

**KEYWORDS**—Estimation theory, sliding innovation filter, extended Kalman filter, maneuvering, uncertainties, complex road.

## 1. INTRODUCTION

Maneuvering and navigation in 2D and 3D environments are known to be tricky, time consuming, and sometimes difficult to achieve. To estimate the vehicle's states including its positions and velocities in Cartesian Coordinates, a filter is required. The filter is an estimation algorithm that tries to recover the hidden information from the direct information that is linked to the measurement signals while reducing the noise effects. For that purpose, some filters use a mathematical model that represents the system and then excites it by the given input. These filters are known as the model-based-filters [1-10]. The pioneer work of Kalman in early 1960s is considered the largest milestone in the field. His filter (KF) finds the optimal solution of well-known and well-modelled linear system, simulated under white Gaussian noise signals. The filter solves the least squared error estimation. For given

assumptions, the filter is considered the optimal solution. No other filter may give a better performance. However, once one of the assumptions (at least) is violated, then the filter's performance starts to fall apart [11-14].

To overcome the KF performance's issues, several works have been developed like linearizing the nonlinear model using the Jacobian matrices as in the Extended KF (EKF) [15-17], or using summation of weighted particles as in the Sigma-point KF (SPKF) [18-22]. SPKF includes the pioneer work of Unscented KF (UKF) [19], and the Cubature KF (CKF) [23-24]. These versions of KF make the filter applicable for nonlinear systems. Other works combined the KF or its variants to another stable and robust filter like the Smooth Variable Structure filter (SVSF) [25-28] and Sliding Innovation Filter (SIF) [29-37].

SIF, which was formulated in 2020 as a robust/stable filter, defines a hyperplane from the measurements that represent the true trajectories and forces the estimates to remain on its neighborhood [35]. The filter shares similar principles as the SVSF and Sliding Mode Observer. These filters belong on another category of filtering that target the stability of the filter and making its robust against the uncertainties.

In this brief work, the SIF is combined with the EKF to achieve the benefit of the two filters in term of optimality and robustness/stability. The proposed filter is applied to a nonlinear target tracking application, where a vehicle is maneuvering in a complex road with several turns. However, the new strategy may be applied to any nonlinear estimation problem. The application model is illustrated in Section 2, while the filters and the proposed filter are formulated in Section 3. The results are discussed and concluded in Section 4 and Section 5, respectively.

## 2. SYSTEM MODEL

A vehicle is assumed to move in a complex road with several turns (up to 10 turns) as shown in Figure 1.

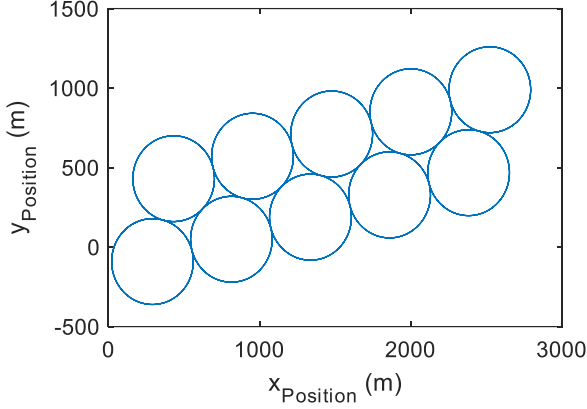


Figure 1. The complex road model.

The equation of the model is derived as follows [37]:

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \\ x_{4,k+1} \\ x_{5,k+1} \end{bmatrix} = \begin{bmatrix} x_{1,k} + M_k x_{2,k} - N_k x_{4,k} \\ c x_{2,k} - s x_{4,k} \\ x_{3,k} + N_k x_{2,k} - M_k x_{4,k} \\ s x_{2,k} + c x_{4,k} \\ x_{5,k} \end{bmatrix} + \begin{bmatrix} w_{1,k} \\ w_{2,k} \\ w_{3,k} \\ w_{4,k} \\ w_{5,k} \end{bmatrix} \quad (1)$$

where:

$c = \cos(x_{5,k}T)$ ,  $s = \sin(x_{5,k}T)$ ,  $M_k = \frac{\sin(x_{5,k}T)}{x_{5,k}}$  and  $N_k = \frac{1 - \cos(x_{5,k}T)}{x_{5,k}}$ ,  $x_{1,k}$  and  $x_{3,k}$  are the position in x- and y-axes, respectively,  $x_{3,k}$  and  $x_{4,k}$  are the velocity in x- and y-axes, respectively, and  $x_{5,k}$  is the maneuvering angular speed.  $w$  is the system noise and disturbances.  $T$  is the sampling time, and the subscript  $k$  and  $k + 1$  represent the time step. The system equation can be simplified as:

$$\rightarrow x_{k+1} = f(x_k) + w_k \quad (2)$$

The measurement signals are defined as:

$$\begin{bmatrix} z_{1,k+1} \\ z_{2,k+1} \\ z_{3,k+1} \\ z_{4,k+1} \\ z_{5,k+1} \end{bmatrix} = \begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \\ x_{4,k+1} \\ x_{5,k+1} \end{bmatrix} + \begin{bmatrix} v_{1,k+1} \\ v_{2,k+1} \\ v_{3,k+1} \\ v_{4,k+1} \\ v_{5,k+1} \end{bmatrix} \quad (3)$$

where  $z$  is the measurement signal and  $v$  is the measurement noise and uncertainties. The measurement equation can be simplified as:

$$\rightarrow z_{k+1} = x_{k+1} + v_{k+1} \quad (4)$$

The system is considered to have nonlinear model with linear sensor model. Due to the nonlinearity, the KF cannot be applied and a nonlinear form of the filter is needed. EKF

is going to be used in this paper. However, the Jacobian cannot capture the high nonlinearity; therefore, the system noise covariance matrix,  $Q$ , will be increased to minimize the error. The Jacobian of the system,  $F$ , is defined as the following:

$$F = \left. \frac{\partial f}{\partial x} \right|_{x=x_k} = \begin{bmatrix} 1 & M_k & 0 & -N_k & F_1 \\ 0 & c & 0 & -s & F_2 \\ 0 & N_k & 1 & -M_k & F_3 \\ 0 & s & 0 & c & F_4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

where:

$$F_1 = \frac{(Tx_{5,k}c-s)x_{2,k}}{x_{5,k}^2} - \frac{x_{4,k}(Tx_{5,k}c-1+c)}{x_{5,k}^2} \quad (6)$$

$$F_2 = -sx_{2,k}T - Tcx_{4,k} \quad (7)$$

$$F_3 = \frac{x_{2,k}(Tx_{5,k}c-1+c)}{x_{5,k}^2} - \frac{(Tx_{5,k}c-s)x_{4,k}}{x_{5,k}^2} \quad (8)$$

$$F_4 = Tcx_{2,k} - Tsx_{4,k} \quad (9)$$

The measurement matrix is already a linear matrix of the form:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

The system and noise measurement vectors are Gaussian with zero mean and the following covariance matrices:

$$Q = 3.3 \times \begin{bmatrix} 10^{-5} & 0 & 0 & 0 & 0 \\ 0 & 10^{-7} & 0 & 0 & 0 \\ 0 & 0 & 10^{-5} & 0 & 0 \\ 0 & 0 & 0 & 10^{-7} & 0 \\ 0 & 0 & 0 & 0 & 10^{-9} \end{bmatrix} \quad (11)$$

And

$$R = 3.3 \times \begin{bmatrix} 10^{-3} & 0 & 0 & 0 & 0 \\ 0 & 10^{-7} & 0 & 0 & 0 \\ 0 & 0 & 10^{-3} & 0 & 0 \\ 0 & 0 & 0 & 10^{-7} & 0 \\ 0 & 0 & 0 & 0 & 10^{-9} \end{bmatrix} \quad (12)$$

The initial condition for the state is:

$$x_0 = [100 \ 10 \ 100 \ 10 \ 0.1]^T \quad (13)$$

The system will have a constant value for  $x_{5,k}$  unless there is a turn, and the filter needs to estimate it.

### 3. METHODOLOGY

#### 3.1 SIF algorithm

The SIF estimate an unrefined estimate during a stage refereed to as the prediction stage. The estimate is called the a priori estimate ( $\hat{\mathbf{x}}_{k|k-1}$ ). Then the filter refine the estimate to the a posteriori estimate ( $\hat{\mathbf{x}}_{k|k}$ ) during the update stage. These steps are summarized as follows:

##### 1- Prediction Stage,

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} &= \hat{f}(\hat{\mathbf{x}}_{k-1|k-1}), \\ \hat{\mathbf{z}}_{k|k-1} &= \hat{\mathbf{x}}_{k|k-1}\end{aligned}\quad (14)$$

##### 2- Update Stage,

$$\begin{aligned}\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + [S_k(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1})], \\ \hat{\mathbf{z}}_{k|k} &= \hat{\mathbf{x}}_{k|k}\end{aligned}\quad (15)$$

where  $S_k$  is a square diagonal matrix with elements defined as:

$$S_{i,k} = \begin{cases} \frac{|z_{i,k} - \hat{z}_{i,k|k-1}|}{\psi_{i,k}} & |z_{i,k} - \hat{z}_{i,k|k-1}| < \psi_{i,k} \\ 1 & |z_{i,k} - \hat{z}_{i,k|k-1}| \geq \psi_{i,k} \end{cases} \quad (16)$$

where  $\psi_{i,k}$  is the boundary layer for the  $i^{th}$  measurement.

#### 3.2 EKF algorithm

The EKF is also a predictor and corrector filter. However, the further calculation is required to obtain the error covariance matrix in it's a priori and a posteriori forms. These will be later used to calculate the correction gain as summarized here:

##### 1- Prediction Stage,

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} &= \hat{f}(\hat{\mathbf{x}}_{k-1|k-1}), \\ \hat{\mathbf{z}}_{k|k-1} &= \hat{\mathbf{x}}_{k|k-1}\end{aligned}\quad (17)$$

##### 2- Update Stage,

$$\begin{aligned}\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + K_k(\mathbf{z}_k - \hat{\mathbf{z}}_{k|k-1}), \\ \hat{\mathbf{z}}_{k|k} &= \hat{\mathbf{x}}_{k|k}\end{aligned}\quad (18)$$

where:

$$\mathbf{K}_k = \mathbf{P}_{k|k-1}(\mathbf{P}_{k|k-1} + \mathbf{R}_k)^{-1} \quad (19)$$

and

$$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k-1}^T + \mathbf{Q}_{k-1} \quad (20)$$

#### 3.3 EKF/SIF algorithm

The proposed filter combines the EKF with the SIF as follows:

##### 1- Prediction Stage,

$$\begin{aligned}\hat{\mathbf{x}}_{k|k-1} &= \hat{f}(\hat{\mathbf{x}}_{k-1|k-1}), \\ \hat{\mathbf{z}}_{k|k-1} &= \hat{\mathbf{x}}_{k|k-1}\end{aligned}\quad (21)$$

##### 2- Update Stage,

$$\text{If } |\mathbf{z}_{i,k} - \hat{\mathbf{z}}_{i,k|k-1}| > \gamma_i \quad (22)$$

Then

use (15) and (16)

reset the value of the  $\mathbf{P}_{0|0}$

Else

use (18) – (20)

Where  $\mathbf{P}_{0|0}$  is 10 times the identity matrix,  $\psi = [29 \ 0.01 \ 29 \ 0.01 \ 0.1]^T \times 10^{-2}$  and  $\gamma = [16 \ 1.6 \ 16 \ 1.6 \ 16]^T \times 10^{-2}$ .

### 4. SIMULATION RESULTS

The EKF, SIF and EKF/SIF are applied to the system in Section (2). Monte Carlo Simulation are conducted with repeated simulation up to 1000 times. Two cases were considered, case 1 for the model of (1) and case (2) by assuming  $x_{5,k}$  is very small to make the system linear. The sampling time is 1 sec. Each simulation has a different noise generation. Bothe the root mean squared error (RMSE) and the maximum absolute error (MAE) are calculated using (23) and (24), respectively, and where recorded for case 1 and case 2 in Table 1 and 2, respectively.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{ns} (x_{Actual,i} - x_{Prediction,i})^2}{ns}} \quad (23)$$

$$MAE = \max(|x_{Actual} - x_{Prediction}|) \quad (24)$$

The Monte Carlo performance of the filters is plotted in Figure 2, and the error performances are plotted in Figure 3 for case 1 and Figure 4 for case 2.

The results show a superior performance for the proposed method compared to the EKF and SIF. Due to the nonlinearity in the system, the SIF performed better then EKF in both cases. This is due to the fact that the Jacobian method does not approximate the high nonlinearity in the system.

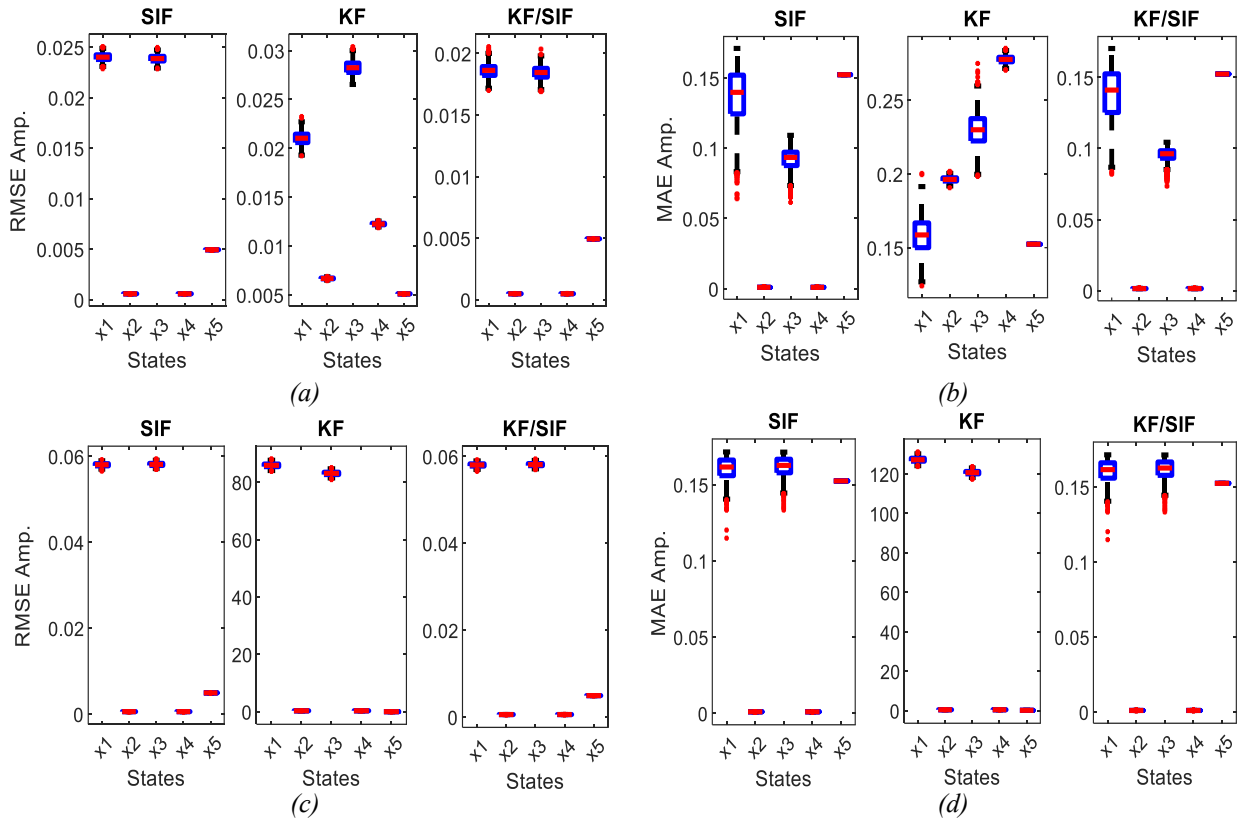
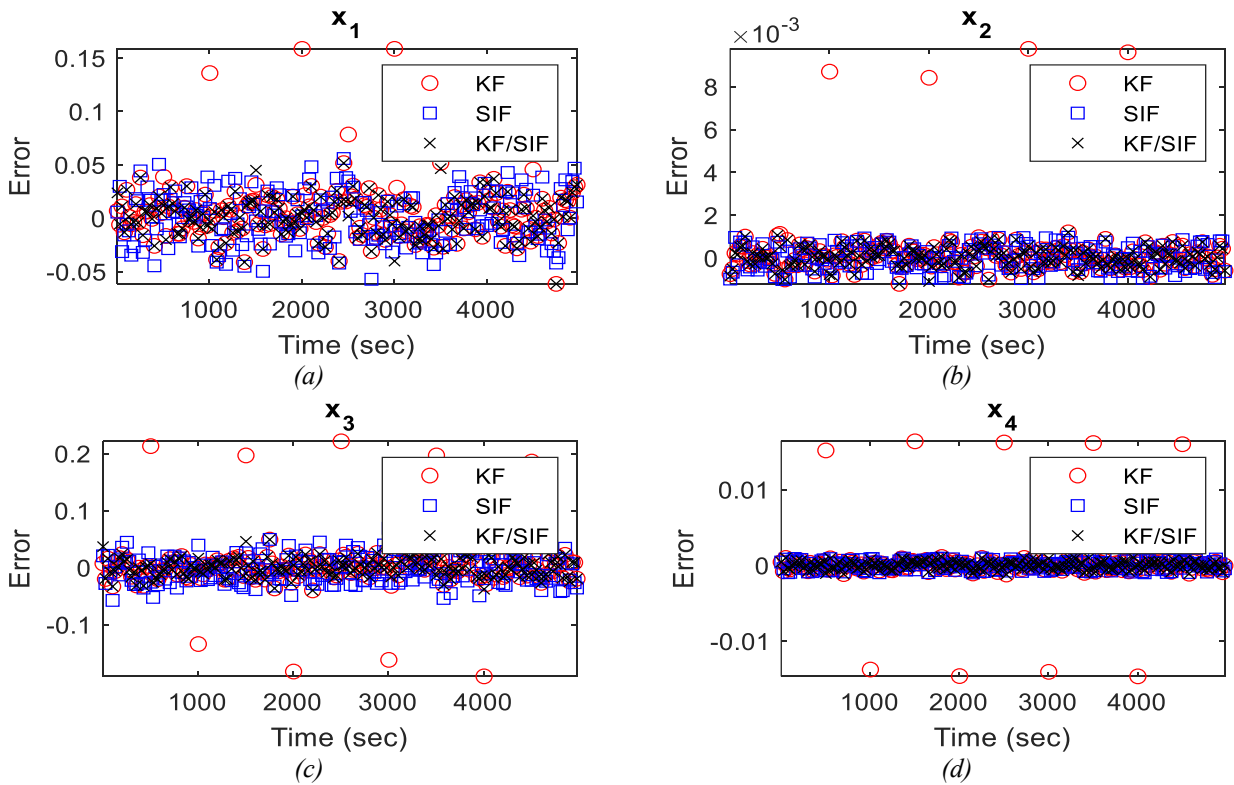


Figure 2. The Monte Carlo Simulation results for (a) RMSE and (b) MAE for case 1, and for (c) RMSE and (d) MAE for case 2.



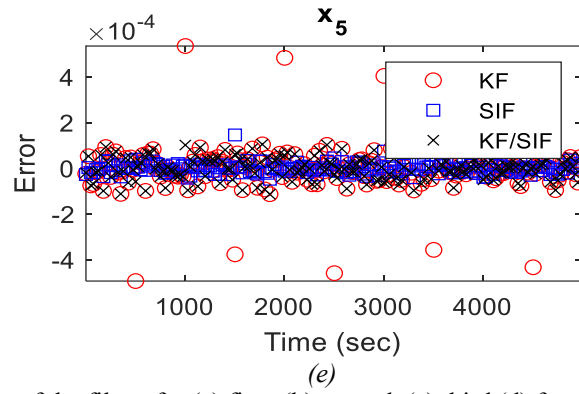


Figure 3. The performance of the filters for (a) first, (b) second, (c) third (d) fourth and (e) fifth states for case 1.

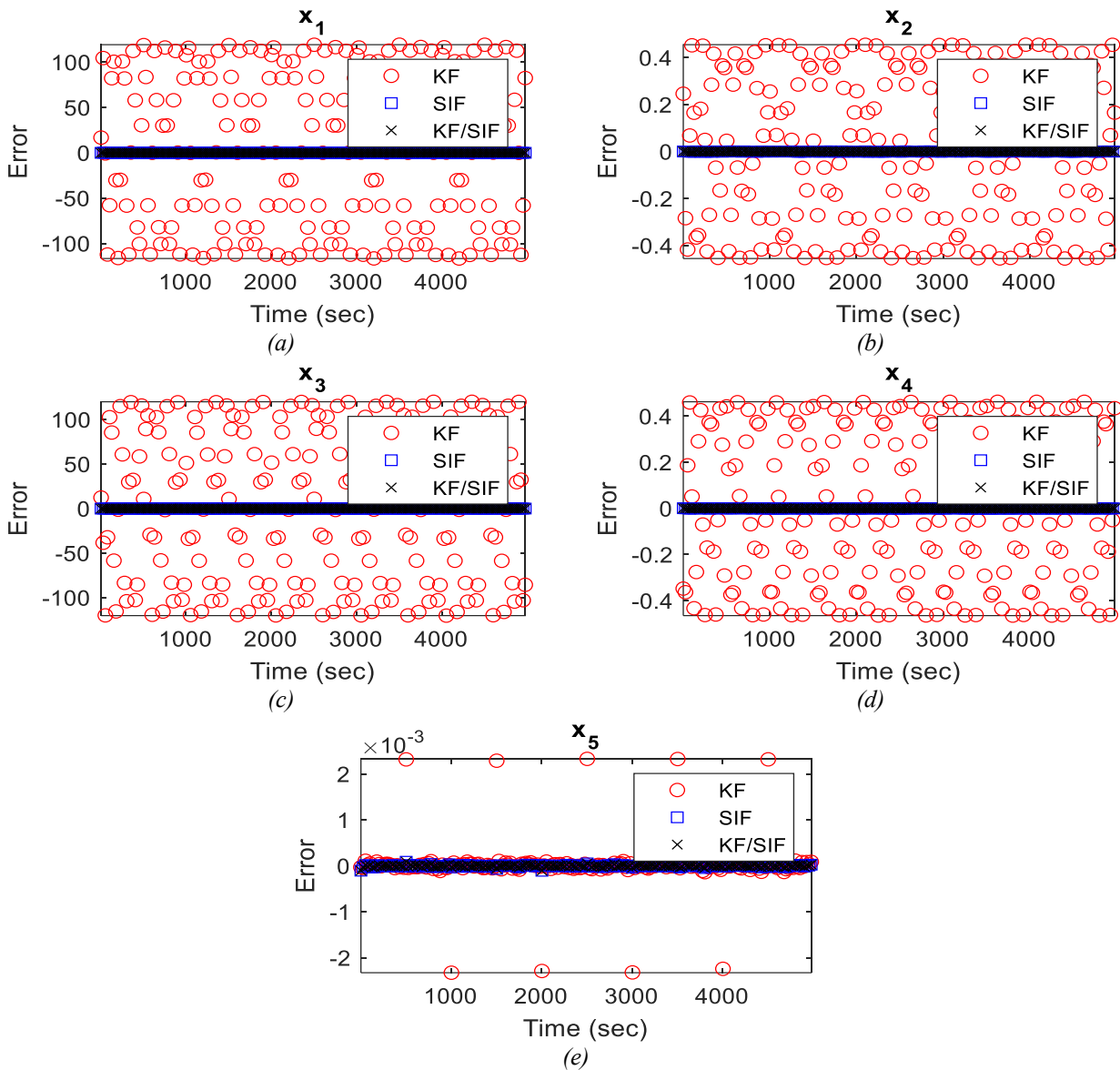


Figure 4. The performance of the filters for (a) first, (b) second, (c) third (d) fourth and (e) fifth states for case 2.

Table 1. RMSE and MAE for case 1

	RMSE in			MAE in		
	SIF	EKF	EKF/SIF	SIF	EKF	EKF/SIF
$x_1(m)$	$2.4 \times 10^{-02}$	$2.0 \times 10^{-02}$	$1.8 \times 10^{-02}$	$1.8 \times 10^{-01}$	$1.6 \times 10^{-01}$	$1.7 \times 10^{-01}$
$x_2(m/s)$	$6.0 \times 10^{-04}$	$6.7 \times 10^{-03}$	$5.0 \times 10^{-04}$	$1.0 \times 10^{-03}$	$2.0 \times 10^{-01}$	$1.5 \times 10^{-03}$
$x_3(m)$	$2.4 \times 10^{-02}$	$2.7 \times 10^{-02}$	$1.7 \times 10^{-02}$	$1.7 \times 10^{-02}$	$2.2 \times 10^{-01}$	$9.2 \times 10^{-02}$
$x_4(m/s)$	$6.0 \times 10^{-04}$	$1.2 \times 10^{-02}$	$5.0 \times 10^{-04}$	$1.0 \times 10^{-03}$	$2.8 \times 10^{-01}$	$1.3 \times 10^{-03}$
$x_5(rad/s)$	$4.9 \times 10^{-03}$	$5.1 \times 10^{-03}$	$4.9 \times 10^{-03}$	$1.5 \times 10^{-01}$	$1.5 \times 10^{-01}$	$1.5 \times 10^{-01}$

Table 2. RMSE and MAE for case 2

	RMSE in			MAE in		
	SIF	EKF	EKF/SIF	SIF	EKF	EKF/SIF
$x_1(m)$	$5.7 \times 10^{-02}$	$8.4 \times 10^{01}$	$5.7 \times 10^{-02}$	$1.7 \times 10^{-01}$	$1.2 \times 10^{03}$	$1.7 \times 10^{-01}$
$x_2(m/s)$	$6.0 \times 10^{-04}$	$3.1 \times 10^{-01}$	$6.0 \times 10^{-04}$	$1.0 \times 10^{-03}$	$4.6 \times 10^{-01}$	$1.0 \times 10^{-03}$
$x_3(m)$	$5.8 \times 10^{-02}$	$8.4 \times 10^{-02}$	$5.8 \times 10^{-02}$	$1.6 \times 10^{-02}$	$1.2 \times 10^{02}$	$1.6 \times 10^{-02}$
$x_4(m/s)$	$6.0 \times 10^{-04}$	$3.3 \times 10^{-01}$	$6.0 \times 10^{-04}$	$1.0 \times 10^{-03}$	$4.7 \times 10^{-01}$	$1.0 \times 10^{-03}$
$x_5(rad/s)$	$4.9 \times 10^{-03}$	$5.1 \times 10^{-03}$	$4.9 \times 10^{-03}$	$1.5 \times 10^{-01}$	$1.5 \times 10^{-01}$	$1.5 \times 10^{-01}$

## 5. CONCLUSION

In this brief paper, a new method that combines the EKF with the SIF is proposed. The resulting filter was tested by estimating a vehicle's position and velocity on a very complex road. The results were compared to both classical strategies (the EKF and the SIF). The results demonstrate a strong performance of the proposed EKF/SIF over the standard filters, even when modeling uncertainties were present. In future work, the algorithm will be tested on higher-order systems with hidden states that have been built for experimentation.

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