RISK MANAGEMENT FOR SUPPLY CHAIN FINANCE AND RAILWAY TRANSPORTATION

MITIGATING DISRUPTION RISKS IN SUPPLY CHAIN FINANCING AND RAILWAY TRANSPORTATION NETWORKS

By

SEYYED HOSSEIN ALAVI, M.S., B.S.,

A Thesis Submitted to the School of Graduate Studies in Partial Fulfilment of the Requirements for the Degree of Doctor of Philosophy
McMaster University

DOCTOR OF PHILOSOPHY (2023)

Business Administration – Management Science

DeGroote School of Business

McMaster University

Hamilton, Ontario, Canada

TITLE: Mitigating Disruption Risks in Supply Chain Financing and Railway Transportation Networks

AUTHOR: Seyyed Hossein Alavi, M.S., B.S.

SUPERVISOR: Dr. Manish Verma

SUPERVISORY COMMITTEE MEMBERS: Dr. Prakash Abad, Dr. Yun Zhou

NUMBER OF PAGES: ix, 188

Supplication

O’ Lord, I thank you for my acquaintance with Cyrus the Great, from whom I learned that respect for others enhances my own dignity.

O’ Lord, I thank you for gifting me the name Hossein, inspiring me to embrace the virtues of freedom and courage from Hossein ibn Ali.

O’ Lord, I thank you for the wisdom gained from knowing Gandhi, teaching me that with firm belief in my abilities, I can develop the capacity to achieve my goals.

O’ Lord, I thank you for the opportunity to travel the world, enriching my life with a love for all humanity.

O’ Lord, I thank you for blessing me with a beautiful and kind sister, who is the author of beautiful stories in my life.

Seyyed Hossein Alavi

با احترام فراوان،

ثمره پنج سال سختی و غربت را به خانواده دوست‌داشتنی‌ام تقدیم می‌کنم که همواره انگیزه‌ی اصلی ادامه‌ی مسیرم بوده‌اند،

و همچنین با افتخار به خاک پاک میهنم، ایران.

With respect,

I dedicate the fruit of five years of hardship to my beloved family, who have always been my main motivation to continue,

and also proudly to my country, Iran.

Abstract:

This dissertation examines the challenges associated with disruptions in supply chain financing and the railway transportation network. The study is divided into six chapters:

In Chapter 1, we introduce the core problems under investigation. Chapter 2 investigates supply chain financing, emphasizing trade credit and bank credit—two predominant external financing mechanisms. Given the inherent uncertainties in demand, interest rates, and supplier credit ratings, this chapter introduces a stochastic programming model accounting for demand uncertainty. Subsequently, a robust optimization program is applied, whose complexity demands a specialized solution methodology. By analyzing a case study centered around a prominent U.S. retailer, the research reveals key insights into decision-making processes related to financing, the effects of bargaining power on portfolio mix and profits, and the relative importance of interest rate uncertainties over supplier credit ratings.

Chapter 3 introduces a game-theoretical model designed to hedge financing risks in supply chains, with a focus on the application of insurance for both trade and bank credits. To support the design of effective supply chain finance contracts, three distinct contracts are developed, aiming to synchronize both financial and material flows within the supply chain. A significant feature of this chapter is the data-driven approach employed to address the potential bankruptcy risks that can arise from borrowing loans. Alongside this, a novel solution algorithm is introduced to solve the proposed non-convex models. A case study involving Ford Motor Company and a Chicago-based retailer enriches the research with real-world context. The findings offer several managerial insights: the strategic advantages of different insurance services vary based on the risk attitudes and profit margins of participants. For example, when a retailer operates with a lower profit

margin, the use of Trade Credit Insurance (TCI) is recommended in conjunction with a risk-seeking retailer, while a risk-averse retailer might diminish the benefits of TCI. Conversely, with high profit margin retailers, the adoption of Payment Protection Insurance (PPI) is advised under all conditions.

In Chapter 4, a game-theoretical model for risk mitigation within railway transportation is introduced. This model addresses random disruptions by employing strategies like repair, re-routing, third-party services, and leasing capacity from competing rail companies. Through a U.S. case study, the efficacy of these strategies is examined, with renting railcars emerging as a particularly potent approach to enhance resilience and reduce third-party expenses. The research further suggests that negotiations extending delivery dates can significantly diminish post-disruption costs.

Finally, Chapter 5 summarizes the primary contributions of this research, laying the groundwork for prospective studies in this domain.

Table of Contents

Tables List

Figures List

Chapter I: Introduction

This thesis encompasses an in-depth analysis of supply chain disruption and transportation problems, providing a systematic framework for handling challenges in these domains. It unfolds in five distinct chapters. This introductory chapter outlines the structure of the book. In the second chapter, we explore the intersection of supply chain network planning and corporate finance risks, providing a foundation for their integrated management. The third chapter introduces a risk management methodology that utilizes insurance services to mitigate the combined risks of supply chain and corporate finance. The fourth chapter discusses strategies for enhancing railway networks following disruptions, with a focus on reducing transportation risk. We wrap up the thesis with a concluding chapter that synthesizes the main findings and insights from the discussions presented in the earlier chapters.

The second chapter focuses on disruption in financial and material flows within the supply chain. There is a critical emphasis on the unpredictable nature of financial elements such as bank interest rates and supplier reliability over different periods. Due to the absence of a predictable pattern in these factors, devising a method that enables buyers to manage financial disruption risks becomes imperative. In this chapter, the focus shifts towards the management of financial resources in long-term planning within supply chains. Recent statistical analyses have illuminated the growing reliance of supply chains, especially Small and Medium-sized Enterprises, on external loans such as trade credit and bank credit. Over the past ten years, there has been a growing trend of using external borrowing to fund supply chains, as evidenced by recent data. For example, in 2017, in Ontario, Canada, external financing was utilized by over 48 percent of small and medium-sized businesses (SMEs). Despite the associated risks, such as potential delayed repayments, reputational damage, and financial disruption to the supplier, trade credit emerges as a favored choice for external financing due to its flexible policies. Large-scale buyers like Walmart leverage their bargaining power to influence supplier behavior, benefiting from trade credit financing. Bank credit is another important source of external financing, and its prevalence is evinced by data from 2009 indicating that approximately 72% of Canadian SMEs utilized it. Sodhi and Tang (2009) and Hahn and Kuhn (2012) proposed quantitative techniques like stochastic programming models for managing these risks. The more recent trend has been addressing financial disruption risk, with specific attention to on-time loan repayments and designing contracts to mitigate bankruptcy risk (Yang et al., 2021). Our study adds a new dimension by considering disruptions from financial variation and trade credit affordability concurrently.

Moreover, the second chapter outlines the challenges accompanying the use of external financing. There exists a potential risk of disruption, and in extreme cases, bankruptcy, if customer demand does not meet the buyer's order volume. The chapter seeks to answer research questions concerning large buyers' financing strategies, management of uncertain financing, the impact of supplier reliability, and the motives behind borrowing excessively.

The third chapter revolves around the impact of unpaid trade credit and bank credit, which can potentially lead to bankruptcy, causing severe consequences for businesses and their supply chains. To mitigate such risks, we explore different financing methods like trade credit insurance and payment protection insurance and their implications. According to a report by the World Bank, in 2008, approximately €5.3 billion in global credit insurance premiums safeguarded €2.6 trillion worth of sales through Trade Credit Insurance (Jones, 2010).

In terms of insurance, we examine Trade Credit Insurance (TCI) and Payment Protection Insurance (PPI), each playing a pivotal role in mitigating financial disruptions. TCI can serve as a safeguard against disruptions when a retailer fails to repay trade credit (Yang et al., 2016). Our research explores this further by considering an optimal TCI contract that not only mitigates disruptions for both parties but also empowers suppliers with the choice to purchase and partially insure the offered trade credit.

PPI, though encouraged for insuring bank credit repayments, has not been extensively studied in the context of supply chain finance disruptions. Our work is among the first to employ PPI to secure the supply chain players' assets, enabling them to hedge against potential bankruptcy disruptions (Ashton and Hudson, 2011).

Furthermore, we define risk as the likelihood of asset loss due to bankruptcy, an often-overlooked disruption in supply chain finance. Previous studies mainly associated bankruptcy with uncertain demand (Yang and Birge, 2018; Dada and Hu, 2008). To address this disruption, we propose a data-driven Value at Risk (VaR) that efficiently manages risk even with scarce data points.

The Hebei company in China stands as an example of successful risk management, having used trade credit insurance to recover losses stemming from a German company's bankruptcy. This chapter aims to provide a comprehensive framework for supply chain players considering loan insurances and introduces three innovative contracts to manage the risk of borrowing loans. We also incorporate a data-driven Value at Risk method to measure risk and meet the retailer's profit target.

The fourth chapter shifts the spotlight towards transportation issues, specifically the random disruption in railway transportation networks which represented approximately 28% of all domestic freight transportation (U.S. Department of Transportation, Federal Railroad Administration, n.d.). We touch upon railway disruption management, specifically the uncertain repair times. Despite several studies on disruption management (Tan et al., 2020; Mohammadi et al., 2017; Fotuhi and Huynh, 2018), the impact of uncertain repair times on mitigation strategies has not been thoroughly understood. Our research uniquely contributes to this understanding, thereby filling an important research gap in this field. It highlights the U.S. freight rail network, which accounts for approximately 28% of all domestic freight transportation. However, these networks are susceptible to disruptions due to natural events, and the mitigation strategies often depend on uncertain repair costs. A Distributionally Robust Optimization approach is suggested to manage such uncertainties. This chapter seeks to determine the optimal contract between railway companies for railcar rent, the ideal policy for managing service leg disruptions, and handling repair cost uncertainty and its impact on the resilience of the railway transportation network.

This research contributes a comprehensive framework for managing financial resources in supply chains and strategies for railway disruption management. It offers insightful strategies that aid supply chain partners and railway operators in their decision-making processes, facilitating improved resilience against disruptions and financial stability. The final chapter concludes the study by proposing various strategies to reduce the risk of disruption in supply chain finance and transportation problems.

Chapter II: a Robust Optimization Approach to Managing Disruption in a Capital-Constrained Supply Chain

2.1. Introduction

Financial resources are vital for the smooth operation of supply chains, which are networks of organizations involved in producing, distributing, and selling goods. These supply chains need to finance production, distribution, and meeting customer demands, all of which impose costs. A primary method to finance these costs is through working capital, defined as an organization's short-term assets minus its short-term liabilities. However, when working capital is not sufficient, external financing becomes necessary. External financing refers to acquiring funds from sources outside of the organization, such as loans or investors. It is interesting that major corporations such as Walmart leverages external financing for its purchasing needs (Zhang and Zhang, 2022). Walmart regularly employs trade credit financing to procure products from its suppliers (Breza and Liberman, 2017), and in fact favors over the capital invested by shareholders (Tsao, 2011). There are two popular types of external loans to finance buyer requirements: trade credit, and bank credit (Razavian et al., 2021; Alavi and Jabbarzadeh, 2018). Each of these financing mechanisms offers distinct advantages, contributing positively to the financial stability of the supply chain.

However, it is crucial to recognize that engaging in these borrowing options also introduces associated risks that need to be carefully managed.

The reliance on external loans to finance supply chains has increased over the last decade, supported by recent statistics. For instance, in Ontario, Canada, more than 48 percent of small and medium-sized enterprises (SMEs) used external financing in 2017 (Statistics Canada, 2018b).

Two popular types of external loans used to finance buyer requirements are trade credit and bank credit. Trade credit is short-term credit offered by suppliers to their customers. It is particularly popular among SMEs and often comes in two forms: net term and cash discount policies. A net term policy allows the buyer to delay repayment up to a predetermined date, while a cash discount encourages early repayment by offering time-dependent discounts.

To mitigate the risk of delays in repayment, insurance companies like Euler Hermes, Atradius, and EDC offer trade credit insurance. This insurance covers the risk of the buyer failing to make timely payments, although it usually only covers a portion of the extended loan. Delay in repayment affects both the buyer’s credit rating, which is a measure of their creditworthiness, and the financial stability of the supplier.

Bank credit involves banks extending loans based on a borrower’s credit rating and is another popular financing method. For example, around 72% of Canadian SMEs had access to bank credit in 2009. It’s worth noting that larger corporations also commonly use external sources of financing. For example, Walmart has utilized trade credit financing to source products from suppliers.

When a buyer has significant bargaining power, it can influence terms and conditions. Suppliers often prefer to collaborate with large companies due to the advantages of timely payments, the opportunity to introduce themselves to new customers, reduced demand uncertainty, and the ability to compete more effectively with other competitors. Hence, it may lead suppliers to follow their decisions. For example, Walmart pharmacies renewed their contract with CVS Health's Prescription Benefit Manager (PBM) to maintain low prices after a pricing dispute. This study is also relevant for SMEs and can be applied to a range of products.

One major concern with external financing is the obligation to repay the borrowed funds, which can be disrupted if customer demand falls short of the number of products ordered by the buyer. Financial disruption for a buyer can stem from two sources. The first is a change in interest rates, the cost of borrowing money. Although fluctuations in interest rates have become common during the post-COVID period, it was expected to observe some changes in interest rates each year, even before the COVID era. The second is uncertainty in a supplier's rating, which measures the reliability and performance of a supplier.

Predicting financial factors such as bank interest rates and the reliability of a supplier in offering trade credit can be complex, especially in multi-period planning, a strategy that spans multiple time periods. Banks might adjust loan interest rates based on various circumstances, making it difficult to estimate this factor using a probability distribution function, a mathematical function that provides the probabilities of different outcomes. Likewise, supplier reliability may change over time, making it equally difficult to estimate.

Given these complexities, we used a robust optimization approach in our study. Robust optimization is a form of optimization that is designed to perform well under various kinds of uncertainty. It's particularly useful when it's difficult to fit a probability distribution function for financial factors. This approach also considers the buyer's attitude toward risk, or their willingness to take or avoid risk, which is crucial for managing financial disruption in supply chains. The inclusion of the buyer's risk attitude in our analysis is crucial, as it directly influences the approach to managing financial disruptions in the supply chain. Different buyers may exhibit varying degrees of risk tolerance, impacting their preferences for financing options and strategies for mitigating financial risks. A risk-averse buyer, for instance, might prefer stable financing sources, even if they are costlier, to minimize the risk of disruption. Conversely, a risk-tolerant buyer could choose more volatile financing options, accepting higher risk in exchange for potential cost savings.

The proposed study focuses on financial disruptions within supply chain networks, specifically focusing on the challenges faced by a buyer. Typically, buyers have interactions with a variety of suppliers and financial institutions. This situation turns our study focus into a broader supply chain network problem, where we need to make decisions about both physical goods and finances. Our study highlights the importance for buyers to not only obtain sufficient financing but also to implement strategies that strengthen the supply chain's resilience against financial disruptions.

Our research is focused on answering some important questions about how big companies, with multiple retail stores, manage their money. We want to know:

- What strategies do these big companies use to make sure they have enough money to keep their stores stocked?
- How can they protect themselves financially over longer periods of time?

- How important is it to work with reliable suppliers who can offer flexible payment terms?
- Why might a company choose to borrow more money than it immediately needs?

The main goal of our study is to find ways to make supply chains—essentially, the whole process of getting products from manufacturers to store shelves—as financially stable and cost-effective as possible. We explore several strategies, such as: 1) planning ahead to deal with financial uncertainties, 2) setting aside money for future needs, 3) partnering with dependable suppliers who offer flexible payment options, 4) creating a flexible system that can handle financial ups and downs.

We're breaking new ground by developing a unified approach for big companies to manage the finances of their retail stores. Our study particularly focuses on big companies that have more negotiating power when it comes to terms and conditions with their suppliers.

To dig into these issues, we used some complex mathematical models. These models help us simulate what happens in supply chains when you consider both the flow of money and the flow of products, all while taking into account uncertainties like fluctuating demand or unreliable suppliers. Because these models can get complicated, we've developed some specialized techniques to solve them.

We applied these models to real-world data from Walmart stores in Pennsylvania. This allowed us to study how a company's approach to financial risk affects its choices between different types of loans. We also looked at how much influence a big company like Walmart has in negotiations, and how changing interest rates compare to the reliability of suppliers in terms of importance.

A brief summery of our findings is presented as follows. We show that a risk-averse buyer prefers bank credit, while the risk-seeker seeks more trade credit. A risk-seeker decision maker adopts longer net date, whereas the risk-averse prefers longer repayment period to payoff bank credit. In addition, this study represents that the proposed robust optimization method can set better wholesale price ceiling compared to stochastic programming. According to the results, higher bargaining power reduces the buyer required financial loans. It is also suggested that the cooperation with higher credit rated suppliers could help the buyer financially.

The rest of the chapter is organized as follows. Section 2.2 reviews the relevant literature, while Section 2.3 contains a schematic of the research question and the explicit managerial problem. Section 2.4 develops the stochastic programming model and the robust optimization programs, while the need to develop a solution methodology and the resulting steps are outlined in Section 2.5. Several problem instances developed using the realistic infrastructure is analyzed and pertinent insights are developed in Section 2.6, while Section 2.7 provides the conclusions, contributions, and directions of future research.

2.2. Literature Review

For expositional reasons and to position this work in the context of the existing literature, the relevant literature is organized under two themes: first, disruption risk strategies in supply chain finance; and second, intersection of finance and supply chain planning.

2.2.1. Disruption risk strategies in supply chain finance

Our study contributes to developing financial disruption risk strategies in supply chain. We categorize the related literature as follows.

The first stream focuses on financial risk in a supply chain. There are two types of financial risk in a supply chain: first, financial flow risk that is associated with variation in demand, exchange rate, etc.; and second, financial disruption risk triggered by the inability of the buyer/supplier to meet the financial obligations. An array of quantitative methods has been developed to tackle financial flow risk. Sodhi and Tang (2009) proposed a stochastic programming model for matching financial flow and liabilities in a supply chain under uncertain demand. Hahn and Kuhn (2012) used the technique to tackle demand variation to make better financial decisions – including investment, cash flow management, and satisfying financial requirements. Besides, financial disruption risk has recently attracted a lot of attention since supply chains are interested in on-time loan repayments, which in turn motivated companies to design a contract to decrease bankruptcy risk. Yang et al., (2021) designed a contract between a supplier and an insurer who provides trade credit insurance. The insurer evaluates the buyer ability to meet the loan obligation. Differently, our study considers both the financial risks in supply chain. It allows buyer to manage financial variation and trade credit affordability simultaneously.

The second related stream investigates the buyer risk attitude regarding supply chain finance. Yousefi and Pishvaee (2018) looked at west Asia and applied fuzzy programming to capture uncertain exchange rates affected by wars and political issues. Their methodology allows the decision maker to adjust risk by changing fuzzy numbers. Guan et al., (2022) applied a hybrid robust and stochastic optimization for transporting fresh products when uncertain factors such as transportation cost, demand and deterioration rate enables buyer to tackle product flow uncertainty. Yunzhang et al., (2021) modeled the uncertainty of account receivables by a multi-period trinomial tree to calculate the probability of default. Our work differs from theirs by developing a hybrid robust optimization that manages both financial and product flows uncertainty for various risk attitudes.

The third related stream concentrates on saving strategy to face financial disruption in supply chain. Companies conserve money and inventories in the real world to manage their requirements in the future. Chen and Zhang (2021) proposed a stochastic optimization for financing a multi-period multi-product problem. Although their model benefits from ordered inventory to fulfill the subsequent demands, it does not save money for future expenses. Wang and Huang (2019) designed a supply chain network by financing the facilities and saving money for the upcoming expenses. However, their applied saving model is independent of on-time borrowing and reimbursing loans. Hence, the saved money cannot be used to reduce the loans interest. To improve the buyer efficiency, our work presents a mathematical programming model that allow buyer to borrow loans for making profit and save the acquired profit for the subsequent expenses. It also relates saving money and inventory holding simultaneously to satisfy demand in a multi-period planning.

The next related strategy is supplier reliability. Trade credit affordability depends on supplier financial situation. A high credit rated supplier may continue supporting buyer financially by offering trade credit. In contrast, a low credit rated supplier may cease providing trade credit to reduce the risk of unpaid trade credit reimbursement. Jiang et al., (2022) measure the trustworthy of SME using fuzzy numbers. Their study assists SME when bank is reluctant to lend loan to low credit rated companies. A noteworthy engagement in supply chain finance is by Kouvelis and Zhao (2018), who investigated the impact of credit rating on the risk of bankruptcy for both the buyer and the supplier in a single period planning. Recently, Zhu et al., (2019) and Wang et al. (2022) made use of machine learning methods to forecast SME credit risk for upcoming period. However, to the best of our knowledge, the role of credit rating uncertainty in a multi-period setting has not been explored. For the first time, this study tackles the disruption risk caused by supplier inability to extend trade credit in a multi-period planning.

2.2.2. Intersection of finance and supply chain planning

Given the context, we review works that have focused on the integration of finance and supply chain planning.

First, this study develops a centralized framework for managing retail outlets of a buyer financially. Zhong et al., (2018) proposed a mathematical framework to reduce the inventory cost of a supply chain network design problem which contains trade credit cost. Razavian et al., (2021) integrated raw material procurement, production, inventory and financing decisions in a supply chain network design problem. However, they assumed that the supply chain facilities are responsible for taking financial decisions independently. Differently, our study contributes to the literature by managing financial decisions of a buyer who distributes the products to multiple retail outlets.

Second, we study the optimal time of loan repayment in a multi-period planning. Alavi and Jabbarzadeh (2018) developed a mixed integer programming model to find repayment times for the first time. Recently, Hua et al., (2022) discussed loan discount rate and optimal repayment time for a dynamic discount contract. However, we analyze the appropriate bank credit and trade credit repayment times based on buyer risk attitude.

This study also investigates the impact of uncertain interest rate on buyer purchases. Zhao et al., (2022) investigates demand recovery investment for a capital constrained retailer when loans interest rate change after meeting disruption. They supposed that the interest rate values take predetermined numbers. Arani and Torabi (2018) developed a multi-period financial planning for a supply chain network. However, they assumed that the interest rate remains the same during planning horizon. For the first time, we present a robust optimization model to tackle uncertain interest rate for financing a capital constrained buyer.

The next stream is concentrated on bargaining power of supply chain players. Financial superiority of a supply chain party over other components is not observed in the existing literature. Kouvelis and Zhao (2012) designed a contract between a supplier and a retailer to manage bank credit and trade credit in supply chain. Kouvelis and Zhao (2016) presented a revenue sharing contract where bank credit is available for financing the supply chain players. Taleizadeh et al., (2022) proposed a model to consider bargaining power of a low-quality manufacture compared to high quality one when the financial resources are limited. Zhang et al., (2022) evaluated the bargaining power of a small or medium sized manufacturer for cooperating with a capital constrained or capital-sufficient retailer. Our work differs from theirs by considering a large size buyer as decision maker who has more bargaining power.

Finally, we study financing of transportation and inventory holding cost. Inventory holding cost is one of the major drivers of supply chain profit (Solyali et al., 2012; Emtehani et al, 2021). Note that offering trade credit can assist a supply chain financially, it does transfer the associated inventory holding cost to the buyer. Though Serrano et al. (2012) commented on the role of inventory policy on supply chain finance, financing the inventory holding has been neglected in the literature. Moreover, most of works in the field of transportation finance are related to road and infrastructure financing, such as Bröcker et al., (2010). To the best of our knowledge, bank credit has not used for transportation in supply chain planning.

2.3. Problem description

Figure 2-1 depicts the schematic of the managerial problem being investigated, where the uncertain and deterministic flows are represented by dash and straight lines, respectively. For the problem of interest, the supply chain consists of a bank, suppliers, customers, and a buyer (with distribution centers and retailer outlets). There are multiple retail outlets where customers can fulfill their demands. However, considering direct shipping from suppliers to retail outlets increases the complexity of the problem and makes the organization's operations more challenging. In addition to the payment flow arrows, the product and repayment flows are represented by the solid circle end and solid arrow end, respectively. At the beginning of *Period 1*, the buyer can use trade credit and bank credit to finance purchases and satisfy retailer demand using the products received from the suppliers. At the end of the period, the remaining inventory and cash are transferred to the next period. The periodical cash and inventory transfers are shown by solid square end flows in Figure 2-1. Note that the buyer must repay the received bank credit, trade credit and any accrued interests in the future periods. The network complexity increases in each subsequent *period 2* to *n-1*, where *n* is the number of periods in the planning horizon. For example, if the buyer receives trade credits from *period 1* up to *period n-1*, it should select one of the several periods between *2* to *n* for the repayment. Hence, the buyer cannot receive any loan in *period n*, and should repay all the loans and associated interest before the end of *period n*. Thus, the managerial problem of interest is to determine the optimal mix of trade credit and bank credit over a multi-period setting – given uncertainty in demand, interest rate, and supplier credit rating, such that the profit of the supply chain is maximized.

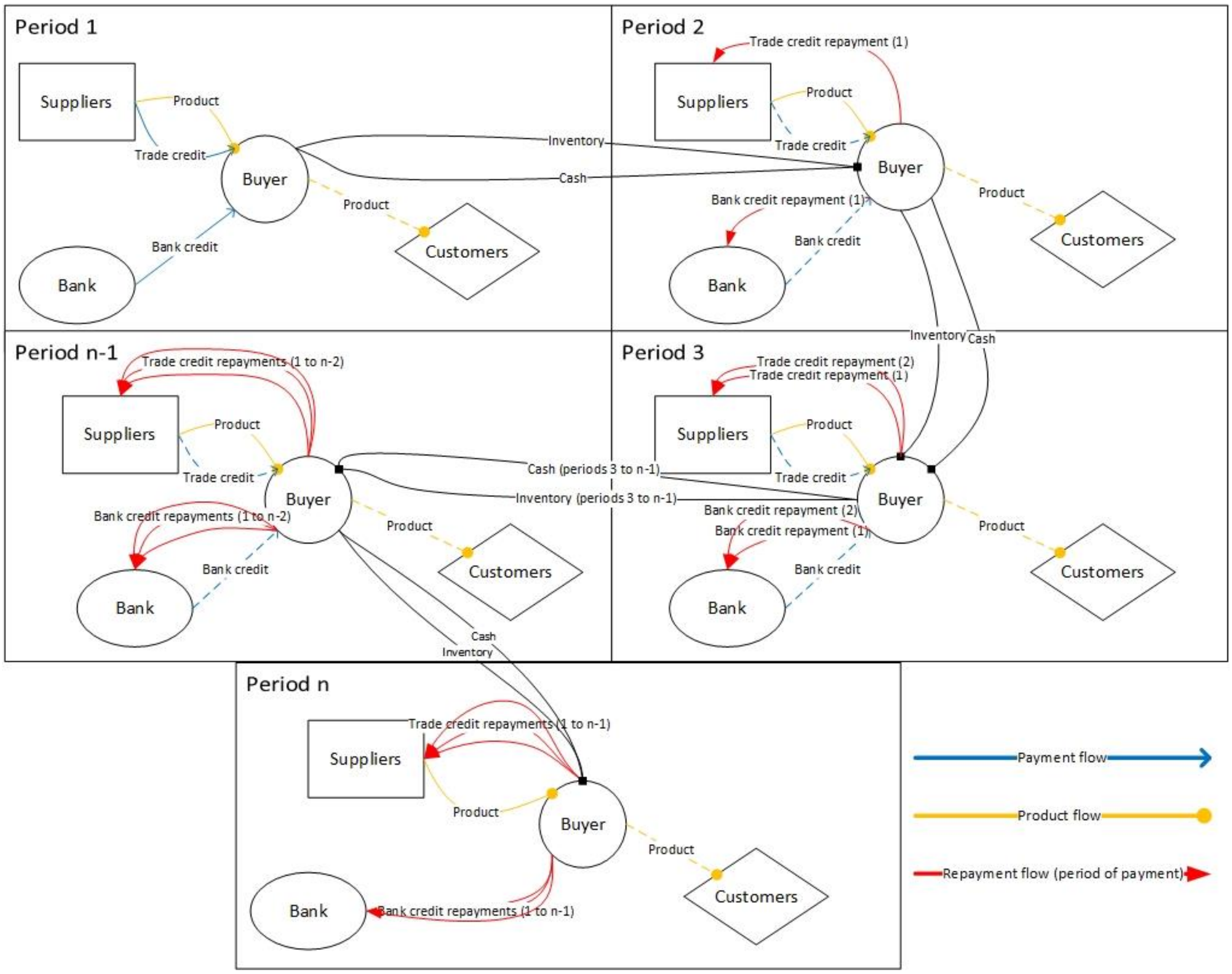


Figure 2-1: schematic of material and financial flows in a supply chain network

2.4. Mathematical Program

In this section, we first develop a stochastic model to represent the managerial problem given demand uncertainty, and then propose a hybrid optimization model to tackle the two types of uncertainty related to supply chain finance.

2.4.1. Stochastic supply chain finance model

In this subsection, a mixed-integer programming model is proposed from the perspective of the buyer, and one that aims to improve supply chain efficiency by integrating the financial and product flows given demand uncertainty. We make the following assumptions: first, the buyer can select the customers, but needs to ensure that demand is satisfied from a buyer site; second, the buyer incurs inventory holding cost, and can utilize the inventory to meet future demands; third, supplier trade credit does not entail cash advances but only product delivery (and sales), and cannot exceed what is financially viable for the supplier; fourth, buyer has access to bank credit through financial institutions, and should pay interest expense at the start of a period, which is based on forecasted rates; fifth, bank credit can be used to pay inventory holding, transportation and procurement costs, while trade credit can only be used to procure products; and sixth, Given the complexity of fitting a probability distribution to the proposed mathematical programming model, we can alternatively consider a set of scenarios that approximate the demand probability distribution based on available data, experts opinion and etc.,. We next introduce the notations, and then outline the stochastic model.

Sets and indices

I: set of suppliers, indexed by *i*;

J: set of distribution centers, indexed by *j*;

K: set of retail outlets, indexed by *k*;

T: set of planning horizons, indexed by *t* and *p*;

Ω: set of scenarios, indexed by ξ.

Parameters

n: number of planning periods;

f_{ij}: fixed cost of transportation from supplier *i* to distribution center *j*;

c_{jk}: fixed cost of transportation from distribution center *j* to retail outlet *k*;

ε_j: per unit inventory holding cost at distribution center *j*;

l_k: per unit selling price of product at retail outlet *k*;

e_i: unit purchase cost from supplier *i*;

b_i: credit rating of supplier *i*;

$\Psi_i(b_i)$: the probability of supplier *i* going bankrupt, which is a decreasing function of b_i;

u_i: penalty rate for delay in repayment of trade credit received from supplier *i*;

o_i: maximum trade credit offered by supplier *i*;

h_i: permissible delay in repayment without interest charges offered by supplier *i*;

r: bank interest rate;

d_t^{ξ}: demand in period *t* under scenario ξ;

φ_k: fraction of total demand at retail outlet *k*;

a_j: capacity of distribution center *j*;

π^{ξ}: probability of scenario ξ.

Decision Variables

$$G_{ijt}^{\xi} = \begin{cases} 1 \text{ if product is shipped from supplier } i \text{ to distribution center } j \text{ in period } t \text{ under scenario } \xi \\ 0 \text{ otherwise} \end{cases};$$

$S_{jkt}^{\xi} =$

$\begin{cases} 1 \text{ if product is shipped from distribution center } j \text{ to retail outlet k in period } t \text{ under scenario } \xi \\ 0 \text{ otherwise} \end{cases};$

$Q_{jt}^{\xi} =$ inventory at the end of period *t* in distribution center *j* under scenario ξ;

$X_{it}^{\xi} =$ trade credit extended by supplier *i* at the end of period *t* under scenario ξ;

$Y_{itp}^{\xi} =$ amount of repayment to supplier *i* in period *p* for trade credit received in period *t* under scenario ξ;

$W_{ijt}^{\xi} =$ amount shipped from supplier *i* to distribution center *j* in period *t* under scenario ξ;

$Z_{t}^{\xi} =$ bank credit received to purchase products in period *t* under scenario ξ;

$V_{t}^{\xi} =$ bank credit received towards transportation and inventory in period *t* under scenario ξ;

$\Gamma_{t}^{\xi} =$ total bank credit received in period *t* under scenario ξ;

$M_{tp}^{\xi} =$ amount of repayment to bank in period *p* for trade credit received in period *t* under scenario ξ;

$\Theta_{t}^{\xi} =$ cash flow assigned to purchase products in period *t* under scenario ξ;

$\Xi_{t}^{\xi} =$ cash flow assigned towards transportation and inventory in period *t* under scenario ξ.

The components of the objective function are as follows:

Total Revenue: $\sum_{j\in J}\sum_{k\in K}\sum_{t\in T}\sum_{\xi\in\Omega} l_k \varphi_k d_t^{\xi} S_{jkt}^{\xi}$ (2-1)

Variable Costs: $\sum_{t\in T}\sum_{\xi\in\Omega}\left(\Theta_t^{\xi} + \Xi_t^{\xi}\right)$ (2-2)

Trade credit repayment: $\sum_{i\in I}\sum_{t\in T|t<p}\sum_{p\in T}\sum_{\xi\in\Omega} Y_{itp}^{\xi}$ (2-3)

Bank credit repayment: $\sum_{t\in T|t<p}\sum_{p\in T}\sum_{\xi\in\Omega} M_{tp}^{\xi}$ (2-4)

Equation (2-1) represents the total revenue obtained by satisfying demand at the selected retail outlets, while (2-2) depicts the total cash assigned to purchase products from the suppliers and pay for the transportation and inventory holding. Equations (2-3) and (2-4) account for the repayment of trade and bank credit, respectively. Note that in both instances the actual credit (loan) is extended in period *t* while the repayment takes place in period *p*, and where the latter must chronologically be after the former. Incorporating the probability of various demand scenarios, the total profit from the resulting supply chain network could be depicted as follows:

(SP)

$$Max \quad \sum_{\xi\in\Omega}\pi^{\xi}\left[\sum_{t\in T}\left(\sum_{j\in J}\sum_{k\in K}l_k\varphi_k d_t^{\xi}S_{jkt}^{\xi}-\Theta_t^{\xi}-\Xi_t^{\xi}\right)-\sum_{t\in T|t<p}\sum_{p\in T}\left(M_{tp}^{\xi}+\sum_{i\in I}Y_{itp}^{\xi}\right)\right] \tag{2-5}$$

Subject to:

$$\sum_{j\in J}S_{jkt}^{\xi}\le 1 \qquad k\in K, t\in T, \xi\in\Omega \tag{2-6}$$

$$Q_{jt}^{\xi}+\sum_{k\in K}\varphi_k d_t^{\xi}S_{jkt}^{\xi}\le Q_{j,t-1}^{\xi}+\sum_{i\in I}W_{ijt}^{\xi} \qquad j\in J, t\in T, \xi\in\Omega \tag{2-7}$$

$$\Theta_p^{\xi}+\Xi_p^{\xi}\le\Gamma_p^{\xi}+\sum_{t\in T|t<p}\left(\Gamma_t^{\xi}+\sum_{j\in J}\sum_{k\in K}l_k\varphi_k d_t^{\xi}S_{jkt}^{\xi}-\sum_{i\in I}Y_{itp}^{\xi}-M_{tp}^{\xi}-\Theta_t^{\xi}-\Xi_t^{\xi}\right)$$
$$p\in T, \xi\in\Omega \tag{2-8}$$

$$\sum_{i\in I}\sum_{j\in J}e_iW_{ijt}^{\xi}\le\sum_{i\in I}(1-\Psi_i(b_i))X_{it}^{\xi}+Z_t^{\xi}+\Theta_t^{\xi} \qquad t\in T, \xi\in\Omega \tag{2-9}$$

$$\sum_{j\in J}\left(\sum_{i\in I}f_{ij}G_{ijt}^{\xi}+\sum_{k\in K}c_{jk}S_{jkt}^{\xi}+\varepsilon_jQ_{jt}^{\xi}\right)\le V_t^{\xi}+\Xi_t^{\xi} \qquad t\in T, \xi\in\Omega \tag{2-10}$$

$$X_{it}^{\xi}\le o_{it} \qquad i\in I, t\in T, \xi\in\Omega \tag{2-11}$$

$$X_{it}^{\xi}\le\sum_{p\in P|p>t}Y_{itp}^{\xi}\left(\frac{1}{1+u_i}\right)^{max(0,p-t-h_i)} \qquad i\in I, t\in T, \xi\in\Omega \tag{2-12}$$

$$\Gamma_t^\xi \leq \sum_{p\in P|p>t} M_{tp}^\xi \left(\frac{1}{1+r}\right)^{p-t} \qquad t \in T, \xi \in \Omega \qquad (2\text{-}13)$$

$$\sum_{i\in I} W_{ijt}^\xi \leq \sum_{i\in I} a_j G_{ijt}^\xi \qquad j \in J, t \in T, \xi \in \Omega \qquad (2\text{-}14)$$

$$Z_t^\xi + V_t^\xi = \Gamma_t^\xi \qquad t \in T, \xi \in \Omega \qquad (2\text{-}15)$$

$$G_{ijt}^\xi \in \{0,1\} \qquad i \in I, j \in J, t \in T, \xi \in \Omega \qquad (2\text{-}16)$$

$$S_{jkt}^\xi \in \{0,1\} \qquad j \in J, k \in K,\ t \in T, \xi \in \Omega \qquad (2\text{-}17)$$

$$Q_{jt}^\xi \geq 0 \qquad j \in J, t \in T, \xi \in \Omega \qquad (2\text{-}18)$$

$$W_{ijt}^\xi \geq 0 \qquad i \in I, j \in J, t \in T, \xi \in \Omega \qquad (2\text{-}19)$$

$$X_{it}^\xi \geq 0 \qquad i \in I, t \in T, \xi \in \Omega \qquad (2\text{-}20)$$

$$Z_t^\xi \geq 0 \qquad t \in T, \xi \in \Omega \qquad (2\text{-}21)$$

$$V_t^\xi \geq 0 \qquad t \in T, \xi \in \Omega \qquad (2\text{-}22)$$

$$M_{tp}^\xi \geq 0 \qquad t \in T, p \in T, \xi \in \Omega \qquad (2\text{-}23)$$

$$Y_{itp}^\xi \geq 0 \qquad i \in I, t \in T, p \in T, \xi \in \Omega \qquad (2\text{-}24)$$

$$\Theta_t^\xi \geq 0 \qquad t \in T, \xi \in \Omega \qquad (2\text{-}25)$$

$$\Xi_t^\xi \geq 0 \qquad t \in T, \xi \in \Omega \qquad (2\text{-}26)$$

$$\Gamma_t^\xi \geq 0 \qquad t \in T, \xi \in \Omega \qquad (2\text{-}27)$$

Constraints set (2-6) assigns a distribution center to each retail outlet. Constraints set (2-7) ensures that the demand at the retail outlet and the ending inventory in each period is does not exceed the opening inventory and the purchases made in that period. (2-8) ensures that the amount spent on paying for purchases, transportation and inventory does not exceed the available budget. (2-9)-(2-13) and (2-15) represent corporate financing constraints and assist the buyer in managing cash flows. (2-9) forces the model to pay for the purchasing cost using the received trade credit, bank credit and assigned money to purchase products. The assigned money comes from the revenue of selling products in past periods. Due to bargaining power, the buyer can influence the price e_i and negotiate an agreement with supplier i. The term $(1 - \Psi_i(b_i))X_{it}^{\xi}$ is the expected value of affordable trade credit by each supplier, where the higher value of supplier bankruptcy probability $\Psi_i(b_i)$ increase the risk of supplier inability in a multi-period planning, thereby limiting available trade credit for buyer. (2-10) ensures that payment for transportation and inventory is done through bank credit and revenue from product sales from past periods. (2-11) bounds the trade credit offered by the supplier in any period by the maximum, while (2-12) calculates the trade credit repayment to the supplier. (2-12) impose the penalty rate for delay in trade credit repayment if the repayment time exceeds the permissible delay in repayment (i.e., $(p - t - h_i)$ takes positive values). (2-13) enforces the repayment of bank credit. (2-12)-(2-13) consider time value of money for the loan repayments. (2-14) places a capacity on the distribution centers, and (2-15) determines the amount of bank credit being used for purchasing and transportation & inventory. Finally, constraints (2-16)-(2-27) show the domain of the variables.

2.4.2. Robust optimization model

The uncertain demand can follow a probability distribution, however, the same might not be possible for uncertainty associated with the credit rating of the supplier and the interest rates. Hence, in this section, we make use of robust optimization to capture the two types of uncertainties, which in turn would augment the stochastic programming model outlined in Section 2.4.1. Thus, we end up with a hybrid robust optimization framework Jabbarzadeh et al., (2016). The method is termed hybrid robust optimization because it employs both stochastic programming and robust optimization simultaneously to address uncertainty. The robust optimization models independent uncertain factors (i.e., credit rating and interest rate) which could be managed by the buyer risk attitude separately. We make use of the methodology developed by Bertsimas and Sim (2003, 2004) to overcome uncertainty where the distribution function can be estimated by uncertainty set. For expositional reasons, we have provided the pertinent details in Appendix A. We next outline the formulation of uncertainty around credit rating of the supplier, and then that around interest rates.

2.4.2.1. Formulating credit rating uncertainty

The uncertain parameter, $\Psi_i(b_i)$, assumes a value over the interval $\left[\widetilde{\Psi}_i(b_i) - \widehat{\Psi}_i(b_i), \widetilde{\Psi}_i(b_i) + \widehat{\Psi}_i(b_i)\right]$ for each supplier, where $\widetilde{\Psi}_i(b_i)$ and $\widehat{\Psi}_i(b_i)$ are considered as the nominal values of $\Psi_i(b_i)$ and the deviation of the nominal value respectively. The budget uncertainty χ^b, i.e., conservatism degree, assumes values between zero and the number of suppliers (see Appendix A for more details). The budget uncertainty χ^b enables us to integrate the buyer's risk preference into the model concerning the supplier credit rating. Credit rating uncertainty is captured in constraints set (2-9) in **(SP)** and can be replaced by the following four constraints in the robust formulation.

$$\sum_{i\in I}\sum_{j\in J} e_i W_{ijt}^{\xi} + \sum_{i\in I}\mu_i^1 + \beta_{t\xi}^1\chi^b \le \sum_{i\in I}\left(1-\widetilde{\Psi}_i(b_i)\right)X_{it}^{\xi} + Z_t^{\xi} + \Theta_t^{\xi} \qquad t\in T, \xi\in \Omega \tag{2-9a}$$

$$\beta_{t\xi}^1 + \mu_i^1 \ge \widehat{\Psi}_i(b_i)X_{it}^{\xi} \qquad i\in I, t\in T, \xi\in\Omega \tag{2-9b}$$

$$\beta_{t\xi}^1 \ge 0 \qquad t\in T, \xi\in\Omega \tag{2-9c}$$

$$\mu_i^1 \ge 0 \qquad i\in I \tag{2-9d}$$

2.4.2.2. Formulating interest rate uncertainty

The second uncertain parameter, interest rate r , assumes a value over the interval $[\tilde{r}-\hat{r},\tilde{r}+\hat{r}]$, where $\tilde{r}$ represents the nominal values of r, while $\hat{r}$ denotes the deviation from these nominal values. The budget uncertainty χ^r assumes a value between zero and one. Interest rate uncertainty is captured in constraints set (2-13) in **(SP)** and can replaced by the following four constraints in the robust formulation.

$$\Gamma_t^{\xi} + \mu^2 + \beta_{t\xi}^2 \Gamma^r \leq \sum_{p\in P|p>t} M_{tp}^{\xi}\left(\frac{1}{1+\tilde{r}}\right)^{p-t} \qquad t\in T, \xi\in \Omega \qquad \text{(2-13a)}$$

$$\beta_{t\xi}^2 + \mu^2 \geq M_{tp}^{\xi}\left(\frac{1}{1+\hat{r}}\right)^{p-t} \qquad t\in T, p\in T, \xi\in \Omega \qquad \text{(2-13b)}$$

$$\beta_{t\xi}^2 \geq 0 \qquad t\in T, \xi\in \Omega \qquad \text{(2-13c)}$$

$$\mu^2 \geq 0 \qquad \text{(2-13d)}$$

2.4.2.3. Robust formulation

Hence, the robust formulation for the stochastic programming model outlined in Section 2.4.1 would be as follows:

(RO)

$$Max \quad \sum_{\xi\in\Omega} \pi^{\xi}\left[\sum_{t\in T}\left(\sum_{j\in J}\sum_{k\in K} l_k \varphi_k d_t^{\xi} S_{jkt}^{\xi} - \Theta_t^{\xi} - \Xi_t^{\xi}\right) - \sum_{t\in T|t<p}\sum_{p\in T}\left(M_{tp}^{\xi} + \sum_{i\in I} Y_{itp}^{\xi}\right)\right] \qquad \text{(3-5)}$$

Subject to:

(3-6)-(3-8), (3-9a)-(3-9d), (3-10)-(3-12), (3-13a)-(3-13d), (3-14)-(3-27).

2.5. Solution methodology

In this section, we first discuss the mathematical structure of **(RO)**, and then outline the solution methodology. We have developed Proposition 1 to substantiate the application of the solution algorithm to our problem. Proposition 1 demonstrates that our problem falls into the category of NP-hard problems, highlighting its inherent complexity and the challenges associated with solving it.

Proposition 1: **(RO)** is NP-hard.

Proof: We make the following assumptions to simply the model: *first*, transportation cost between a distribution center and a retail outlet is ignored, which in turn makes subscript k redundant and removes the associated variable S_{jkt}^{ξ}; *second*, capacity is large enough to satisfy demand, i.e., $(a_j = \underset{\xi\in\Omega}{Max} \sum_{t\in T} d_t^{\xi}, \forall j \in J))$; and *third*, banks and suppliers offer unlimited financial support with negligible interest and penalty rate, which implies $\sum_{p\in P|p>t} Y_{itp}^{\xi} = X_{it}^{\xi}$, $\sum_{p\in P|p>t} M_{tp}^{\xi} = \Gamma_t^{\xi}$, removes constraints (2-11) – (2-13), and renders constraint (2-8) redundant and hence it could be dropped. In addition, examining constraints (2-9), (2-10) and (2-15), the objective function (2-5) is minimized by substituting $\sum_{i\in I}\sum_{j\in J} e_i W_{ijt}^{\xi}$ and $\sum_{j\in J}\left(\sum_{i\in I} f_{ij} G_{ijt}^{\xi} + \varepsilon_j Q_{jt}^{\xi}\right)$ instead of the financial terms. The above imply the following model.

$$\underset{W,G,Q}{Min} \sum_{i\in I}\sum_{j\in J}\sum_{t\in T}\left(e_i W_{ijt}^{\xi} + \varepsilon_j Q_{jt}^{\xi} + f_{ij} G_{ijt}^{\xi}\right) \qquad (2\text{-}28)$$

Subject to:

$$Q_{jt}^{\xi} \le \sum_i W_{ijt}^{\xi} - d_t^{\xi} \qquad j \in J, t \in T, \xi \in \Omega \qquad (2\text{-}29)$$

$$W_{ijt}^{\xi} \le d_t^{\xi} G_{ijt}^{\xi} \qquad i \in I, j \in J, t \in T, \xi \in \Omega \qquad (2\text{-}30)$$

(2-16), (2-18), (2-19)

The reduced model is similar to the inventory lot-sizing problem with supplier selection, which is NP-hard (see, Basnet and Leung, 2005). Therefore, a fortiori **(RO)** is NP-hard.

(RO) is NP-hard, and hence the optimization packages cannot find solutions for large problem instances in reasonable time. We propose a Lagrangian relaxation approach to alleviate the computational complexity of the model (Fisher, 2004), and note that several applications in supply chain have made use of this technique (Gendron et al., 2016; Jabbarzadeh et al., 2016). In general, Lagrangian relaxation algorithm has three steps: *first*, an upper bound is found by removing the complicating constraint and adding a penalty function to the objective value; *second*, a lower bound is generated by solving a feasible problem; and *third*, a sub-gradient optimization approach is utilized to converge the two bounds to reach an acceptable threshold (Fisher, 2004), and higher iteration might be needed to reach high-quality solutions. We next outline how each step is being implemented for **(RO)**.

2.5.1.1. *Finding an upper bound*

Constraint (2-10), which integrates transportation and inventory holding decisions with the financial requirements, is the complicating constraint in (**RO**). By separating the transportation and inventory holding decisions from the financial requirements, we can relax the problem into a smaller-sized problem that is more tractable and easier to solve. This relaxation is achieved by incorporating Constraint (2-10) into the objective function using a penalty term, specifically the Lagrangian multiplier, denoted as, λ_t^{ξ}. The resulting Lagrangian dual program (LDP) is as follows:

$$\underset{\lambda}{MinMax} \sum_{\xi\in\Omega}\pi^{\xi}\left(\sum_{t\in T}\left(\sum_{j\in J}\sum_{k\in K}(l_j-\lambda_t)\varphi_k d_t^{\xi} S_{jkt}^{\xi}-\Theta_t^{\xi}-\Xi_t^{\xi}\right)-\right.$$

$$\left.\sum_{t\in T|t<p}\sum_{p\in T}\left(M_{tp}^{\xi}+\sum_{i\in I}Y_{itp}^{\xi}\right)\right)-$$

$$\sum_{\xi\in\Omega}\sum_{t\in T}\lambda_t^{\xi}\left(\sum_{j\in J}\left(\sum_{i\in I}f_{ij}G_{ijt}^{\xi}+\sum_{k\in K}c_{jk}S_{jkt}^{\xi}+\varepsilon_j Q_{jt}^{\xi}\right)-\right.$$

$$\left.V_t^{\xi}-\Xi_t^{\xi}\right) \tag{2-31}$$

Subject to:

(2-6)-(2-8), (2-9a)-(2-9d), (2-11)-(3-12), (2-13a)-(2-13d), (2-14)-(2-27).

Proposition 2: Constraint (2-32) is a valid inequality for the (LDP), where Q'^{ξ}_{j} is the initial inventory at distribution center *j* under scenario ξ.

$$Q_{jp}^{\xi}+\sum_{k\in K}\sum_{t\in T}\varphi_k d_t^{\xi}S_{jkt}^{\xi}\le Q'^{\xi}_{j}+\sum_{i\in I}W_{ijt}^{\xi} \qquad j\in J, p=n, \xi\in\Omega \tag{2-32}$$

Proof: It is sufficient to show that the inequality in (2-32) is obtained by aggregating both sides of the constraint (2-7) over all periods. Thus, the new constraint is a valid inequality for the (LDP).

Proposition 3: Constraint (2-33) is a valid inequality for the (LDP).

$$\Xi_t^{\xi}\le\sum_{j\in J}\left(\sum_{i\in I}f_{ij}G_{ijt}^{\xi}+\sum_{k\in K}c_{jk}S_{jkt}^{\xi}+\varepsilon_j Q_{jt}^{\xi}\right) \qquad t\in T,\xi\in\Omega \tag{2-33}$$

Proof: Suppose that $\Xi^{*\xi}_{t}>\sum_{j\in J}\left(\sum_{i\in I}f_{ij}G^{*\xi}_{ijt}+\sum_{k\in K}c_{jk}S^{*\xi}_{jkt}+\varepsilon_j Q^{*\xi}_{jt}\right)$. It means that the assigned cash flow for the transportation and inventory holding cost is greater than the required money in period *t* under scenario ξ. It is a contradiction because the objective function (2-5) minimizes the term Ξ_t^{ξ}.

2.5.1.2. *Finding a lower bound*

By removing the complicating constraint, it is probable that the solution gained from solving the (LDP) is infeasible to the original model. To generate a feasible solution, S_{jkt}^{ξ} values are taken from solving the (LDP), which is then embedded in **(RO)** to ensure a lower bound for the optimum solution.

2.5.1.3. *Converging the bounds*

The lower and upper bounds obtained should be enhanced iteratively to decrease the gap between them. In the proposed study, a permissible gap is specified, and a suitable solution is found when the difference between the bounds is smaller. We make use of (2-34)-(2-36) to update the multiplier λ_t.

$$\lambda_t^{\xi,\delta+1} = Max\left(0, \lambda_t^{\xi,\delta} + \theta^{\delta}\left(\sum_{j\in J}\left(\sum_{i\in I} f_{ij} G_{ijt}^{\xi} + \sum_{k\in K} c_{jk} S_{jkt}^{\xi} + \varepsilon_j Q_{jt}^{\xi}\right) - V_t^{\xi} - \Xi_t^{\xi}\right)\right)$$

$$\forall t \in T, \xi \in \Omega \tag{2-34}$$

where, δ is the number of iterations and θ^{δ} is computed as follows.

$$\theta^{\delta} = \frac{\eta^{\delta}(UB^{\delta} - GLB)}{\sum_{\xi\in\Omega}\sum_{t\in T}\left(\sum_{j\in J}\left(\sum_{i\in I} f_{ij} G_{ijt}^{\xi} + \sum_{k\in K} c_{jk} S_{jkt}^{\xi} + \varepsilon_j Q_{jt}^{\xi}\right) - V_t^{\xi} - \Xi_t^{\xi}\right)^2} \tag{2-35}$$

$$GLB = Min\{LB^{\delta}\}; \delta = 1,2,3,\ldots \tag{2-36}$$

It is suggested that the initial value of η^{δ} is taken from the interval [0,2]. If no enhancement occurs after 4 consecutive iterations, η^{δ} reduces to the half. The updating procedure continues until the condition between the permissible gap and upper and lower bound gap is satisfied.

2.6. Computational experiments

In this section, the proposed analytical model is applied to a case study that has been built using the realistic infrastructure of Walmart in Pennsylvania (United States). We first outline the specifics of the case study, followed by the analyses of the solution, and then underscore the performance of the Lagrangian relaxation solution method on datasets of varying size.

2.6.1. Case study setting

Walmart, the largest retailer in the world, prefers to source grocery from local suppliers to ensure reliable sourcing of quality merchandise (Walmart, 2020). Figure 2-2 depicts the region of interest and provides the geographic location of 136 retail outlets depicted by circle, 2 distribution centers depicted by diamonds, and the 6 canned vegetable supplier locations as depicted through United States Department of Agriculture (https://www.ams.usda.gov/local-food-directories/foodhubs).

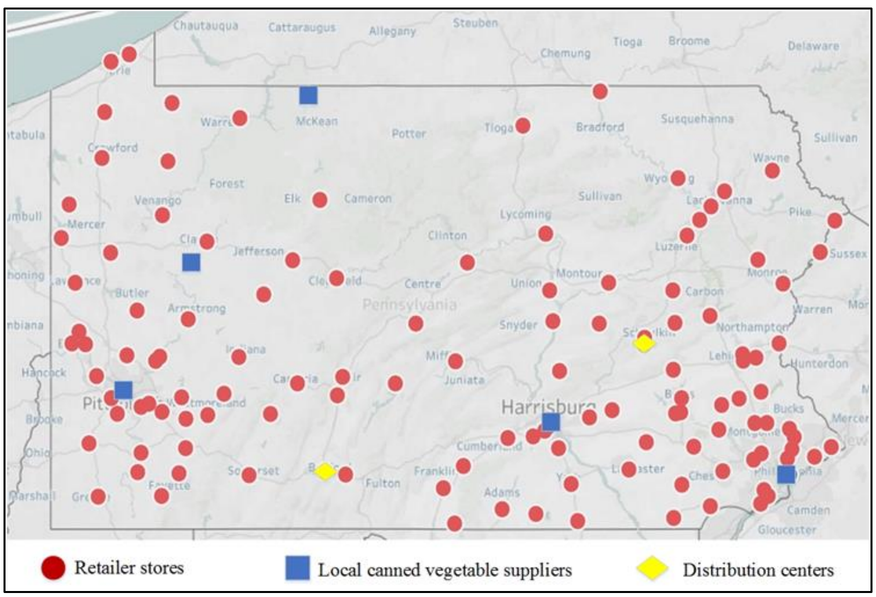


Figure 2-2: Region of interest

<u>Product flow related parameters:</u> Demand for canned vegetable is calculated using the target population, the weekly consumption of canned vegetables, and the market share of Walmart. More specifically, the average annual canned vegetables per capita demand from 1970 to 2017 follows a normal distribution with mean of 100.30 and standard deviation of 6.21 (Source:

https://www.ers.usda.gov/data-products/food-availability-per-capita-data-system/), and Walmart controls around 26% of the grocery market share (https://www.statista.com/statistics/818602/), which yields a weekly mean demand of 0.49 and standard deviation of 0.03. For each retail outlet, we look at what proportion of the population of Pennsylvania resides in the catchment area, which in turn generates the demand for the specific store. The average marginal truck transportation cost in northeast United States is \$3.15 per kilometer (Murray and Seth, 2019), which when multiplied by distance yields the transportation cost between a pair of nodes. The selling price for 27 canned vegetables produced by Walmart was collected from the website, whose average selling price is \$2.19 per ounce. We assume a 14% rate of return to calculate wholesale price (Burroughs and Harper, 2002), which after necessary manipulations result in the normal distribution for wholesale price per pound of canned vegetables, i.e., mean of 1.52 and standard deviation of 0.10.

Financial flow related parameters: For bank credit, we assume a nominal interest rate $\tilde{r}$ of 3.25% for highly rated companies such as Walmart, and a deviation of 1.5% (see https://www.jpmorganchase.com/about/our-business/historical-prime-rate). The penalty rate u_i is a function of the opportunity cost, and we base it on the prime interest rate plus 0.5% interest rate on certificates of deposits, i.e., uniformly distributed between 3.25 and 3.75. For supplier's credit rating, we assume that Walmart will select the ones very high on the FICO rating which ranges between 300 and 850 (Wiley, 2011). More specifically, we assume that suppliers whose rating is at least 800 will be chosen, which translates into a normalized credit rating score with a mean of 0.95 and a standard deviation of 0.015. Doing so ensures that the supplier's credit rating will always be exceptional, since the probability of bankruptcy will be $min\{\Psi_i(b_i), 1 - \Psi_i(b_i)\}$. The permissible delay in payment h_i is generated by an integer uniform distribution, i.e., *uniform*(0, 4) weeks (Sarker et al., 2000). Finally, as the maximum trade credit offered o_i is unknown, the supplier's historical trade credit information is used to set the available trade credit. In other words, suppliers with higher trade credit can offer more financial support. Hence, the extended trade credit X_{it}^{ξ} is less than or equal to $(1 - \Psi_i(b_i)) \sum_{j \in J} e_i W_{ijt}^{\xi}$ in constraint (2-11). To tackle the credit rating uncertainty, constraint (2-11) can be substituted by the following four constraints:

$$X_{it}^{\xi} + \mu_i^3 + \beta_{it\xi}^3 \leq \left(1 - \widetilde{\Psi}_i(b_i)\right) \sum_{j \in J} e_i W_{ijt}^{\xi} \qquad i \in I, t \in T, \xi \in \Omega \qquad \text{(2-11a)}$$

$$\mu_i^3 + \beta_{it\xi}^3 \geq \widehat{\Psi}_i(b_i) e_i W_{ijt}^{\xi} \qquad i \in I, j \in J, t \in T, \xi \in \Omega \qquad \text{(2-11b)}$$

$$\mu_i^3 \geq 0 \qquad i \in I \qquad \text{(2-11c)}$$

$$\beta_{it\xi}^3 \qquad i \in I, t \in T, \xi \in \Omega \qquad \text{(2-11d)}$$

All the problem instances were coded in GAMS using CPLEX on a laptop with Corei5 CPU and 4GB of RAM.

2.6.2. Solution and Analyses

2.6.2.1. Base case

To capture the entire gamut of possible decisions within a robust optimization context, we organize the discussion around the risk-attitude of the decision maker. Note that the budget uncertainty in the proposed robust optimization model **(RO)** is the indicator of risk-attitude, and higher values represent more aversion to risk. The concept of budget uncertainty, as presented in our robust optimization model, is especially pivotal in addressing financial disruptions during unpredictable economic climates. When the broader economic environment is volatile, uncertainties in interest rates can lead to significant financial disruption for the buyer. By incorporating a budget uncertainty for interest rates, our model inherently captures these fluctuations, thereby safeguarding against potential financial disruptions. Conversely, when considering supplier credit rating, a lower budget uncertainty is indicative of a more reliable supplier. In such cases, the risks associated with trade credit are minimized, as the buyer can anticipate consistent credit terms and behaviors from the supplier. However, in cases where there's a higher budget uncertainty associated with the supplier credit rating, our model underscores the importance of preparing for potential inconsistencies in trade credit availability. Such proactive measures help in mitigating the financial disruptions that might arise from sudden changes in a supplier's credit rating, ensuring a smoother and more resilient financial operation within the supply chain. To set up the baseline, we borrow representative values from the extant literature (Jabbarzadeh et al., 2016), and assume $\chi^r = 0.2$ and $\chi^b = 0.2$ for *risk-seeker* decision maker; $\chi^r = 0.4$ and $\chi^b = 0.8$ for *risk-neutral* decision maker; and, $\chi^r = 0.8$ and $\chi^b = 2.5$ for *risk-averse* decision maker. Table 2-1 provides an overview of the financial flow over the time horizon of interest for the three types of decision makers.

Risk attitude	**Financial Activity**	**Periods**									
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Risk seeker	*TC Received*	2.389	--	--	--	--	--	--	--	--	--
	TC Repay	--	--	--	--	2.389	--	--	--	--	--
	BC Received	2.151	--	--	--	--	--	--	--	--	--
	BC Repay	--	0.601	0.610	0.619	0.628	--	--	--	--	--

Risk neutral	*TC Received*	2.388	--	--	--	--	--	--	--	--	--
	TC Repay	--	--	--	--	2.388	--	--	--	--	--
	BC Received	2.508	--	--	--	--	--	--	--	--	--
	BC Repay	--	0.585	0.594	0.603	0.612	0.621	--	--	--	--
Risk averse	*TC Received*	--	--	--	--	--	--	--	--	--	--
	TC Repay	--	--	--	--	--	--	--	--	--	--
	BC Received	1.337	--	--	--	--	--	--	--	--	--
	BC Repay		0.205	0.208	0.211	0.215	0.218	0.221	0.224	0.228	--

Table 2-1: Financial flow ($ millions) for different risk attitudes

It is clear from Table 2-1 that both the risk-neutral and risk-seeker decision makers would like to use the interest-free nature of trade credit (*TC*) to finance their requirements as much as possible. However, a risk-averse decision maker insists on getting bank credit (*BC*) because higher budget uncertainty implies increased supplier unreliability and thus the increased reliance on reliable financing. On the other hand, for lower degree of risk aversion, the decision maker would prefer trade credit financing because the suppliers are more reliable. Note that both the risk-neutral and risk-seeker decision makers delay repaying the trade credit until the end of the permissible delay in payment because they can make money on borrowed credit. For bank credit, the risk-averse decision maker makes the repayment over a longer period and the amount increases in each period.

<table>
<tr><th rowspan="2">DC</th><th rowspan="2">Risk attitude</th><th colspan="10">Periods</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th><th>10</th></tr>
<tr><td rowspan="3">Bedford</td><td>Risk seeker</td><td rowspan="2">0.56</td><td rowspan="3">0.52</td><td rowspan="3">0.59</td><td rowspan="3">0.49</td><td rowspan="3">0.64</td><td rowspan="3">0.52</td><td rowspan="3">0.51</td><td rowspan="3">0.56</td><td rowspan="3">0.47</td><td rowspan="3">0.48</td></tr>
<tr><td>Risk neutral</td></tr>
<tr><td>Risk averse</td><td>0.97</td></tr>
<tr><td rowspan="3">Pottsville</td><td>Risk seeker</td><td rowspan="2">1.39</td><td rowspan="3">1.33</td><td rowspan="3">1.40</td><td rowspan="3">1.26</td><td rowspan="3">1.37</td><td rowspan="3">1.32</td><td rowspan="3">1.31</td><td rowspan="3">1.40</td><td rowspan="3">1.20</td><td rowspan="3">1.23</td></tr>
<tr><td>Risk neutral</td></tr>
<tr><td>Risk averse</td><td>0.98</td></tr>
</table>

Table 2-2: Material flow (million units) to distribution centers (DC) under different risk-attitudes

Table 2-2 provides information about the product flow under the three risk attitudes over the time periods of interest. The order quantity of a risk-averse retailer differs from those of other risk attitudes in the initial period. However, in subsequent periods, the risk attitude does not influence the order quantity. It is pertinent that the supplier in Pittsburgh (Pennsylvania) is chosen to satisfy the requirements at the distribution centers (DC), and that 75% of sourced products transit the Pottsville DC to get to the retail outlets. Note that 133 of the 136 retail outlets of Walmart in the state of Pennsylvania can satisfy customer demand in each period. The locations at Gibsonia, Milford and Belle Vernon are unable to meet customer demand under any of the three risk attitudes because their population is so small that there is no economic justification for paying the supply chain costs, such as transportation and inventory holding costs.

2.6.2.2. How uncertainty budgets impact risk-attitude of decision makers?

The conservatism of a decision-maker is not uniform across the range of values for the two uncertain factors of interest, i.e., supplier credit rating, and interest rates. To understand how the behavior would change, we ran 28 problem instances where the two uncertainty budgets were varied. Table 2-3 reports the highlight of the results, and where the second column indicates the interest rate uncertainty while the supplier credit rating uncertainty is indicated in the third column. The next four columns are associated with bank credit received, how it was spent on transportation & inventory and on purchasing products, and the repayment. The eight and ninth columns are associated with trade credit details, while the last column indicates the profit for the supply chain.

It is clear from Table 2-3 that a given amount of received bank credit can be used in several ways, i.e., spent on transportation and inventory financing and/or purchasing products without preferring one over the other. In general, for smaller values associated with interest rate uncertainty, the buyer chooses to rely only on bank credit and does not avail the trade credit offering. However, for higher budget uncertainty associated with interest rates, the buyer also starts using trade credit financing, but the usage is zero for larger credit rating uncertainty. The latter point also has a bearing on the supply chain profit, which decreases for higher values for both types of uncertainty budgets. Next, we develop several corollaries to better emphasize the relationship between uncertainty budgets and conservatism of the decision-maker.

<u>*Corollary 1:*</u> By raising the conservatism associated with supplier rating or trade credit, i.e., χ^b, getting bank credit is preferred for a risk-averse decision-maker, while trade credit has more benefits for a risk-seeking decision-maker. If there is reduced uncertainty regarding credit ratings,

leading to lower budget uncertainty associated with these ratings, then trade credit may be more preferable over bank credit. However, raising the conservatism associated with bank credit, i.e., χ^r, by itself would not suggest distinct actions for different decision makers.

Corollary 2: The cost of an uncertain factor could be determined as the change in profit for the most conservative and the least conservative attitudes. For instance, the cost of uncertainty associated with bank interest rate is $331K, i.e., OFV associated with the least conservative attitude is 15.764 while that for most conservative attitude is 15.433. Similarly, the cost of uncertainty associated with supplier rating is $221K (=15.654-15.433 that are respectively the least conservative and the most conservative stances). Thus, the impact of interest rate uncertainty is more on supply chain profit vis-à-vis that of supplier rating. This implies that when planning and making decisions, a business might want to pay more attention and possibly allocate more resources to managing and mitigating the risks associated with interest rates than those associated with supplier ratings.

<table>
<tr><th rowspan="2">Problem Instances</th><th rowspan="2">χ^r</th><th rowspan="2">χ^b</th><th>BC Recd.</th><th>T&I finance</th><th>Purchase Finance</th><th>BC Repay</th><th>TC Recd.</th><th>TC Repay</th><th>OFV</th></tr>
<tr><th colspan="7">($ millions)</th></tr>
<tr><td>1</td><td rowspan="4">0.05</td><td>0.20</td><td rowspan="4">1.338</td><td>0.0317</td><td>1.306</td><td rowspan="4">1.396</td><td>--</td><td>--</td><td rowspan="4">15.764</td></tr>
<tr><td>2</td><td>0.80</td><td>--</td><td rowspan="3">1.338</td><td>--</td><td>--</td></tr>
<tr><td>3</td><td>1.10</td><td>--</td><td>--</td><td>--</td></tr>
<tr><td>4</td><td>2.50</td><td>--</td><td>--</td><td>--</td></tr>
<tr><td>5</td><td>0.20</td><td>0.20</td><td>0.215</td><td>0.006</td><td>0.209</td><td>0.246</td><td>2.389</td><td>2.389</td><td>15.693</td></tr>
</table>

6		0.80		0.025	1.312		--	--	
7		1.10	1.338	0.038	1.299	1.529	--	--	15.676
8		2.50		--	1.338		--	--	
9	0.30	0.20	0.215	0.006	0.209	0.253	2.389	2.389	15.686
10		0.80		--			--	--	
11		1.10	1.337	--	1.337	1.571	--	--	15.633
12		2.50		--			--	--	
13	0.40	0.20	0.215	0.044	0.171	0.258	2.389	2.389	15.680
14		0.80	0.251	0.006	0.245	0.301	2.388	2.388	15.601
15		1.10	1.338	--	1.338	1.608	--	--	15.597
16		2.50		--	1.338		--	--	
17	0.60	0.20	0.211	0.006	0.205	0.264	2.397	2.397	15.669
18		0.80	0.251	0.006	0.244	0.314	2.389	2.389	15.589
19		1.10	0.268	--	0.269	0.336			15.548
20		2.50	1.337	--	1.337	1.673	--	--	15.531
21	0.80	0.20	0.222	--	0.222	0.287	2.399	2.399	15.633
22		0.80	0.251	0.019	0.231	0.324	2.389	2.389	15.578
23		1.10	0.268	0.006	0.262	0.347	2.388	2.388	15.537
24		2.50	1.337	--	1.337	1.731	--	--	15.473
25	0.95	0.20	0.207	0.025	0.182	0.275	2.405	2.405	15.654
26		0.80	0.247	0.006	0.241	0.327	2.396	2.396	15.568
27		1.10	0.268	--	0.268	0.355	2.389	2.389	15.529
28		2.50	1.337	0.019	1.318	1.771	--	--	15.433

Table 2-3: Snapshot of different problem instances

2.6.2.3. Bargaining power analysis

As alluded, Walmart prefers to use local suppliers, and can empower the latter by offering more money for the products. On the other hand, Walmart can exert its clout to demand lower prices. In this subsection, we analyze the role wholesale price plays on loan repayment for the three types of decision makers. To this end, we generate wholesale prices within (0.5, 1.5) of the values used in the base case setting, and then solved the resulting problem instances using two techniques: *first*, stochastic programming formulation, i.e., **(SP)**, where only demand varies, and both the interest rate and the supplier rating were fixed at their nominal values; and *second*, robust optimization formulation, i.e., **(RO),** where both types of uncertainty could vary over respective intervals described in the subsection 2.5.2.1. Table 2-5 (and Figure 2-3) provide the highlight of the resulting financial obligations, and in organized to provide information on bank credit when using stochastic programming methodology (i.e., *BC: SP*), bank credit repayment using robust optimization formulation (i.e., *BC Repay: RO*) and trade credit repayment using robust formulation (i.e., *TC Repay: RO*).

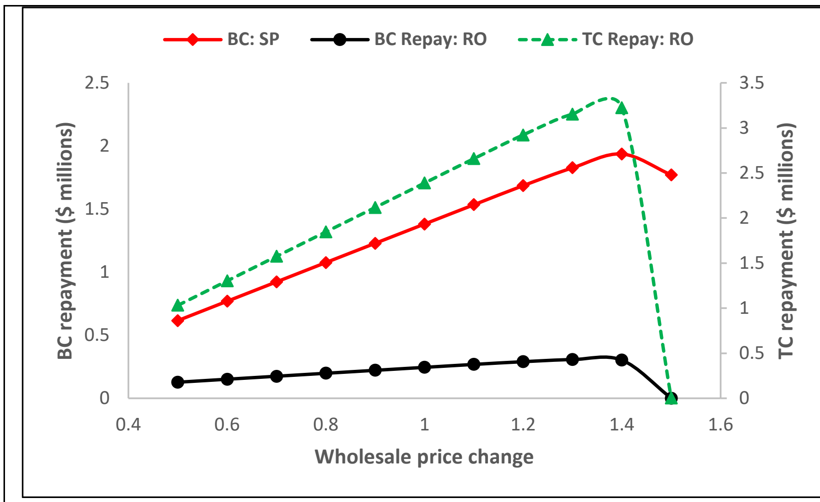
Risk attitude	**Repay legends**	**Multiplier of original wholesale price**										
		0.5	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**
Risk-seeker	*BC: SP*	0.62	0.77	0.92	1.07	1.23	1.38	1.53	1.69	1.83	1.94	1.77
	BC Repay: RO	0.13	0.15	0.18	0.20	0.22	0.25	0.27	0.29	0.31	0.30	0.01
	TC Repay: RO	1.03	1.30	1.58	1.85	2.12	2.39	2.66	2.92	3.15	3.22	0.01
Risk-	*BC: SP*	0.62	0.77	0.92	1.07	1.23	1.38	1.53	1.69	1.83	1.94	1.77

neutral	*BC Repay: RO*	0.15	0.18	0.21	0.24	0.27	0.30	0.33	0.36	0.37	0.29	0.01
	TC Repay: RO	1.03	1.30	1.58	1.85	2.12	2.39	2.66	2.91	3.11	2.86	0.01
Risk-averse	*BC: SP*	0.62	0.77	0.92	1.07	1.23	1.38	1.53	1.69	1.83	1.94	1.77
	BC Repay: RO	0.77	0.96	1.16	1.35	1.54	1.73	1.92	2.08	2.13	0.01	0.01
	TC Repay: RO	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.01

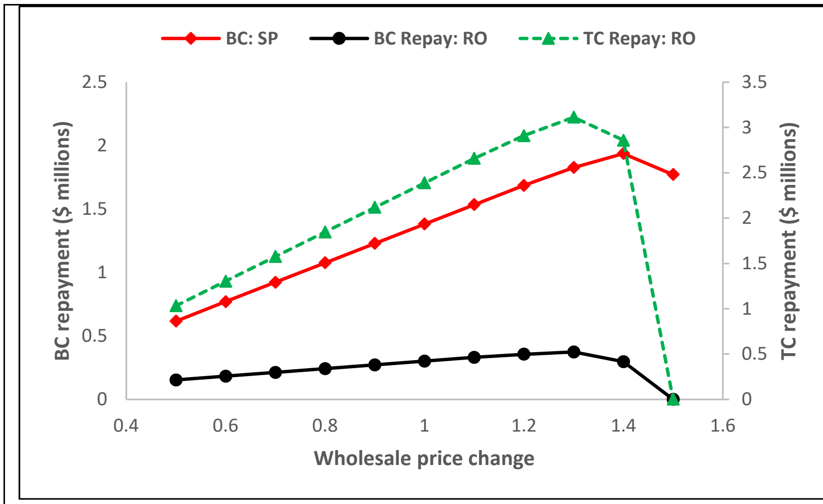
Table 2-4: Loan repayments ($ millions) for various levels of bargaining power

Corollary 3: Both Table 2-4 and Figure 2-3 show that the behavior of both the risk-seeker and risk-neutral decision makers are rather comparable with both relying mostly on trade credit, however, the exact peaks are happening at different points. For instance, for risk-seeker decision maker, the bank credit repayment peaks when wholesale price change is 1.3 times the base-case setting while the equivalent level for trade credit repayment is 1.4. On the other hand, for risk-neutral decision maker, both types of repayment peaks at 1.3 times the wholesale price of the base case. Finally, the risk-averse decision maker is reluctant to finance its requirements using trade credit, and almost exclusively relies on bank credit. Thus, it is recommended that a risk-averse decision maker ignore trade credit for financing.

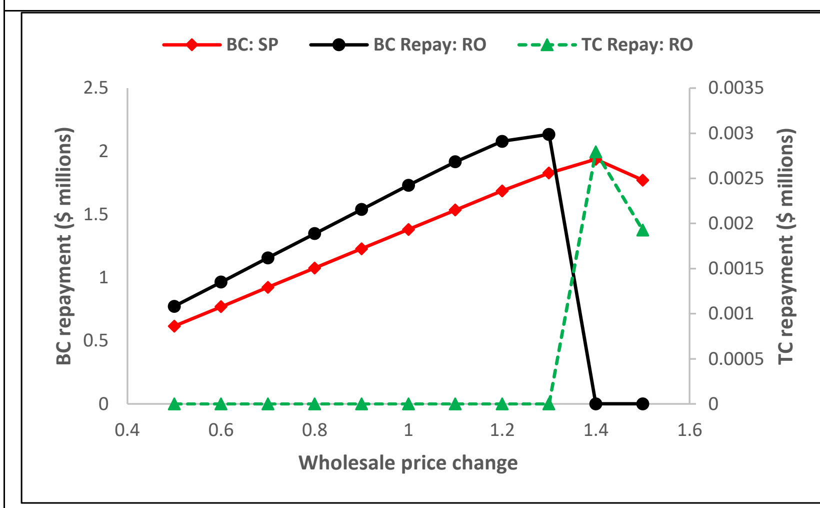
Corollary 4: The proposed stochastic programming formulation **(SP)** maximizes the mean value of profit, while the robust optimization formulation **(RO)** maximizes the worst-case solutions thereby decreasing the optimal profit. From Table 2-4 and Figure 2-3, it is clear that the repayment plan peaks around 1.4 times the base case wholesale price, which in turn suggests that **(RO)** will yield negligible profit beyond the peak. In contrast, **(SP)** does yield profit beyond the peak, and would enable the decision maker to run business. It implies that **(RO)** can set better wholesale price ceiling for the decision maker. Thus **(SP)** provides a more optimistic and flexible approach, potentially yielding profits in a broader range of scenarios, but it might expose the decision maker to greater risk. On the other hand, **(RO)** offers a more conservative and secure strategy, protecting the decision maker from worst-case scenarios but potentially limiting profitability in more favorable conditions. For a decision maker, choosing between these two approaches would depend on their risk tolerance and specific business needs.



a) Risk-seeker decision maker



b) Risk-neutral decision maker



c) Risk-averse decision maker

Figure 2-3: Loan repayments for various levels of bargaining power

<u>*Corollary 5:*</u> If bargaining power is used to reduce purchasing price, then irrespective of risk-attitude, the financial repayments are lower. When the purchasing price of goods or services is reduced due to higher bargaining power, the total amount of money that needs to be spent is also reduced. Consequently, if a business is financing the purchases through loans, the total amount required for these financial obligations would also be lower. Thus, higher bargaining power will lower required financial loans.

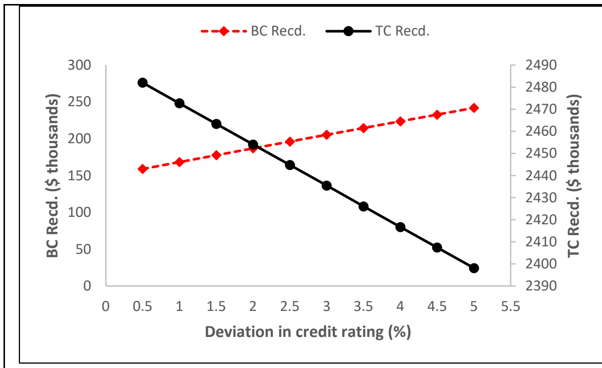
Corollary 6: It is clear from Figures 2-3(a) and 2-3(b) that risk-seeker and risk-neutral decision makers are interested in getting trade credit. It is expected that the risk-seeker decision maker is more eager to utilize trade credit. Thus, finding suppliers with high credit rating can benefit the buyer corporations financially.

2.6.2.4. Benefits of supplier credit rating analysis

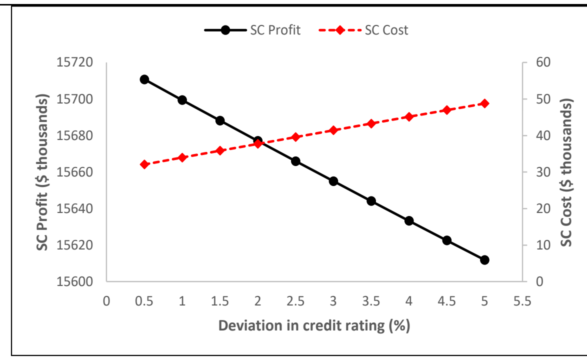
In this subsection, we ascertain how the uncertainty associated with credit rating would impact the three types of decision makers. To this end, we establish a range over which the credit rating could deviate from the nominal values, and this range is from 0.5% to 5.0%. For each value of deviation, we re-run the optimization program, and extract the pertinent information as per the risk attitude of the decision makers. For instance, Figure 2-4 (a) and (b) is concerned with the risk-seeker and, respectively, summarize the information on the two modes of financing, and the supply chain profit and cost. Similarly, (c) and (d) deals with the risk-neutral decision maker; and (e) and (f) is concerned with the risk-averse decision maker.

Corollary 7: By looking at the risk-averse behaviour in Figure 2-4(e), a deviation exceeding 4% implies reliance on bank credit to meet the financing requirements, which in turn increases the supply chain cost thereby driving down the profit (Figure 2-4(f)). Hence, it is suggested that a risk-averse decision maker should use trade credit only if the deviation is less than 4%.

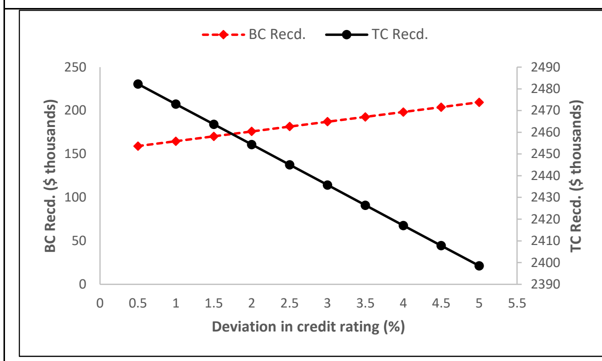
Corollary 8: The slope of BC finance for the risk-seeker is more than that for a risk-neutral decision maker (Figures 2-4(a) and 2-4(c)), however, the decreasing slops of TC financing are the same for both. Thus, the dependence on BC for a risk-neutral is more than that for a risk-seeker, which in turn results in higher supply chain cost and lower profit for the former (Figures 2-4(b) and 2-4(d)).



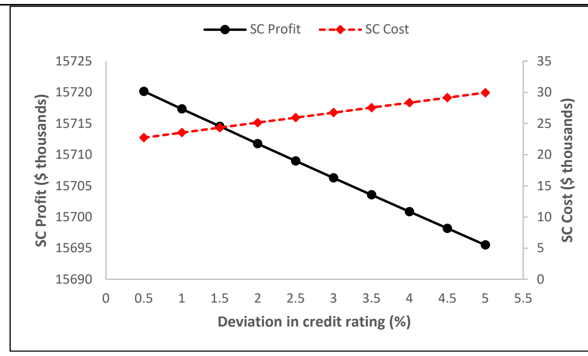
a) Risk-seeker: BC and TC



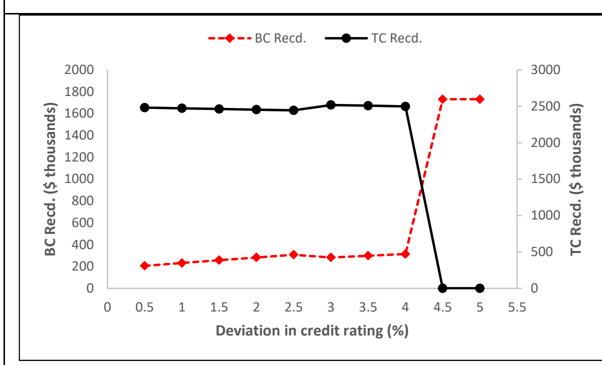
b) Risk-seeker: SC Profit and Cost



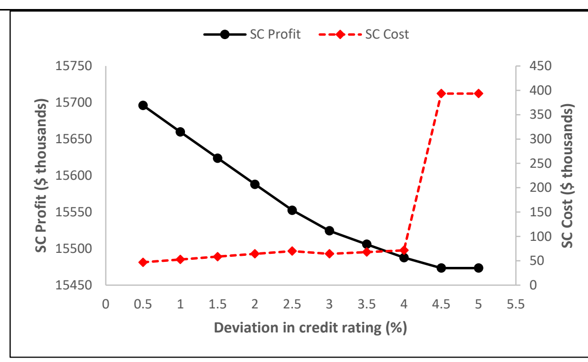
c) Risk-neutral: BC and TC



d) Risk-neutral: SC Profit and Cost



e) Risk-averse: BC and TC



f) Risk-averse: SC Profit and Cost

Figure 2-4: Impact of deviation in credit rating

2.6.3. Performance of Lagrangian relaxation

To demonstrate the performance of the proposed Lagrangian relaxation method on different data sets, we generated 15 large problem instances (i.e., 5 each for the three risk-attitudes). In each problem instance, the transportation costs have a uniform distribution between (1000, 5000), and each retail outlet can satisfy up to 20,000 customers. For convergence of upper and lower bound, a tolerance level of 1% was assumed. Table 6 depicts the pertinent attributes of the 15 large problem instances plus the equivalent case study (CS) setting for the three types of risk-attitudes. In here, “Gap” is determined as [{(upper bound – lower bound)/lower bound} *100]. To evaluate performance, the same problem instances were solved by GAMS (version 25.0.2) where the termination time was 4000 seconds, i.e., if optimal solutions could not be found then GAMS reports a gap.

Datasets		**Size**				**Lagrangian Relaxation**				**GAMS**	
		I	*J*	*K*	*T*	*LB*	*UB*	*Gap(%)*	*Cpu (s)*	*Gap(%)*	*Cpu (s)*
CS	**Risk-seeker**	6	2	137	10	15,599,763	15,754,621	0.9	22	0.00	30
#1		20	2	160	13	12,215,114	12,332,426		500	0.05	>4,000
#2		25	3	170	15	16,513,159	16,666,790		1,145	0.08	
#3		35	4	240	22	37,012,542	37,357,882		3,828	4.92	
#4		30	3	200	20	22,536,184	22,752,790		2,361	58.6	
#5		20	10	200	25	30,284,358	30,582,649		11,510	NA	NA
CS	**Risk-neutral**	6	2	137	10	15,691,407	15,846,165		23	0.00	26
#1		20	2	160	13	12,263,096	12,379,979		496	0.05	>4,000
#2		25	3	170	15	16,563,513	16,726,826		1,032	0.02	
#3		35	4	240	22	37,126,053	37,471,686		3,803	4.87	
#4		30	3	200	20	22,590,417	22,807,408		2,190	23.1	
#5		20	10	200	25	30,316,661	30,614,358		8,287	NA	NA
CS	**Risk-**	6	2	137	10	15,471,982	15,615,265		23	0.00	33
#1		20	2	160	13	12,128,451	12,246,932		420	0.05	>4,000

#2		25	3	170	15	16,411,000	16,564,714		835	0.02	
#3		35	4	240	22	36,848,725	37,192,867		3,991	NA	NA
#4		30	3	200	20	22,429,594	22,647,626		1,920	0.13	>4,000
#5		20	10	200	25	30,183,324	30,455,816		8,388	NA	NA

Table 2-5: Performance of Lagrangian relaxation v/s GAMS

It is clear from Table 2-5 that the proposed Lagrangian relaxation method solves the problem instances of varying size rather efficiently vis-à-vis the performance of GAMS. While the average gap is less than 1% for the proposed relaxation technique, the range of gap was rather significant when using GAMS. It is noteworthy that GAMS was unable to solve 4 problem instances, irrespective of the risk-attitude, and took more than 4,000 seconds to solve 11 of the 15 problem instances. In other words, GAMS was able to solve only the case-study (CS) settings within the specified time limit. Thus, it is possible to conclude that the proposed relaxation technique will work better than GAMS on problem instances of all sizes.

2.7. Conclusion

Supply chains primarily rely on working capital and external financing to procure products and pay for the transportation & inventory costs. External financing generally takes the form of bank credit and trade credit, both of which have inherent uncertainty and unfavorable movement could cause disruption. We make the first attempt to study the uncertainty in interest rate and supplier credit rating in a multi-period setting, emphasizing how saving and borrowing strategies can affect loan interests across different periods. To this end, a mixed-integer programming model that integrates the financial and product flows given demand uncertainty is first developed and is then augmented to develop a robust optimization program that incorporates the two types of uncertainties. The resulting complexity motivated the development of two valid inequalities, which in turn made the program amenable to the development of a Lagrangian relaxation solution methodology.

The proposed optimization programs were used to study numerous problem instances generated using the realistic infrastructure of Walmart in Pennsylvania (United States) and enables us to make the following conclusions. *First*, for small uncertainty in interest rate the buyer, irrespective of the risk attitude, will rely only on bank credit but will start considering trade credit when uncertainty increases. *Second*, a risk-averse decision maker turns towards reliable bank financing when the supplier unreliability increases but would prefer trade credit financing if the uncertainty associated with supplier credit rating decreases. On the other hand, both the risk-neutral and risk-seeker decision makers would gravitate towards interest-free nature of trade credits as much as possible and would choose to rely on suppliers with high credit rating. *Third*, the bargaining power of the buyer can impact the mix of the financing vehicles, i.e., trade credit and bank credit, which in turn will have a bearing on the supply chain profit. *Fourth*, the impact of interest rate uncertainty is more on supply chain profit vis-à-vis that of supplier rating. *Fifth*, Buyers often borrow beyond their immediate needs, anticipating financial benefits from the surplus funds. While trade credit permits repayment deferment until the end of the permissible delay. Furthermore, risk-averse decision-makers typically favor smaller, prolonged bank credit repayments. *Finally*, the proposed solution methodology can efficiently solve realistic size problem instances and performs better than standard optimization packages.

This research presents several limitations, particularly concerning the simplifications made about the framework of the trade credit, including pre-determined wholesale prices and interest rates. In real-world operations, the dynamic between the supplier and the buyer often mirrors a game-theoretic problem, where both parties seek to design a mutually beneficial contract.

Additionally, the limitation of the proposed robust optimization to uncertain interest rates and the supplier credit rating neglects the real-world possibility of these elements being correlated with information from previous periods. Such correlations could serve to diminish the uncertainty over multiple planning periods.

There are several directions of future research. First, investing resources to fortify unreliable facilities could minimize both material and financial flow disruption. Second, creditors may demand collateral in lieu of financing, and how that would impact transactions. Third, considering the role of 3PL to handle transportation and inventory, and its impact on the supply chain performance.

Declaration

Conflict of Interest: No conflict of interest exists in this research project.

Appendix 2A

Bertsimas and Sim (2003, 2004) offer a method to overcome uncertainty when the distribution function of uncertain parameters can be estimated by uncertainty set. This feature assists us to tackle existing uncertainty in interest rate and supplier credit rating. Consider (2A-1)-(2A-3).

$$\boldsymbol{Min\ c_j' x_j'} \qquad (2A\text{-}1)$$

Subject to:

$$\boldsymbol{\sum_{j \in J} a_{ij}' x_j' \leq b_i'} \qquad \forall \boldsymbol{i = 1, 2, 3, \ldots, m'} \qquad (2A\text{-}2)$$

$$x_j' \geq 0 \qquad \forall j \qquad (2A\text{-}3)$$

Where $a_{ij}^{'}$ is an uncertain parameter and j_i implies the set of uncertain parameters in constraint i. It is assumed that the uncertain parameter $a_{ij}^{'}$ is in the interval $[\tilde{a}_{ij} - \hat{a}_{ij}, \tilde{a}_{ij} + \hat{a}_{ij}]$ where $\tilde{a}_{ij}$ and $\hat{a}_{ij}$ are considered as the nominal values of $a_{ij}^{'}$ and the deviation of the nominal value respectively. The robust counterpart of the model (2A-1)-(2A-3) can be written as the following model by applying the dual theory.

$$Min\ c_j^{'} x_j^{'} \tag{2A-4}$$

Subject to:

$$\sum_{j \in J} \tilde{a}_{ij} x_j^{'} + \beta_i \Gamma_i + \sum_{j \in J} \mu_{ij} \leq b_i^{'} \ \ \forall i \in I \tag{2A-5}$$

$$\beta_i + \mu_{ij} \geq \hat{a}_{ij} x_j^{'} \ \ \forall j \in J \tag{2A-6}$$

$$\beta_i \geq 0\ \forall i \in I \tag{2A-7}$$

$$\mu_{ij} \geq 0\ \forall j \in J \tag{2A-8}$$

$$x_j^{'} \geq 0 \tag{2A-9}$$

Where Γ_i is taken into account as the uncertainty budget in the interval $[0, |j_i|]$. For higher values of Γ_i, the model (2A-4) – (2A-9) offers more conservative solutions. Also, β_i and μ_{ij} are non-negative variables. For more details, refer to Bertimas and Sim (2003, 2004).

References:

Alavi, S. H., & Jabbarzadeh, A. (2018). Supply chain network design using trade credit and bank credit: A robust optimization model with real world application. Computers & Industrial Engineering, 125, 69-86.

Arani, H. V., & Torabi, S. A. (2018). Integrated material-financial supply chain master planning under mixed uncertainty. Information Sciences, 423, 96-114.

Basnet, C., & Leung, J. M. (2005). Inventory lot-sizing with supplier selection. Computers & Operations Research, 32(1), 1-14.

Bertsimas, D., & Sim, M. (2003). Robust discrete optimization and network flows. Mathematical programming, 98(1), 49-71.

Bertsimas, D., & Sim, M. (2004). The price of robustness. Operations research, 52(1), 35-53.

Blasio, G. D. (2005). Does trade credit substitute bank credit? Evidence from firm-level data. Economic notes, 34(1), 85-112.

Breza, E., & Liberman, A. (2017). Financial contracting and organizational form: Evidence from the regulation of trade credit. The Journal of Finance, 72(1), 291-324.

Bröcker, J., Korzhenevych, A., & Schürmann, C. (2010). Assessing spatial equity and efficiency impacts of transport infrastructure projects. Transportation Research Part B: Methodological, 44(7), 795-811.

Burroughs, R., & Harper, D. (2002). An Analysis of Profits within the Canadian Food Processing Sector. Ottawa, Ontario.

Chandler, V. (2009). Determinants of trade credit use by small and medium-sized enterprises in Canada. SME Financing Data Initiative.

Chen, Z., & Zhang, R. Q. (2021). A multi-period multi-product stochastic inventory problem with order-based loan. International Journal of Production Research, 1-14.

DC Velocity Staff, 2021, Report: Companies conserved cash, increased debt during pandemic , https://www.supplychainquarterly.com/articles/5030-report-companies-conserved-cash-increased-debt-during-pandemic

Ding, Y., Jiang, Y., Wu, L., & Zhou, Z. (2021). Two-echelon supply chain network design with trade credit. Computers & Operations Research, 131, 105270.

Emtehani, F., Nahavandi, N., & Rafiei, F. M. (2021). An operations-finance integrated model with financial constraints for a manufacturer in a multi-supplier multi-product supply chain. *Computers & Industrial Engineering*, *153*, 107102.

Fattahi, M., Govindan, K., & Maihami, R. (2020). Stochastic optimization of disruption-driven supply chain network design with a new resilience metric. International Journal of Production Economics, 230, 107755.

Fisher, M. L. (1981). The Lagrangian relaxation method for solving integer programming problems. Management science, 27(1), 1-18.

Forbes (2019), Ending Pharmacy Dispute, Walmart Renews Contract With CVS Heath's PBM, https://www.forbes.com/sites/brucejapsen/2019/01/18/ending-dispute-walmart-renews-multiyear-contract-with-cvs-heaths-pbm/?sh=78ac838f5852

Gendron, B., Khuong, P. V., & Semet, F. (2016). A Lagrangian-based branch-and-bound algorithm for the two-level uncapacitated facility location problem with single-assignment constraints. Transportation Science, 50(4), 1286-1299.

Guan, Z., Mou, Y., & Sun, M. (2022). Hybrid robust and stochastic optimization for a capital-constrained fresh product supply chain integrating risk-aversion behavior and financial strategies. *Computers & Industrial Engineering*, 108224.

Gupta, S., & Dutta, K. (2011). Modeling of financial supply chain. European journal of operational research, 211(1), 47-56.

Han, S., Mo, Y., Chen, L., Luo, Z., Foropon, C. R., & Belal, H. M. (2022). A multi-period closed-loop supply chain network design with circular route planning. Annals of Operations Research, 1-39.

Hahn, G. J., & Kuhn, H. (2012). Value-based performance and risk management in supply chains: A robust optimization approach. International Journal of Production Economics, 139(1), 135-144.

Hasani, A., Zegordi, S. H., & Nikbakhsh, E. (2015). Robust closed-loop global supply chain network design under uncertainty: the case of the medical device industry. International Journal of Production Research, 53(5), 1596-1624.

Homayouni, Z., Pishvaee, M. S., Jahani, H., & Ivanov, D. (2021). A robust-heuristic optimization approach to a green supply chain design with consideration of assorted vehicle types and carbon policies under uncertainty. Annals of Operations Research, 1-41.

Hua, S., Xiaoye, Y., & Yuanfang, S. (2022). Dynamic discounting program of supply chain finance based on a financial information matching platform. *Annals of Operations Research*, 1-30.

Insolvency statistics in Canada- first quarter of 2020, https://www.ic.gc.ca/eic/site/bsf-osb.nsf/eng/h_br04254.html

Jabbarzadeh, A., Fahimnia, B., Sheu, J. B., & Moghadam, H. S. (2016). Designing a supply chain resilient to major disruptions and supply/demand interruptions. Transportation Research Part B: Methodological, 94, 121-149.

Jiang, R., Kang, Y., Liu, Y., Liang, Z., Duan, Y., Sun, Y., & Liu, J. (2022). A trust transitivity model of small and medium-sized manufacturing enterprises under blockchain-based supply chain finance. International Journal of Production Economics, 247, 108469.

Jones, P. M. (2010). Trade Credit Insurance. Washington, DC.

Klebnikov (2022), S&P 500 Briefly Plunges Into Bear Market As Stocks Fall For Seventh Week In A Row, https://www.forbes.com/sites/sergeiklebnikov/2022/05/20/sp-500-plunges-into-bear-market-as-stocks-fall-for-seventh-week-in-a-row/?sh=560a141b722c

Kouvelis, P., & Zhao, W. (2012). Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts. Operations research, 60(3), 566-580.

Kouvelis, P., & Zhao, W. (2016). Supply chain contract design under financial constraints and bankruptcy costs. Management Science, 62(8), 2341-2357.

Kouvelis, P., & Zhao, W. (2018). Who should finance the supply chain? Impact of credit ratings on supply chain decisions. Manufacturing & Service Operations Management, 20(1), 19-35.

Lee, C. H., & Rhee, B. D. (2011). Trade credit for supply chain coordination. European Journal of Operational Research, 214(1), 136-146.

Liu, L., Li, Y., & Jiang, T. (2021). Optimal strategies for financing a three-level supply chain through blockchain platform finance. International Journal of Production Research, 1-18.

Murray, D., & Seth, G. (2019). An Analysis of the Operational Costs of Trucking: 2019 Update. Arlington, Virginia.

Rajan, R. G., & Zingales, L. (1995). What do we know about capital structure? Some evidence from international data. The journal of Finance, 50(5), 1421-1460.

Razavian, E., Tabriz, A. A., Zandieh, M., & Hamidizadeh, M. R. (2021). An integrated material-financial risk-averse resilient supply chain model with a real-world application. Computers & Industrial Engineering, 161, 107629.

Sarker, B. R., Jamal, A. M. M., & Wang, S. (2000). Supply chain models for perishable products under inflation and permissible delay in payment. Computers & Operations Research, 27(1), 59-75.

Seifert, D., Seifert, R. W., & Protopappa-Sieke, M. (2013). A review of trade credit literature: Opportunities for research in operations. European Journal of Operational Research, 231(2), 245-256.

Serrano, A., Kraiselburd, S., & Oliva, R. (2010). A financial perspective on inventory holding cost. Mays Business School Research Paper, (2011-13).

Sodhi, M. S., & Tang, C. S. (2009). Modeling supply-chain planning under demand uncertainty using stochastic programming: A survey motivated by asset–liability management. International Journal of Production Economics, 121(2), 728-738.

SNAP (2022), https://seekingalpha.com/symbol/SNAP/earnings/transcripts

Solyali, O., Cordeau, J. F., & Laporte, G. (2012). Robust inventory routing under demand uncertainty. Transportation Science, 46(3), 327-340.

Statistics Canada. (2018). Survey on financing and growth of small and medium enterprises , 2014 Data tables Released on November 27 , 2015 Statistics Canada, (November).

Statistics Canada. (2018b). SURVEY ON FINANCING AND GROWTH OF SMALL AND MEDIUM ENTERPRISES , 2014 Data tables Released on November 27 , 2015 Statistics Canada, (November).

Taleizadeh, A. A., Safaei, A. Z., Bhattacharya, A., & Amjadian, A. (2022). Online peer-to-peer lending platform and supply chain finance decisions and strategies. *Annals of Operations Research*, 1-31.

Tsao, Y. C. (2011). Replenishment policies considering trade credit and logistics risk. Scientia Iranica, 18(3), 753-758.

Walmart. (2020). Walmart 2020 Annual Report. Washington, D.C. Retrieved from https://s2.q4cdn.com/056532643/files/doc_financials/2020/ar/Walmart_2020_Annual_Report.pdf

Wang, L., Jia, F., Chen, L., & Xu, Q. (2022). Forecasting SMEs' credit risk in supply chain finance with a sampling strategy based on machine learning techniques. Annals of Operations Research, 1-33.

Wang, M., & Huang, H. (2019). The design of a flexible capital-constrained global supply chain by integrating operational and financial strategies. *Omega*, *88*, 40-62.

Wiley, J. (2011). No First Sale for Foreign.

Yang, S. A., Bakshi, N., & Chen, C. J. (2021). Trade credit insurance: Operational value and contract choice. *Management Science*, *67*(2), 875-891.

Yousefi, A., & Pishvaee, M. S. (2018). A fuzzy optimization approach to integration of physical and financial flows in a global supply chain under exchange rate uncertainty. International Journal of Fuzzy Systems, 20(8), 2415-2439.

Yunzhang, H., Lee, C. K., & Shuzhu, Z. (2021). Trinomial tree based option pricing model in supply chain financing. *Annals of Operations Research*, 1-17.

Zhao, Z. J., & Levinson, D. (2012). Introduction to the special issue on value capture for transportation finance. *Journal of Transport and Land Use*, 5(1), 1-3.

Zhao, Y., Zhou, H., & Leus, R. (2022). Recovery from demand disruption: Two-stage financing strategy for a capital-constrained supply chain under uncertainty. European Journal of Operational Research.

Zhang, L. H., & Zhang, C. (2022). Manufacturer encroachment with capital-constrained competitive retailers. European Journal of Operational Research, 296(3), 1067-1083.

Zhang, L. H., Zhang, C., & Yang, J. (2022). Impacts of power structure and financing choice on manufacturer's encroachment in a supply chain. Annals of Operations Research, 1-47.

Zheng, X. X., Li, D. F., Liu, Z., Jia, F., & Lev, B. (2021). Willingness-to-cede behaviour in sustainable supply chain coordination. International Journal of Production Economics, 240, 108207.

Zhong, Y., Shu, J., Xie, W., & Zhou, Y. W. (2018). Optimal trade credit and replenishment policies for supply chain network design. Omega, 81, 26-37.

Zhu, Y., Zhou, L., Xie, C., Wang, G. J., & Nguyen, T. V. (2019). Forecasting SMEs' credit risk in supply chain finance with an enhanced hybrid ensemble machine learning approach. *International Journal of Production Economics*, *211*, 22-33.

Chapter III: Supply Chain Finance Hedging: Designing Data-Driven Contracts

3.1. Introduction

Can we hedge the risk of bankruptcy caused by loans in a supply chain contract when demand is uncertain? Paying interest on a loan causes a business to fail if it cannot be reimbursed. Circuit City, for example, was unable to survive and went bankrupt after borrowing $2.3 billion (Campbell, 2014). According to the United States courts, approximately 23,000 businesses filed for bankruptcy in the year ending March 2020 alone, with around 15,000 corporations liquidating their assets to pay their obligations under Chapter 7 of the United States bankruptcy legislation (U.S. courts, 2020).

A bankrupt partner, like a falling domino, might trigger a supply chain failure since there are severe economic and social consequences. In other words, the risk of loan repayment affects both the delinquent borrower, and its supply chain partners. General Motors filed for bankruptcy protection in 2009, owing more than $53 billion in debt and laying off more than 20,000 employees. The debts consequences, on the other hand, were far more scary, as late payments put financial pressure on suppliers, raising costs and disrupting the new General Motors Company (Isidore, 2009). According to the Motor and Equipment Manufactures data (2009), around 66 percent of General Motors were also the suppliers of Chrysler. The unpaid loans could eliminate 1.1 to 3.3 million jobs in the automotive industry (Goolsbee and Krueger, 2015). Because of the bankruptcy of Chrysler and General Motors, as well as financial difficulties of Ford, 47 automotive suppliers faced financial challenges in the first nine months of 2009 (Pearce, 2012). In November

2008, the chief executive officers of General Motors, Chrysler and Ford asked $25 billion working capital from the US government to reimburse their debt. As another example, Tailored Brands filed for bankruptcy due to $686 million debt in the year 2020. It revealed intentions for closing 500 stores over time and eliminating 20 percent of its corporate jobs (CNBC, 2020). For small and medium-sized firms, the risk of loan repayment can have disastrous effects, because losing a bankrupt customer reduces revenues, and losing a bankrupt supplier, like a nightmare, might result in supplier switching expenses.

However, firms are forced to take out loans due to liquidation constraints. According to the office of Advocacy, around 80% of U.S. businesses are supported by loans in the years 1993,1998 and 2003, mainly trade credit and bank credit (Cole, 2010). Trade credit is internal financial support in a supply chain offered by suppliers for delay in payment. In other words, if a retailer faces liquidation constraints, trade credit can empower the business and its partners in a supply chain, while bank credit is an external financial support that force to reimburse loan with interest. Trade credit is a short term financing that typically gives permission to the retailer to repay the loan in the range of 30 to 90 days; otherwise, the retailer is penalized by interest (Alavi & Jabbarzadeh, 2018; Berlin, 2003) or lost assets (Kouvelis and Zhao, 2012). Near 60% of U.S. businesses utilize this financing method to tackle their liquidation constraints, whereas higher credit rating firms tend to borrow bank credit (Cole, 2010). If both the financing methods are available, how to reduce the risk on loans is a real and important question.

Trade credit insurance (TCI) can hedge the financial risk of offering trade credit. In August 2014, Hebei company in China offered a trade credit contract to a German company for four batches of solar panels. On December 8, the German company filed the bankruptcy and could not repay the loan entirely. Fortunately, the Hebei company survived from capital loss by receiving reimbursement from purchased TCI (Li et al., 2022). Trade credit reimbursement is counted as an account receivable in supplier financial statements. Around 10 percent of them are delinquent accounts receivable and can lead a supplier to bankruptcy (Dittman, 2015). In real world problems, TCI is an efficient tool that can dramatically hedge the supplier's risk to offer trade credit. According to the world bank report, in the year 2008, around € 5.3 billion of global credit insurance premiums protected € 2.6 trillion of sales by TCI (Jones, 2010). For instance, Euler Hermes and Atradius as trade credit insurers and Swiss Re and AIG, and national export–import banks as general insurers offer TCI to support businesses financially (Yang et al., 2021; Jones, 2010). TCI protects the supplier against a delinquent retailer who received trade credit. Insurer guarantees that cover a percentage of unpaid accounts receivable after receiving a percentage of sales. In fact, supplier transfers the risk of delinquent invoices of retailer to insurer by purchasing TCI.

Moreover, both supplier (he) and retailer's (she) risk of bank credit reimbursements can be hedged by credit insurance. Business owners may be required to purchase payment protection insurance (PPI) to hedge the risk of unpaid bank credit. It helps a firm by reimbursing the loan to the bank when it is unable to reimburse accounts payable more. To clarify, PPI protects a firm's assets and credit rating when reimbursing the bank credit is not feasible. According to the Financial Conduct Authority data, PPI compensates more than £4.3 billion debts in United Kingdom in the year 2019 where the borrowers struggle to repay the loans (Statista, 2019). In addition, banks and financial intermediaries offer PPI to support the borrower financially against financial disruption. For example, CIBC bank offers this insurance to its borrowers in Canada and United States (CIBC, n.d.).

TCI and PPI can help supply chain participants mitigate the risk of supply chain disruptions. For instance, the 2014 ignition switch crisis forced General Motors to pay $2.8 billion in repair costs for recalling approximately 30.4 million cars, as reported by CNN. This expense decreased the profit margin of General Motors for the cars sold. The recall might have led to lost sales costs for retailers, causing financial difficulties in repaying trade credit. It could have also financially disrupted the company's ability to repay bank loans. However, General Motors could have reduced disruption expenses by purchasing credit insurance, as suggested by Fow, vice president of sales at Euler Hermes North America (GlobalAutoIndustry.com, n.d.).

The widespread use of TCI and PPI prompted us to explore the following research questions: What drives suppliers to hedge the risk of offering trade credit through TCI? Why do suppliers and retailers use PPI to hedge the risk of bank credit repayment? With TCI and PPI in place, what are the supply chain finance contract parameters (e.g., wholesale price, interest rate, TCI purchase amount, PPI purchase amount by supplier and retailer, order quantity, received bank credit amount, and offered trade credit amount)? What are the reasons for supply chain participants to hedge disruption risks by insuring loans?

In addition to loan repayment risks, inaccurate demand forecasting can contribute to bankruptcy risk. Overestimating demand can result in financial difficulties for the supply chain due to unpaid trade credit and bank credit obligations, while underestimating demand can harm customer satisfaction. Kouvelis & Zhao (2012) addressed uncertain demand in supply chain contracts using a probability distribution function. However, the increasing availability of data in today's rapidly evolving technological landscape encourages us to utilize this information when designing contracts. Leveraging available data can help decision-makers mitigate bankruptcy risk while maximizing customer satisfaction.

Nevertheless, increased customer satisfaction may lead to overestimated demand, which in turn increases the bankruptcy risk due to a higher volume of loans. In other words, the retailer must order more products to meet higher demand, potentially resulting in unsold inventory and financial strain. Therefore, developing an approach that capitalizes on demand data is crucial to reduce bankruptcy risk.

It remains uncertain whether relying solely on demand data is sufficient. Financial market conditions change periodically, and focusing on demand data without considering these financial factors may result in inaccurate risk estimation for supply chains. Prime interest rate, tariffs, inflation rate and other governmental policies can regulate the bankruptcy risk. For example, Ford CFO announced that the rising inflation rate in year 2022, wiped out the expected profitability of the Mustang Mach-E car (CNBC, 2022). In this study, the financial independent variables that can influence on the customer demand, but they are not of direct interest, are called covariates. These financial covariates and historical demand data can benefit the supply chain players to take better decisions. To do so, retailer estimates the demand by historical demand data and financial covariates and then order required loans to meet pre-set profit target at a confidence level. Supplier takes its production and financial decisions based on the retailer order and offered trade credit. The Value at Risk (VaR) method is an instrument in portfolio management which is capable of considering the retailer's risk attitude (Kouvelis and Li, 2019) and contemplate on the chance of meeting retailer's profit target (Lin et al., 2021). This study makes use of VaR to measure the risk of meeting retailer's profit target based on historical demand data and financial covariates.

The following descriptions will shed light on our contributions to this study. The goal of this study is to provide guidance for supply chain players to decide about purchasing insurance for financial loans. *First*, we design three novel contracts that help firms to hedge the risk of borrowing loans, including: 1) supply chain finance without insurance services, 2) hedging the risk of supply chain finance using PPI, and 3) hedging the risk of supply chain finance using both PPI and TCI.

The following details describe the contributions of these contracts. For the first time, a new mathematical programming model is developed for designing a supply chain finance game. Although the past studies (e.g., Kouvelis & Zhao, 2012; Kouvelis & Zhao, 2018) estimate the interrelationship among the contract factors by uncertain demand, the proposed game theoretic contracts consider the profit target of the retailer based on available data. The second developed contract make use of PPI in supply chain finance area for the first time. The PPI can be purchased for both the players (i.e., supplier and retailer) to retain their assets in face of unpaid bank credit. However, ordering this insurance impose its associated cost to supply chain. The last contract investigates the advantages of both TCI and PPI in a supply chain finance contract. Unlike the PPI, the TCI can only be implemented by supplier to save him against unpaid trade credit repayment. Moreover, for the first time, the assets wipe out as much as the player unpaid debts after bankruptcy in the three proposed contracts, whereas the assets totally wipe out after bankruptcy in the current studies (e.g., Kouvelis & Zhao, 2012; Kouvelis & Zhao, 2018).

Second, for the first time, a data driven VaR is developed to tackle bankruptcy risk in supply chain finance area. To do so, we make use of logistic regression to predict the future demand using lost asset data. This method offers a risk averse solution where demand distribution is unknown.

Third, we make use of the well-known *profit split* method for transfer pricing. Transfer pricing refers to the price of goods and services that are exchanged between two interrelated companies. We found that even though the profit of retailer remains constant due to her profit target, the risk of losing assets to bank transfers from retailer to supplier by applying this transfer pricing method. Hence, the transfer pricing can be used as a financial support from supplier to retailer.

Fourth, we found that trade credit can be more costly compared to bank credit in the absence of insurance services. In our study, retailer generates money by selling the products and gaining the commission fee. We discovered that only PPI can prevent the supplier from going bankrupt, whereas TCI can help the supplier to avoid losing profit. However, purchasing PPI makes sense if the retailer's profit margin is at least as high as the supplier's bankruptcy threshold. Besides, supplier's purchase of PPI might assist the retailer to ignore bank credit financing. Furthermore, purchasing TCI by supplier assists the retailer to preserve her assets if the retailer's profit margin falls below the supplier's TCI threshold.

We also examined the impact of different retailer's risk level on supplier's profitability. Based on our calculations, working with a risk averse retailer can pales the importance of TCI. In addition, purchasing PPI works better than working with a risk averse high profit margin retailer. Also, purchasing PPI diminishes the risk of unmet demand and losing assets simultaneously if the retailer's profit margin exceeds the supplier's bankruptcy threshold.

For the first time, we investigate disruption in supply chain finance contracts. We found that the supplier's profit margin after meeting disruption is always positive if supplier purchases both PPI and TCI at the same time for cooperating with a low profit margin retailer. We noticed that PPI works better than TCI if the retailer is struggling to manage her expenses. Moreover, we found that the profit split transfer pricing method can mitigate the retailer's risk of financing affordability if retailer's profit margin be less than supplier's bankruptcy threshold.

The rest of this study is organized as follows. Section 3.2 describes the existing related literature. Section 3.3 presents the problem definition and associate mathematical modeling, including the three contracts, data-driven VaR and solution approach in Section 3.4. Section 3.5 provides a case study and its managerial insights. Section 3.6 explains the conclusion of this study.

3.1. Related Literature

The literature is divided into two sections: hedging loan risk with insurance services and the intersection of operations management, finance, and risk management. The first section focuses mostly on either TCI (Jones, 2010) or PPI (Shi et al., 2021).

3.1.1. Hedging loan risk with insurance services

Our research relates to both types of insurance. In the literature of TCI in supply chain, supplier benefits from TCI when retailer is unable to repay trade credit at the end of planning horizon if TCI is purchased at the time of contract signing. In other words, the insurer is responsible to compensate financial loss of supplier (Yang et al., 2016). This allows a supplier to make better decisions while balancing the cost of insurance and the risk of offering trade credit. In two ways, our work adds to this body of knowledge. First, the vast majority of models in this field are concerned with creating a TCI contract between an insured party and an insurer. Yang et al., (2021) designed a contract between an insurer and a supplier in the presence of risky retailer. To do so, the insurer gather and analyze related information about retailer's default risk. TCI, on the other hand, is highly dependent on the decisions of both supply chain parties, because an optimal contract must alleviate not only the risk of extending trade credit by supplier, but also the cost of financing the retailer's requirements. *Secondly*, to the best of our knowledge, purchasing TCI by supplier is mandatory in the associated studies. For example, Li et al., (2020) force the supplier to make use of TCI. It compels supplier to take a risk averse financial decision by purchasing TCI in any cases. Differently, our study gives the right to supplier to purchase TCI if it is thrifty. Moreover, it permits the supplier to insure the offered trade credit partially.

By investigating the case where players can borrow bank credit, our study is also related to PPI. In real world, the PPI is purchased to insure the bank credit repayments. To the best of our knowledge, although bank encourages borrower to purchase PPI, the insurance of bank credit has never studied in the existing supply chain finance literature. For example, Ashton and Hudson (2011) states that PPI can be sold jointly with a mortgage while the independent policies for selling PPI have lower premiums. Differently, this is the first study which make use of PPI to secure the supply chain players assets. As such, the supply chain players can hedge the risk of borrowing bank credit using PPI and save their assets in the face of probable bankruptcy risk.

3.1.2. Intersection of operations management, finance, and risk management

In addition to the operational value of insurance services in supply chain, TCI and PPI, our study connects the flourishing interface of operations management, finance and risk management as well. Babich and Sobel (2004) coordinated the operational and financial decisions in a supply chain for the first time. Recently, Wang et al., (2021) presented a literature review focused on the supply chain finance area. We categorize the associated literature as follows.

Partner conflict and supply chain finance contracts: Tang et al., (2018) made use of bank loan and buyer financing to support supplier when a low capital supplier has financial constraints. Babich et al., (2021) investigated the crowdfunding platform in comparison with traditional financing methods such as bank loan. Kouvelis and Zhao (2012) characterize a financing contract in a supply chain using bank credit and trade credit. Similar to the Kouvelis and Zhao (2012), our study is also built on a game-theoric model. However, we address a bilevel Stackelberg game programming. In addition, we study supply chain financing in the presence of TCI and PPI.

Amount of losing assets after bankruptcy: Kouvelis and Zhao (2018) focused on supply chain players credit rating to determine who should take on the financing role. They assumed that the borrower assets would be completely wiped out if the loan is not fully settled which is similar to the settings in Kouvelis and Zhao (2016). Kouvelis and Zhao (2016) developed a revenue sharing contract when bank credit is available for the supply chain players. Our work differs from theirs by considering partial lost assets after bankruptcy. In other words, bankrupted party loss assets as much as unpaid obligations.

Transfer pricing: Transfer pricing determines the price of products or services between two related firms. Katok and Villa (2021) compared decentralized and centralized transfer pricing where the decentralized is calculated by negotiation and the centralized transfer price is calculated by theoretical and behavioral methods. Hammami and Frein (2014) take advantage of transfer pricing in a supply chain redesign problem. Differently, our study determines the transfer price using the financial contribution of supply chain players.

Developing a mathematical programming model: The next related stream is on finding global optimal solution for a non-convex non-linear supply chain finance contract. Jin et al., (2017) developed an optimization model to find retailer's order quantity and its insurance coverage. However, the supplier decisions are given by parameters and the supplier has no control over the retailer's decisions. Devalkar and Krishnan (2019) designed a model in which trade credit is available in the presence of supplier moral hazard. Differently, to the best of our knowledge, our work is the first study in which a global optimal solution in a supply chain finance contract can be found by mathematical programming using real world case study.

Risk management in supply chain finance: In our work, risk is defined as the likelihood of losing assets due to bankruptcy. The existing literature considers uncertain demand as the cause of bankruptcy. Yang and Birge (2018) provides a risk sharing stochastic model under uncertain demand. Dada and Hu (2008) studied a stochastic financing problem where the retailer has to satisfy uncertain demand, facing a profit maximizing bank. However, their model did not focus on the losing assets as the main cause of bankruptcy. Our work is the first attempt to manage the risk of losing assets in the supply chain finance area. To do so, we develop a data-driven VaR, enabling us to find optimal decision for risk averse decision maker. Hosseini and Verma (2017) applied a VaR method for a railway transportation problem. Lin et al., (2021) introduced a data driven VaR to predict demand of new product using mixed integer and linear programming models. They benefit from kNN, KR, CART and RF machine learning methods to weigh the demand data points. However, when data points are scarce and widely distributed, the forecasted demand may be extremely conservative. On the other hand, our data-driven method takes advantage of logistic regression to estimate demand based on probability of losing assets. Consequently, our method can efficiently forecast demand even for scarce and widely distributed data points. Moreover, this is the first study that use historical data and relevant financial covariates to develop a data-driven VaR to mitigate the financial risk of a supply chain, specifically.

3.2. Supply chain finance hedging model

We now describe the loans hedging problem considering operational and financial decisions where a retailer has a single opportunity to order its requirements prior to the selling season (time 0, the ordering, insuring and production time) from a supplier to satisfy demand (time 1, the selling season).

Consider a supply chain, including a supplier and a retailer as decision makers. Due to the competitive market, a bank is considered for financial support and PPI, and an insurer for providing TCI. After forecasting demand, retailer orders trade credit and products from the supplier while bank credit is available to support both supplier and retailer's financial requirements. Similar to the Kouvelis and Zhao (2012), bank credit interest rate is independent of amount borrowed. To avoid bankruptcy, the repayment of loans, bank credit and trade credit can be guaranteed using

PPI and TCI, respectively. A schematic view of the financial flows of the supply chain is represented in Figure 3-1.

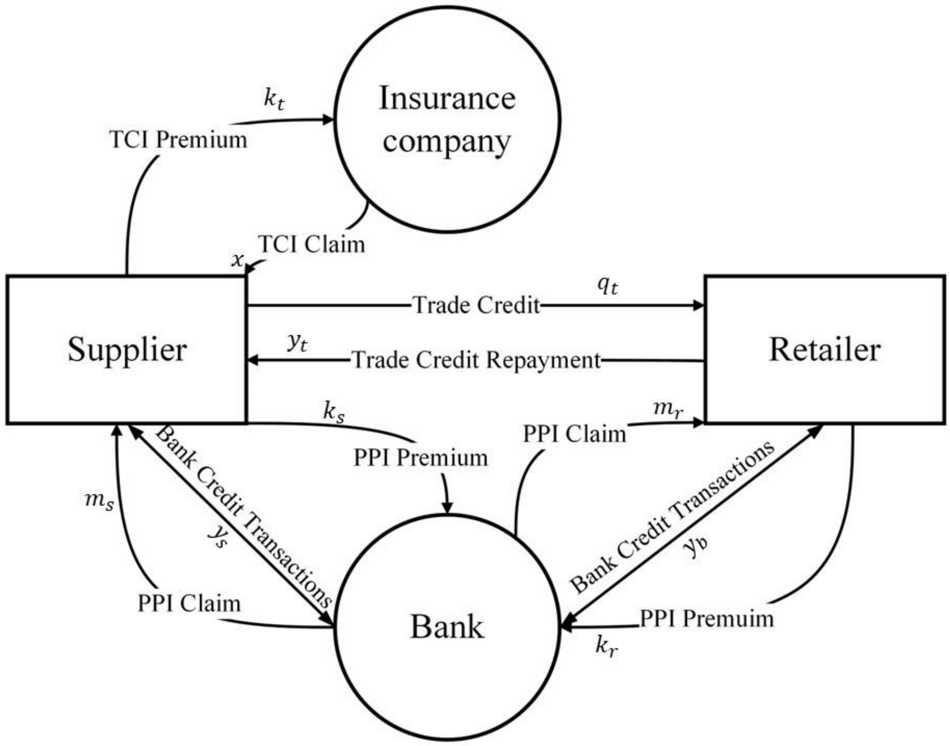


Figure 3-1: Components of the Supply Chain Finance Hedging Problem

Where the transactions consist of the payment and repayment of bank credit and PPI is offered by the bank. For more details, the sequence of events is described in Figure 3-2.

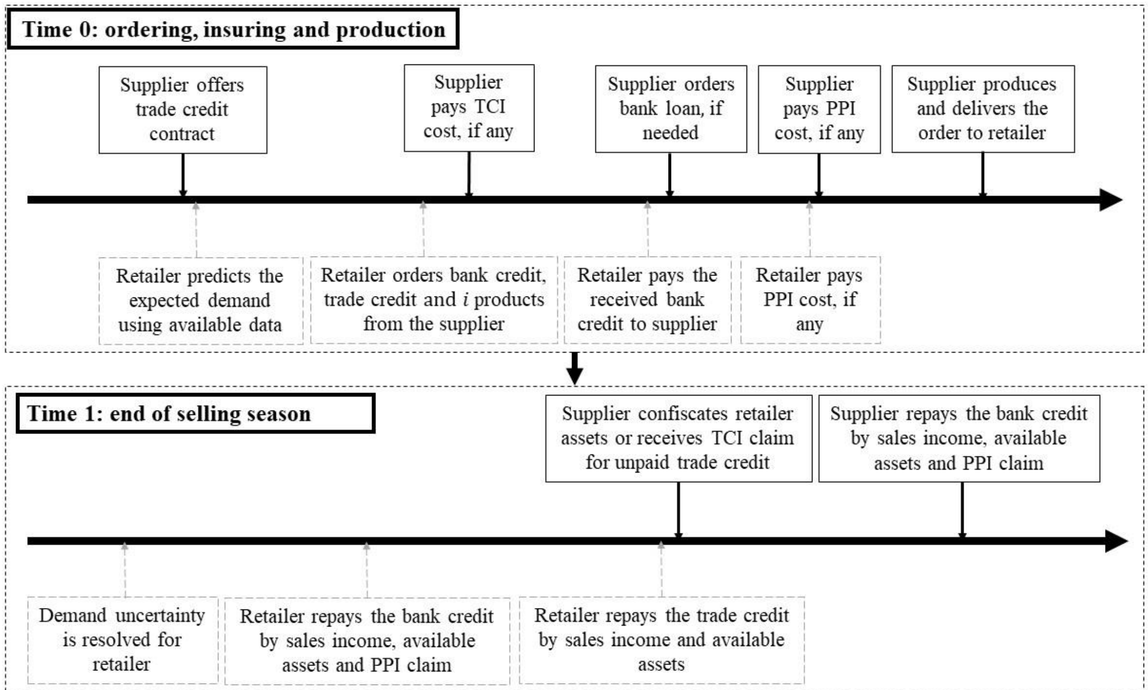


Figure 3-2: The Sequence of Events and Decision Protocol

Supplier offers a well-known *early payment discount contract* (ω, r_t), where ω is the wholesale price at time 0 and r_t is the trade credit interest rate up to time 1. The whole sale price ω is less than the selling price p. The equivalent wholesale price for received trade credit is $\omega(1 + r_t)$. The wholesale price ω is calculated by the financial contribution of players. The OECD organization develop global standards and identify best practices in which governments work together. The OECD guideline manages transfer pricing which is defined as the price of service or product between two interrelated companies. As the financial contribution is intangible, determining the wholesale price ω takes advantage of the *Profit Split* transfer pricing method (OECD, 2018). We make use of the profit split method to determine the wholesale price based on taken financial risk of supply chain players, caused by loan obligations. Furthermore, this policy can ensure financial fairness for players. For example, if the supplier's bargaining power allows for a low commission fee, the retailer can gain a fair portion of product sales. This transfer pricing method is based on taken financial risk of players as their contributions where offering trade credit and supplier's loan repayment is counted as supplier financial contribution and the trade credit interest and retailer's loan repayments are the retailer's financial contribution. If the supplier order TCI at time 0, he can pay θy_t to insurance company where θ is the TCI premium rate for trade credit repayment y_t. TCI insurance can hedge the risk of unpaid trade credit. If the supplier order bank credit y_s with an interest rate r_s, the repayment can be insured by paying PPI cost $\gamma y_s(1 + r_s)$ at time 0, where γ is the PPI premium rate for the supplier bank credit repayment $y_s(1 + r_s)$.

At the end of selling season, retailer repays the bank credit obligation y_b after realizing the actual demand, if possible. On the other hand, retailer assets a_r transfer to bank to pay off the remained obligation or she make use of insurance claim m_r to offset the debt. After paying bank credit obligation, retailer repays trade credit obligation y_t to the supplier, if possible. The retailer must repay the rest of trade credit obligation using her assets a_t to supplier. If the retailer repayment cannot cover the trade credit obligation, the supplier can make use of TCI claim. The supplier repays the bank credit obligation $y_s(1 + r_s)$ by his income. However, supplier transfers his assets a_s to bank to pay off the remained obligation if his income cannot cover bank credit obligation.

Both the supplier and retailer must meet the predicted demand even in the face of losses. At time 0, the exact demand is unknown to everyone. However, at time 1, both the retailer and the supplier become aware of the exact demand. At time 0, they must forecast the demand, and the retailer needs to order the necessary products from the supplier accordingly. The retail price of the product is exogenously specified by market competition. If the retailer accepts the offer, bank loan ωq_b (if needed, with interest rate r_b) and inventory quantity $i = q_t + q_b$ are ordered where

q_t is the purchased products using trade credit and q_b purchased products using bank credit. The retailer makes a payment ωq_b to the supplier at time 0 using the received bank credit. If the retailer decides to hedge the risk of loan using insurance, the PPI cost βy_b is charged, where β is the PPI premium rate for the retailer's bank credit repayment $y_b = \omega q_b(1 + r_b)$. PPI insurance can hedge the risk of unpaid bank credit to protect retailer in the case of less future demand. The supplier's income at time 0 is spent for production cost ci which can be covered by his income ωq_b and bank credit y_s.

Purchasing insurance can hedge the risk of bankruptcy in this case. The insurer guarantees the transferring trade credit risk if it threatens the supplier. Besides, the risk of bank credit can be hedged using PPI, offered by the bank. It means that the bank is responsible for alleviating the bank credit risk by repaying the loan obligation and saving the player's assets if the player purchases the PPI. The following assumptions are assumed in this study. *First*, the repayment of a bank credit obligation takes precedence over a trade credit obligation. *Second*, borrowing loans is the only way to meet the financial requirements of the retailer. *Third*, supplier and retailer lose assets as much as unpaid obligations. *Fourth*, The insured claim fully refunds unpaid loan obligation. *Fifth*, the interest rate is incurred when assets are lost. *Sixth*, the bank credit obligation is less than value of player's assets. *Seventh*, retailer's margin profit takes negative value if operational cost exceeds its commission fee. *Eighth*, price is constant; Maintaining stable prices fosters customer loyalty and trust, ensuring predictable costs for consumers and steady demand for businesses in a fluctuating market. Finally, bank loans are competitively priced.

3.3. Data-driven optimization methodology

This Section develops a mathematical data-driven method to hedge the risk of loans using insurance services. The mathematical models of hedging contracts are described in subsection

4.4.1. A data-driven VaR method is developed to tackle uncertain demand in the subsection 4.4.2. Finally, a solution methodology is presented in subsection 4.4.3.

3.3.1. Mathematical models

This Section develops three contracts in supply chain finance area using a data-driven VaR method. *First*, we develop a model without insurance services. *Second*, hedging the bank credit risk using PPI is added to the supply chain finance model for both the supply chain players. *Third*, the risk of offering trade credit is hedged using TCI. *Finally*, a data-driven VaR is developed to tackle uncertain demand for different risk attitudes. The notation is listed in Table 3-1.

Parameters	
c:	Production cost
p:	Retailer's selling price
r_f:	Interest rate to penalize losing assets
r_s:	Bank credit interest rate for supplier
r_b:	Bank credit interest rate for retailer
$\mathbb{E}_{d\sim\mathcal{D}}[d\|\mathcal{S}]$:	Expected value of uncertain demand d in the presence of covariate $\mathcal{S}$ when d follows the unknown distribution $\mathcal{D}$
a_s^{max}:	Value of supplier's assets
a_r^{max}:	Value of retailer's assets
h:	TCI coverage limit
β:	PPI premium rate for retailer
γ:	PPI premium rate for supplier

θ:	TCI premium rate
π:	Retailer's target profit margin for a product
δ:	Retailer's risk level toward her target profit margin, $\delta \in (0,1)$
Supplier decision variables:	
ω:	Wholesale price
r_t:	Trade credit interest rate
a_s:	Supplier's lost asset value if he does not repay his bank credit obligation
y_s:	Received bank credit by supplier
z_b:	Profit from supplier sale if the retailer borrows bank credit for purchasing
z_t:	Profit from supplier sale if the retailer borrows trade credit for purchasing
ϑ:	Confiscated assets by supplier due to unpaid trade credit obligation
m_s:	PPI claim by supplier
x:	TCI claim by supplier
k_s:	Cost of purchasing PPI by supplier
k_t:	Cost of purchasing TCI by supplier
Ψ:	A binary variable, equal to 1 if supplier purchase PPI; 0, otherwise
Γ:	A binary variable, equal to 1 if supplier purchase TCI; 0, otherwise
Retailer decision variables:	
i:	Purchased inventory by retailer
q_t:	Purchased inventory by retailer using trade credit
q_b:	Purchased inventory by retailer using bank credit
y_b:	Paid bank credit obligation by retailer

y_t:	Paid trade credit obligation by retailer
a_r:	Retailer's lost assets value if she does not repay her bank credit obligation
a_t:	Retailer's lost assets value if she does not repay her trade credit obligation
m_r:	PPI claim by retailer
k_r:	Cost of purchasing PPI by retailer

Table 3-1: Notation

Figure 3-3 illustrates the relationship between supply chain players. The supplier aims to maximize profit from selling products to the retailer, who can decide on the order quantity and financial transactions. In scenarios without insurance services, linear programming models are suggested for both the leader (supplier) and follower (retailer). The supplier, acting as the leader by offering trade credit first, must take responsibility for designing the contract. However, the supplier's model is developed using mixed-integer programming, and the retailer is required to purchase PPI for the second and third contracts.

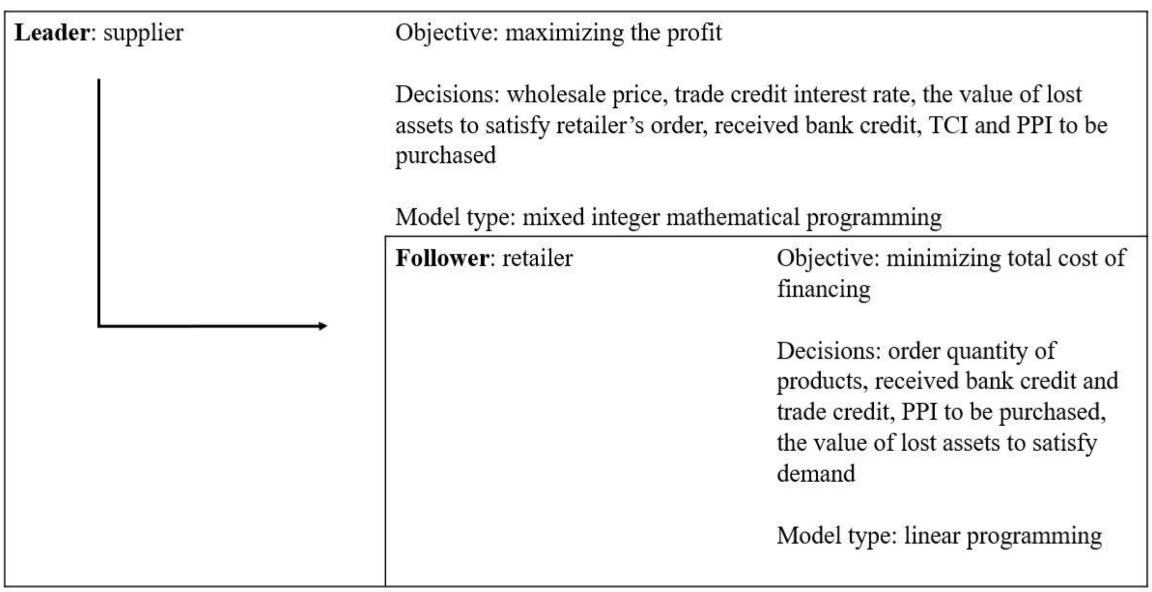


Figure 3-3: Relationship between supply chain players

3.3.1.1. First contract: supply chain finance model without insurance services

This contract finds the optimal value of financial supports without insurance services for the supply chain decision makers. A Stackelberg game problem is developed using a bilevel programming model where supplier and retailer are the leader and follower, respectively. For any given wholesale price ω and trade credit interest rate r_t, the follower problem can be written as follows.

$$\underset{i,q,y_b,y_t,a_t,a_r}{Min} \; \omega r_t q_t + \omega r_b q_b + (a_t + a_r)(1 + r_f) \qquad (3-1a)$$

$S.t.$

$$i = q_t + q_b \qquad (3-1b)$$

$$y_b = \omega q_b (1 + r_b) \qquad (3-1c)$$

$$y_t = \omega q_t (1 + r_t) \qquad (3-1d)$$

$$i = \mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}] \qquad (3-1e)$$

$$y_b - pi \le a_r \qquad (3-1f)$$

$$y_t + y_b - pi - a_r \le a_t \qquad (3-1g)$$

$$a_t \le y_t \qquad (3-1h)$$

$$a_r \le y_b \qquad (3-1i)$$

$$y_b \le a_r^{max} \qquad (3-1j)$$

$$\mathbb{P}_{\mathcal{D}}\Big[pi - \omega q_t(1 + r_t) - \omega q_b(1 + r_b) - (a_t + a_r)(1 + r_f) \ge \pi \, \mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}]\Big] \ge 1 - \delta \qquad (3-1k)$$

$$i, q_t, q_b, y_b, y_t, a_t, a_r \ge 0 \qquad (3-1l)$$

The objective function (3-1a) minimizes retailer's supply chain finance cost including her loan interests and lost assets. In our model, we prioritize minimizing the cost of supply chain finance within the objective function, under the assumption that the price remains constant and the predicted demand is met. Consequently, since the retailer's revenue is fixed, we have excluded it from consideration. Instead of adopting a profit maximization approach, we have opted for a cost minimization objective function. This function is made up of the retailer's paid interest for bank credit and trade credit, as well as the value of lost assets due to unpaid bank credit and trade credit obligations, which is multiplied by the interest rate. The inventories and purchased products are balanced by constraint (3-1b). The loan repayments to the bank and supplier are shown in constraints (3-1c) and (3-1d), respectively. Constraint (3-1e) assumes that demand is a random variable which depends on covariates $\mathcal{S}$ resulting in a stochastic constraint. To determine its deterministic counterpart, we replace the random demand with its expected value. On the other hand, due to the uncertainty in estimation of the expected demand and presence of covariate $\mathcal{S}$, we consider the whole family of probability distributions fitting the observed data, and to model it, we use a data driven approach. The expected demand estimates depend on the samples, and we have used logistic regression to model the expected demand in the whole probability distributions fitting the data set, which is presented in Section 3.4.2. When the bank credit obligation exceeds the retailer's income, constraint (3-1f) requires the model to lose assets to compensate bank credit. When a retailer is unable to repay a trade credit obligation, she loses assets, as shown in constraint (3-1g) to compensate trade credit. Constraints (3-1f) and (3-1g) also force the model to assign bank credit repayment a higher priority. The model forces the retailer to lose assets to bank by constraint (3-1f), if any; then the remained assets can be confiscated by supplier through constraint (3-1g). The maximum lost asset value to compensate trade credit obligation is restricted by trade credit repayment in constraint (3-1h). The amount of lost assets to bank is restricted by bank credit obligation using constraint (3-1i). Constraint (3-1j) ensures that the bank credit obligation is less than the value of retailer's assets. Constraint (3-1k) demonstrates the retailer's risk concern and ensures the probability of meeting target profit $\pi\ \mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}]$, below a level δ. This VaR constraint guarantees that the probability that retailer revenue exceeds target profit $\pi\ \mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}]$ is at least $1-\delta$. Constraints (3-1k) and (3-1e) are connected as the purchased inventory by retailer i is exactly equal to the expected demand $\mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}]$. This connection ensures that the risk attitude of retailer toward revenue is satisfied while the ordered inventory is exactly the same as expected demand. The domain of follower variables are shown in constraint (3-1l).

The supplier determines wholesale price ω and trade credit interest rate r_t. The supplier model can be formulated as follows.

$$\underset{z,\omega,r_t,a_s,y_s,\vartheta}{Max}\ z_b + z_t + \vartheta - y_s r_s - a_s r_f \tag{3-2a}$$

$S.t.$

$$ci - \omega q_b = y_s - z_b \tag{3-2b}$$

$$y_s(1+r_s) - y_t - z_b + a_t - \vartheta \le a_s \tag{3-2c}$$

$$z_b \le q_b(\omega - c) \tag{3-2d}$$

$$z_t \le q_t(\omega(1+r_t) - c) - a_t \tag{3-2e}$$

$$\vartheta \le a_t \tag{3-2f}$$

$$\vartheta \le a_r^{max} - a_r \tag{3-2g}$$

$$\omega(y_b + y_t + y_s(1+r_s)) \le p(\omega q_t + y_s(1+r_s)) \tag{3-2h}$$

$$y_s(1 + r_s) \leq a_s^{max} \qquad (3-2i)$$

$$\omega, r_t, a_t, a_s, y_s, \vartheta \geq 0, z_b, z_t: free \qquad (3-2j)$$

The objective function (3-2a) maximizes the supplier profit which is composed of the profit of selling products to retailer, confiscated assets due to unpaid trade credit, bank credit interest and paid interest for penalizing lost assets. Constraint (3-2b) forces the supplier to receive bank credit if retailer's bank loan is not sufficient for production cost. Constraint (3-2c) forces the supplier to sell his assets if he is unable to reimburse bank credit obligation. The terms $a_t - \vartheta$ show the amount of unpaid trade credit obligation which cannot be compensated by the retailer's assets. Constraints (3-2d) and (3-2e) calculates profit of selling products using bank credit and trade credit, respectively. Constraints (3-2f) and (3-2g) restrict the amount of confiscated assets to retailer's lost assets and its available assets, respectively. Constraint (3-2h) utilize the profit split method for fairness of wholesale price. It determines the maximum value of wholesale price by the financial contribution of supplier. The financial contribution of players is calculated by their taken financial risk. Here, the supplier has to repay the bank credit loan $y_s(1 + r_s)$ and offered trade credit ωq_t. Constraint (3-2i) guarantees that the bank credit obligation is less than the value of supplier's assets. Constraints (3-2j) show the domain of supplier's variables.

Now, we can reformulate the bilevel models (3-1) and (3-2) to a single level optimization problem using the Karush–Kuhn–Tucker (KKT) conditions (Bard, 1988).

3.3.1.2. Second contract: PPI in a supply chain finance model

In this part, we design a contract that not only manages the financial needs of a supply chain, but also incorporates the PPI to mitigate the risk of bank credit obligations. A bilevel programming model is developed to design the contract, similar to the first contract in subsection 3.4.1.1. The retailer problem is represented as follows.

$$\underset{i,q,y_b,y_t,a_t,a_r,k_r,m_r}{\text{Min}} \omega r_t q_t + \omega r_b q_b + (a_t + a_r)\left(1 + r_f\right) + k_r - m_r \qquad (3-3a)$$

$S.t.$

(3-1b) – (3-1l)

$$m_r \leq a_r \qquad (3-3b)$$

$$\beta y_b = k_r \qquad (3-3c)$$

$$k_r, m_r \geq 0 \qquad (3-3d)$$

The follower's paid interest, lost assets, PPI fee, and gained insurance claim are all considered by objective function (3-3a). Constraint (3-3b) restricts the insurance claim by the retailer’s lost assets to the bank. Constraint (3-3c) determines the insurance fee by the associated insurance premium. Constraints (3-3d) show the domain of added variables to the model.

Like the first contract, supplier offers wholesale price ω and trade credit interest rate r_t to the retailer. However, getting PPI may tackle his risk of bankruptcy. The following model is developed to incorporate the PPI in the supplier transactions.

$$\underset{z,\omega,r_t,a_s,y_s,\vartheta,m_s,k_s,\Psi}{Max} z_b + z_t + \vartheta - y_s r_s - a_s r_f + m_s - k_s \qquad (3-4a)$$

$S.t.$

$(3-2b) - (3-2j)$

$$\gamma y_s(1 + r_s) - (1 - \Psi)a_s^{max} \leq k_s \qquad (3-4b)$$

$$m_s \leq \Psi a_s^{max} \tag{3-4c}$$

$$m_s - \tau_r \leq y_s(1 + r_s) - y_t + a_t - \vartheta \tag{3-4d}$$

$$\tau_r \leq (1 - \Psi)a_s^{max} \tag{3-4e}$$

$$\Psi \in \{0,1\} \tag{3-4f}$$

$$m_s, k_s, \tau_r \geq 0 \tag{3-4g}$$

Objective function (3-4a) maximizes the supplier's profit when he can purchase PPI by paying k_s and receiving insurance claim m_s. Constraint (3-4b) determine the PPI fee if supplier purchases the insurance. Constraint (3-4c) assigns insurance claim to the supplier if he purchases PPI. Constraints (3-4d) and (3-4e) avoid receiving insurance claim more than supplier financial loss where τ_r is an auxiliary variable. The constraint (3-4d) permits supplier to receive insurance claim if there is lost assets due to unpaid bank credit obligation. The auxiliary variable τ_r guarantees feasibility of constraint (3-4d) when supplier is profitable. Constraints (3-4f)-(3-4g) show the domain of added variables.

The KKT reformulation can be used to transform the models (3-3)-(3-4) to the single level problem.

3.3.1.3. Third contract: PPI and TCI in a supply chain finance model

This contract hedges the risk of trade credit and bank credit using TCI and PPI, respectively. The application of TCI can improve the financial performance of supplier. The supplier model is presented as follows where the supplier takes a decision on purchasing TCI.

$$\underset{z,\omega,r_t,a_s,y_s,\vartheta,m_s,k_s,\Psi,\mathrm{x},k_t,\Gamma}{\text{Max}} z_b + z_t + \vartheta - y_s r_s - a_s\left(1 + r_f\right) + m_s + x - k_s - k_t \qquad (3-5a)$$

$S.t.$

$(3-2b)-(3-2j),(3-4b)-(3-4g)$

$$\theta y_t - (1-\Gamma)a_s^{max} \leq k_t \qquad (3-5b)$$

$$x \leq a_t - \vartheta \qquad (3-5c)$$

$$x \leq \Gamma h \qquad (3-5d)$$

$$x - \tau_s \leq y_t + y_b - pi - a_r \qquad (3-5e)$$

$$\tau_s \leq (1-\Gamma)a_s^{max} \qquad (3-5f)$$

$$\Gamma \in \{0,1\} \qquad (3-5g)$$

$$x, k_t, \tau_s \geq 0 \qquad (3-5h)$$

The objective function (3-5a) maximizes the profit of supplier where the PPI and TCI fees and their associated claim variables are added to alleviate the risk of borrowed and extended loans, respectively. Constraint (3-5b) determines the TCI fee if supplier purchases the insurance. Constraint (3-5c) restricts the amount of TCI claim by the amount of unpaid trade credit which is not paid by retailer's assets. Constraint (3-5d) shows that TCI claim cannot exceed the insurance coverage. Constraints (3-5e) and (3-5f) avoid receiving TCI claim more than retailer's unpaid debt to supplier where τ_s is an auxiliary variable. Constraint (3-5e) permits supplier to receive TCI claim if there is retailer's lost assets due to unpaid trade credit obligation. The auxiliary variable τ_s guarantees feasibility of constraint (3-5e) when retailer is profitable. Constraints (3-5g) and (3-5h) show the domain of added variables. The follower problem is the same as model (3-4).

3.3.2. Resolving uncertainty

Bankruptcy risk threaten supply chain players when they lose assets due to unpaid loans. The loans are requested based on demand forecasting. Hence, the amount of lost assets is correlated with forecasted demand by retailer. It means that supply chain players take the risk of bankruptcy if borrow loans more than required money to satisfy the demand. By developing data science area, supply chains can benefit from available data to tackle bankruptcy risk. Our data-driven VaR method is related to Lin et al., (2021). One of the limitations of the method proposed by Shen is its ineffective generalization for unseen data. In their approach, new data points are simply assigned to the nearest existing data points. This can be problematic, particularly when the new data falls outside the scope of the available data, resulting in potential underestimation or overestimation of the bankruptcy probability. In contrast, our method employs logistic regression, which estimates parameters based on historical data and allows for more accurate predictions of bankruptcy probability. This approach not only enables us to calculate the probability of bankruptcy for unseen data, but also provides statistical evidence that underscores the significance of the variables involved.

We understand that the demand, denoted by d, follows an unknown distribution $\mathcal{D}$ with respect to covariate $\mathcal{S}$. In order to model the VaR constraint and the conditional constraint (3-1e), we utilize historical data to inform financial decision-making. Suppose there are j data points, where j belongs to the set J of all data points. The characteristics of these data points are represented by $(\hat{d}_j, \hat{\mathcal{S}}_j, \hat{a}_j)$, where $\hat{d}_j$ and $\hat{\mathcal{S}}_j$ correspond to the demand and its associated covariate, respectively, while $\hat{a}_j$ is employed for data classification. To elaborate, $\hat{a}_j$ is equal to 1 if the retailer sells their asset, and 0 otherwise. Let δ symbolize the retailer's risk attitude, defined as $\mathbb{P}_{\mathcal{D}}(a_r + a_t = 0)$, which is determined using the data points $(\hat{d}_j, \hat{\mathcal{S}}_j, \hat{a}_j)$. The risk attitude of the retailer, δ, with respect to the expected demand value $\mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}]$, is calculated employing the Sigmoid function. In this process, the intercept ε and slopes μ_0 and μ_1 are obtained by fitting the logistic regression model to the data points $(\hat{d}_j, \hat{\mathcal{S}}_j, \hat{a}_j)$.

$$\delta = 1 - \frac{1}{1 + e^{-(\mu_0 \mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}] + \mu_1 \mathcal{S} + \varepsilon)}} \qquad (3-6)$$

The function (3-6) assists us to measure the expected value of demand at retailer's risk attitude δ. Using the equation (3-6), the expected value of safe demand at a given risk level δ is represented by formula (3-7).

$$\mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}] = \frac{ln(\frac{1-\delta}{\delta}) - \mu_1 \mathcal{S} - \varepsilon}{\mu_0} \qquad (3-7)$$

Now, we reformulate the developed contracts in subsection 3.4.1. The constraint (3-1e) is substituted by constraint (3-8) as follows.

$$i = \frac{ln(\frac{1-\delta}{\delta}) - \mu_1 \mathcal{S} - \varepsilon}{\mu} \qquad (3-8)$$

The VaR constraint is substituted by constraint (3-9) as follows.

$$pi - \omega q_t(1 + r_t) - \omega q_b(1 + r_b) - (a_t + a_r)(1 + r_f) \geq \pi \ \frac{ln(\frac{1-\delta}{\delta}) - \mu_1 \mathcal{S} - \varepsilon}{\mu} \qquad (3-9)$$

Constraint (3-9) adjust the retailer's demand at level δ to estimate retailer's profit target.

3.3.3. Solution methodology

The proposed contracts are non-convex non-linear models due to bilinear and trilinear terms (i.e., $\omega q_b$and $\omega q_t r_t$) and complementary constraints created by KKT reformulation. Our novel method enables us to find global optimal solution for the single level mathematical programming models. First, we must develop complementary constraints for the follower problem using the dual variables as equation (3-10) where $g_j(x)$ is the follower constraint. The complementary constraints are reformulated by binary variables (Bhavsar and Verma, 2021). Bhavsar and Verma (2021) applied converted the non-linear complementary constraints to a mixed integer programming model by binary variables. Suppose the complementary constraints are shown as follows.

$$\lambda_j g_j(x) = 0 \tag{3-10}$$

Where the Lagrange multipliers λ_j and their related constraints can be separated by the aid of two sets of inequality constraints as follows.

$$\lambda_j \leq \mathcal{M} b_j \tag{3-11a}$$

$$g_j(x) \leq \mathcal{M}(1 - b_j) \tag{3-11b}$$

Where, $\mathcal{M}$ is a big number and b_j is a binary variable for constraint j.

Second, the trilinear terms can be converted to bilinear ones as all of term have the ωr_t multiplication. In real world contracts, the trade credit interest rate r_t set by decimal digit numbers. For example, r_t can be shown by a two decimal digit numbers as 2.35%. Therefore, we can consider a set of possible trade credit interest rates for a wholesale price value. Suppose the set (3-12) contains all values of the possible trade credit interest rate $\hat{r}_t^h$ where $h \in \{1,2,\dots,\bar{r}_t\}$, $\bar{r}_t$ is the maximum value within this set and n is the number of decimal digits.

$$\Lambda = \left\{h \times 10^{-n} | h \in \{1,2,\dots,\bar{r}_t\}\right\} \tag{3-12}$$

Now, the non-linear term ωr_t can be substituted by $\omega_h \hat{r}_t^h$ where $\hat{r}_t^h \in \Lambda$, and following constraints.

$$\omega_h \leq p\Upsilon_h \qquad \forall h \in \{1,2,\dots,\bar{r}_t\} \qquad (3-13a)$$

$$\sum_{h=1}^{\bar{r}_t} \Upsilon_h = 1 \qquad (3-13b)$$

Where Υ_h is a binary variable, equal to 1 if $h \times 10^{-n}$ is selected as trade credit interest rate; otherwise, 0. The new contracts only have bilinear terms which can be solved by Gurobi 9.0 solver. The solver is capable of identifying the global optimal solution for non-convex quadratic objectives and constraints (Gurobi Optimization, n.d.). The proposed models can be found in the Appendix.

3.4. Case study: Ford Motor Company

In this Section, we illustrate the advantages of loans and insurance services for the Ford Motor Company which is ranked among the most valuable automotive manufacturers worldwide. For simplicity, we provide a numerical analysis based on its partnership with a retailer in Chicago metropolitan area, United States. We select the mentioned players because 1) Ford would like to increase its rate of return on debt. The net debt of this automotive company is \$113.2b at the end of September 2021. However, this debt can carry the risk of bankruptcy (Nasdaq, 2021). 2) Ford cooperate with retailers that are independently owned. Consequently, financial decisions are made independently. 3) Ford maintains 12 dealerships in the metropolitan Chicago area (Ford Motor Company, n.d.), which cater to the demands of over 9.5 million residents in the region. The retailers are located in Chicago, Aurora, Orland Park, Oak Lawn, Naperville, Elgin, Downers Grove, Schaumburg, and Arlington Heights. A schematic view of retailers location are represented in Figure 3-4. Satisfying the demand of third largest cities in U.S. by a limited number of retailers can increase their financial needs. On the other hand, failure to meet demand could reduce Ford's market share and damages its reputation in the U.S. competitive market.

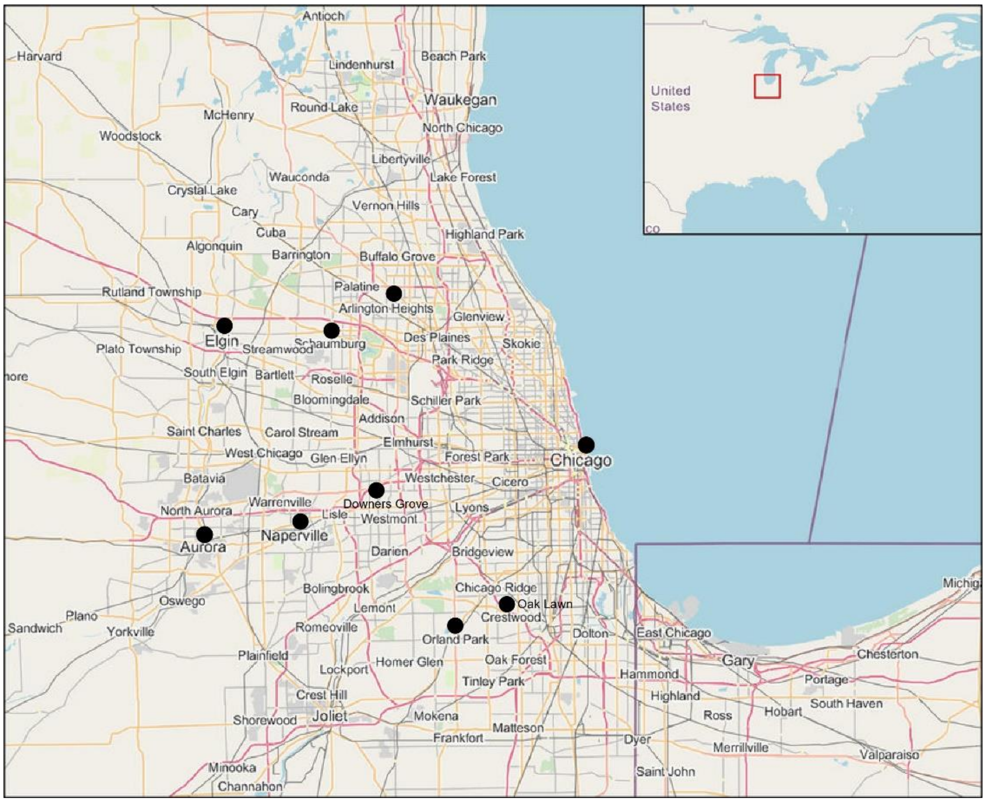


Figure 3-4: A schematic view of Ford retailers location

We would like to improve Ford's profit in this partnership, minimizing the risk of bankruptcy simultaneously. Here, the risk of bankruptcy is defined by losing assets in the presence of inflation rate as covariate. In other words, we are going to attack the risk of bankruptcy caused by ordering products more than the upcoming demand where the inflation rate, as a covariate, might change the customers behavior and purchasing power. The required data for our case study is presented in the below Subsection. We provide a sensitivity analysis on the benefits of developed contracts in this study. *First*, we investigate the advantages of developed supply chain finance contracts by observing the base case solutions. *Second*, we go through various retailer's risk attitude and its impact on the supplier's profitability. *Third*, we discuss how to tackle financial disruption in supply chain. *Finally*, we represent the advantages of profit split transfer pricing in our study.

Here, we provide a summery of parameters presented in Section 3-3. We forecast demand of a single product, Ford Edge, by a retailer in Chicago metropolitan. The monthly demand data for the North America is provided by Good Car Bad Car (GCBC) sales dataset (see https://www.goodcarbadcar.net/ford-edge-sales-figures/). We make use of the reported demand data in United States to forecast the demand for a retailer in metropolitan Chicago. It is assumed that the metropolitan Chicago demand is distributed among retailers uniformly. Now, we estimate the metropolitan Chicago demand by multiplying the data values by the population rate of the region in United States. Therefore, the monthly demand of a retailer is estimated by dividing the monthly metropolitan Chicago demand by the number of retailers in the region. However, these values do not reveal any information about the inflation rate as the covariate. We collected the related historical inflation rate data from U.S. inflation calculator database (see https://www.usinflationcalculator.com/inflation/historical-inflation-rates/). The predicted inflation rate is 4.71, determined by the last reported data in February 2022 (https://tradingeconomics.com/united-states/inflation-cpi).

Rising demand and inflation rates are expected to result in asset losses. After normalizing the demand and inflation rate data, we multiply the normalized demand and inflation rate. Assume the retailer's default rate is between 18.5% (Wang et al., 2018; Boissay and Gropp, 2007) and 24% (Kaur 2019). Among 144 available data points, the highest 18.5% of the multiplied values accounts for lost assets data points while the datapoints in the range of 18.5% to 24% are considered as lost assets randomly. The rest of datapoints are considered as non-lost assets data points. Figure 3-5 shows the lost assets and non-lost assets data points.

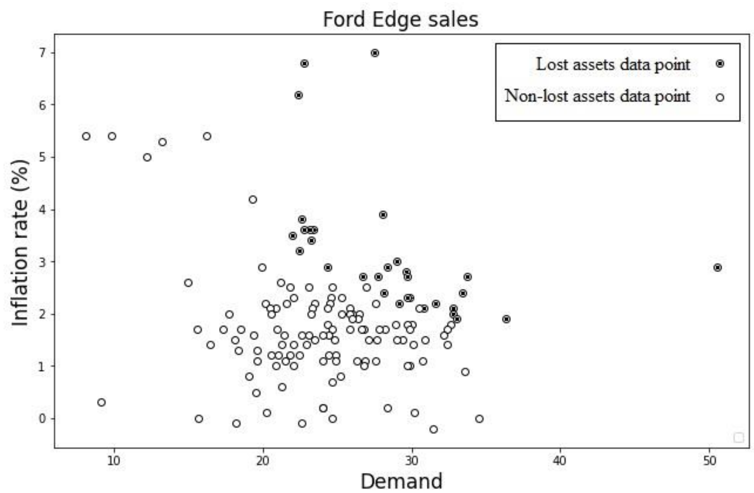


Figure 3-5: Gathered bankruptcy data

We utilize logistic regression method to classify the data. The last 30 percent of data is considered as test data and others are considered as train data. The predicted intercept is -19.9 where the demand and inflation rate coefficients are 0.49 and 2.4, respectively. In other words, the demand coefficient is twice as significant as the inflation rate. By substituting the obtained value from classification, the expected demand is calculated by $\mathbb{E}_{d\sim\mathcal{D}}[d|\mathcal{S}] = \frac{ln(\frac{1-\delta}{\delta})-2.4\mathcal{S}-19.9}{0.49}$. Figure 3-6 reports the test data confusion matrix. This indicates that in 97% of the data points in the test set, logistic regression was able to accurately classify between non-performing and performing assets.

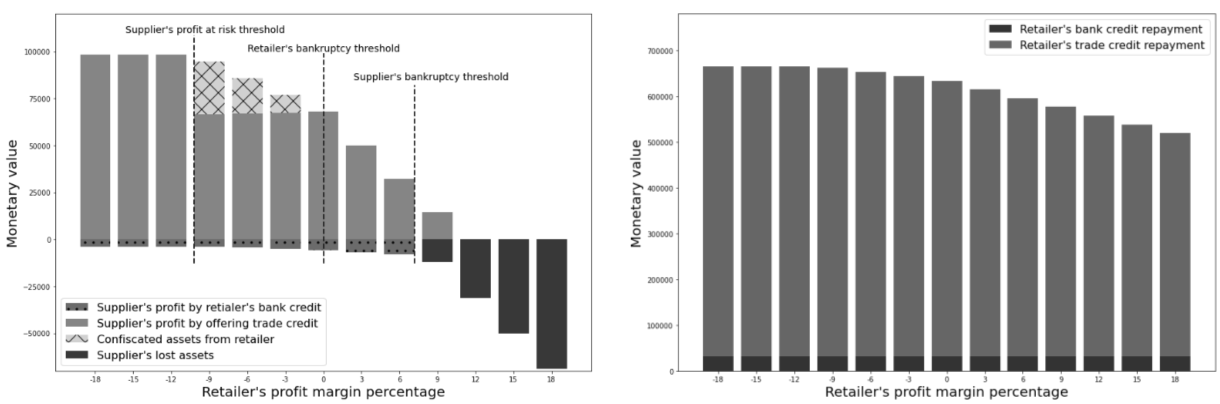
a) Test data b) Training data

Figure 3-6: classification confusion matrix

It is supposed that the product will be sold at starting price. The basic model's manufacturer's suggested retail price is \$36,145 (See https://www.ford.com/suvs-crossovers/edge/) , and the Ford's gross margin for selling a product is 10% (See https://www.llctlc.com/post/how-much-does-it-actually-cost-manufacturers-to-make-a-car). As a result, the car's production cost is around \$32,530. Both the supplier and the retailer are supposed to be AAA credit rated, and the bank will charge them the prime interest rate for the offered Bank credit. Thus, according to JP Morgan dataset, the bank credit interest rate is 3.25%. According to the Euler Hermes company, the conservative TCI premium is 0.25% (See https://www.eulerhermes.com/en_US/what-is-trade-credit-insurance/credit-insurance-cost.html) where the PPI premium offered by CIBC is 0.99% (See https://www.cibc.com/en/personal-banking/insurance/creditor/credit-card-payments.html). It is assumed that Ford's company have enough assets to participate in this contract. However, the value of retailer's assets covers only a portion of the retailer's requirements. In this case, we assume that the retailer's collateral assets constitute 5% of their sales revenue. This is expressed as $p\frac{ln(\frac{1-\delta}{\delta})-2.4\delta-19.9}{9.8}$, which represents the product of 5% and the expected demand. This assumption facilitates the measurement of the contribution of collateral assets to the retailer's overall financial standing, offering crucial insights for effective risk management and financial decision-making. To find maximum value within the trade credit interest rate set (3-12), we make use of the cost of trade credit. Cost of trade credit can estimate the actual cost of this financing scheme (See https://www.thebalancesmb.com/the-cost-of-trade-credit-accounts-payable-392835 for more information). The cost of trade credit formula is calculated by $\left(\text{Discount Percentage} \div (1 - \text{Discount \%})\right) \times \left(\frac{360}{\text{Full allowed payment days} - \text{Discount days}}\right)$. We benefit from the well-known "*2/10 net 30 trade credit*" contract to estimate the cost of trade credit. According to this contract, the supplier offers a 2 percent discount if retailer repays the loan in 10 days; otherwise, the bill due is in 30 days. The cost of trade credit for 2/10 net 30 trade credit contract can set the maximum trade credit interest rate factor. For 360 nominal days in a year, the cost of trade credit is 36%. Finally, the retailer's profit margin is calculated by the retailer's commission fee and operational cost which are not identical in general.

3.4.1. Advantages of financing insurance

Figure 3-7 evaluates different values of profit margin on supply chain decisions when the insurance services are not available.



(a) Supplier's gain (b) Retailer's repayments

Figure 3-7: sensitivity of financial decisions in supply chain without insurance services

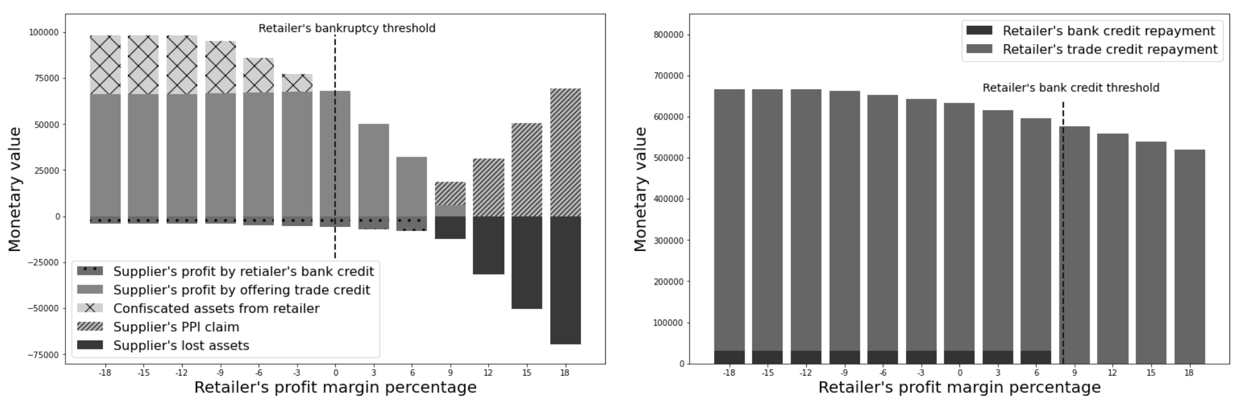
Figure 3-7a bars represent the supplier's profit and lost assets, in the absence of insurance services. Let's consider a negative profit margin retailer. The supplier makes more profit by offering trade credit and confiscating the retailer's assets. We consider supplier's profit at risk threshold as the maximum retailer's debt to supplier that can be paid by her assets. Unexpectedly, if the retailer's profit margin be less than the supplier's profit at risk threshold, the retailer repays the bank loan by her assets and the supplier cannot confiscate the low profit margin retailer assets. In fact, the retailer considers her assets to offset bank credit obligation when her profit margin is awful. The supplier receives the trade credit repayment from the retailer's income. In other words, the supplier's profit at risk threshold determines a level that the supplier cannot take into account the retailer's assets. The supplier's profit at risk threshold is the juncture at which the supplier stands to lose assets, should the retailer fail to repay the trade credit loan. Hence, he is exposed to the risk of losing profit if the retailer is unable to sell the ordered products.

If the retailer's profit margin is more than the supplier's profit at risk threshold, the retailer repays the bank loan by its income and transfer her assets to the supplier. Even for higher profit margin retailer, offering trade credit is more profitable for supplier unless the retailer's profit margin surpasses the supplier's bankruptcy threshold. Surprisingly, figure 3-7b represents that a higher profit margin retailer is reluctant to borrow trade credit as borrowing bank credit prevents paying highly rated trade credit interest. In other words, the retailer maintains the bank credit loan even though she reduces the required trade credit. Interestingly, the win-win game outcomes occur when the profit margin of the retailer be in interval 0 to 7.2 percent, shown by retailer's bankruptcy threshold and supplier's bankruptcy threshold, respectively. Within this interval, neither the supplier nor the retailer faces the risk of bankruptcy. In other words, a win-win contract is designed when the difference between retailer's commission fee and operational cost belongs to this interval.

For more than 7.2 percent retailer's profit margin, supplier starts to lose assets, even though trade credit profit is positive at supplier's bankruptcy threshold. The supplier's bankruptcy threshold is the point at which the supplier can no longer generate a profit and begins to incur losses. However, for more profit margin values, offering trade credit could be costly to supplier as well. Thus, he goes bankrupt by cooperating with a higher profit margin retailer.

PPI:

The risk of bankruptcy might be hedged by PPI. Figure 3-8 represents the impact of PPI on the supply chain components.



(a) Supplier's gain

(b) Retailer's repayments

Figure 3-8: sensitivity of financial decisions in supply chain with PPI

Surprisingly, Figure 3-8a represents that although banks might encourage the borrower to purchase PPI, it is not always efficient. Even when working with a retailer with a positive profit margin, purchasing PPI might be wasteful. However, the supplier's bankruptcy threshold is removed as the lost assets are covered by the PPI claim. Thus, PPI can justify working with a higher profit margin retailer. Moreover, the supplier profit at risk threshold is eliminated as retailer purchased PPI. In the presence of PPI, retailer's assign her assets to supplier as the insurance covers the risk of unpaid bank credit obligation. Hence, the supplier diminishes the risk of profit losing. Here, PPI assists the supplier to prevent the risk of bankruptcy if the retailer's profit margin be more than 7.2 percent. However, it imposes a risk-free interest rate to the supplier to recover the lost assets. In other words, the supplier is no longer at risk of bankruptcy, but the expense of recovering the lost assets is imposed on the supplier for future actions.

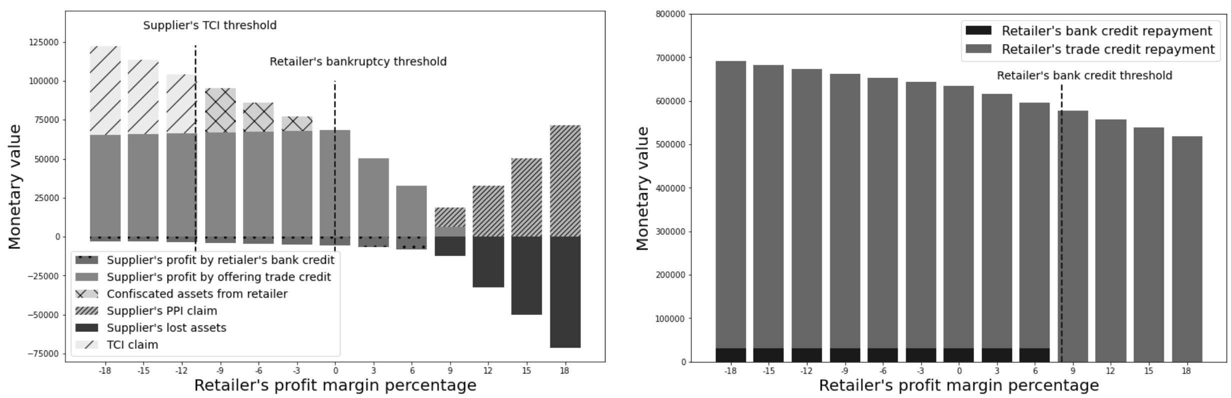
The PPI insurer may consider the moral hazard from the supplier if the retailer profit margin is more than the supplier bankruptcy threshold. In this case, the insurer may be reluctant to offer insurance or increase the PPI premium as much as supplier's PPI claim. The PPI claim covers the value of lost assets. Thus, the insurer set the PPI premium γ' by the optimal value of variables in formula (3-14) where the insurer income must be equal to the lost asset of supplier.

$$\gamma' = \frac{a_s - k_s}{y_s(1 + r_s)} \tag{3-14}$$

On the other hand, the purchased PPI by supplier cannot improve the retailer's bankruptcy threshold. Unexpectedly, figure 3-8b shows that purchasing PPI by supplier helps the retailer to ignore bank credit if the retailer's profit margin be more than 8 percent. We nominate the maximum retailer's profit margin who is interested in borrowing bank credit as retailer's bank credit threshold. In this situation, the supplier is not strict to offer inexpensive trade credit and retailer prefers to borrow more trade credit instead of bank credit. Therefore, the PPI insurance by supplier can be an opportunity and financial aid to the retailer who is unable to present collateral asset for borrowing bank credit. Interestingly, a positive profit margin retailer does not receive PPI claim as there is no confiscated assets by bank. Therefore, the retailer can ignore purchasing PPI if the supplier's risk of profit losing does not matter.

PPI and TCI:

In addition to the PPI, purchasing TCI can benefit the supply chain. Figure 3-9 will shed light on the advantages and disadvantages of purchasing TCI.



(a) Supplier's gain

(b) Retailer's repayments

Figure 3-9: sensitivity of financial decisions in supply chain with PPI and TCI

Figure 3-9a represents that purchasing TCI can improve the supplier's gain significantly. Moreover, supplier can practice dual sourcing to reduce the risk of revenue reduction. Here, purchasing TCI makes sense if the retailer's profit margin be less than -11 percent which is shown by supplier's TCI threshold. The retailer does not lose assets if her profit margin be less than supplier's TCI threshold. Surprisingly, purchasing TCI is not always efficient. Figure 3-9b represents that the retailer's behavior is similar to the second contract where the only available insurance service is PPI. However, the retailer's trade credit repayment is more when her profit margin be less than supplier's TCI threshold as the wholesale price takes more value.

The insurer can increase the TCI premium rate to tackle moral hazard from the supply chain players if the retailer's profit margin be less than the supplier TCI threshold. In this situation, the insurer may avoid offering TCI or increase the TCI premium as much as confiscated assets from the retailer. However, the TCI contract is independent of the retailer decisions. Hence, the insurer can add the value of retailer's assets a_r^{max} to the cost of purchasing insurance k_t to find the TCI premium $\acute{\theta}$as follows.

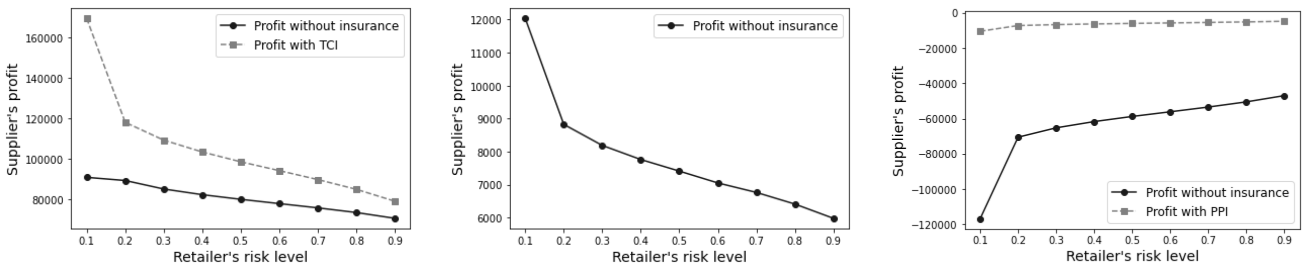
$$\acute{\theta} = \frac{a_r \ - k_t}{y_t} \tag{3-15}$$

Summary of insights:

- In the absence of PPI and TCI, supplier is exposed to the risk of losing profit if retailer's profit margin be less than supplier's profit at risk threshold.
- In the absence of PPI and TCI, a win-win game is designed when retailer's profit margin is between retailer's bankruptcy threshold and supplier's bankruptcy threshold.
- PPI justify working with high profit margin retailer.
- PPI extremely reduce the risk of supplier bankruptcy.
- TCI can significantly improve the supplier profit for working with a risk seeker retailer.

3.4.2. Data-driven risk aversion in supply chain finance

The above discussions are based on a risk neutral retailer where the risk level $\delta = 0.5$ determines the customer's demand. The retailer's attitude toward risk is controlled by the risk level δ. However, the concern is how to hedge the risk against various retailer's risk attitudes. There are three hedging strategies including PPI, TCI, and working with a conservative retailer. The risk level δ determines the retailer's risk attitude where the higher level implies more conservative retailer. Figure 3-10 demonstrates the supplier profit for different risk attitudes.



a) TCI vs. without insurance b) without insurance c) PPI vs. without insurance

Figure 3-10: supplier profit for different retailer's risk level

Figure 3-10a compares the TCI for supplier with the supply chain without insurance services for different risk levels. Here, the retailer's profit margin is supposed to be -16 percent. Surprisingly, supplier benefits from TCI claim to increase his profit. By purchasing TCI, the retailer purchases the products with a higher wholesale price. It increases the supplier's profit by TCI claim and confiscating the retailer's assets. However, the supplier's profit decrease for more conservative retailer. Remarkably, cooperating with a risk averse retailer can devalue purchasing TCI. A more conservative retailer orders fewer products. It reduces the amount of lost assets to the supplier and the TCI claim. Thus, the supplier can ignore TCI for a risk averse retailer.

Figure 3-10b represents the supplier profit when the retailer's profit margin is between supplier's TCI and bankruptcy thresholds. The profit margin for the retailer is fixed at 6 percent. In this case, purchasing insurance services is not recommended. Therefore, the only hedging strategy is cooperating with a risk averse retailer and the supply chain profit decreases for a more conservative retailer.

Figure 3-10c represents the supplier's profit against a high profit margin retailer. Here, we compare the objective functions of first and second contracts for a 16 percent profit margin retailer. The supplier benefits from a risk averse retailer in the absence of insurance services, thereby losing less assets. However, it could not prevent the risk of bankruptcy. On the other hand, purchasing PPI reduce the amount of supplier's lost assets significantly. Clearly, the worst-case solution by purchasing PPI occurs when the retailer is risk seeker. In addition, the best-case solution without PPI occurs when the retailer is risk averse. Surprisingly, purchasing PPI at least works 5.6 times better than cooperating with a risk averse retailer.

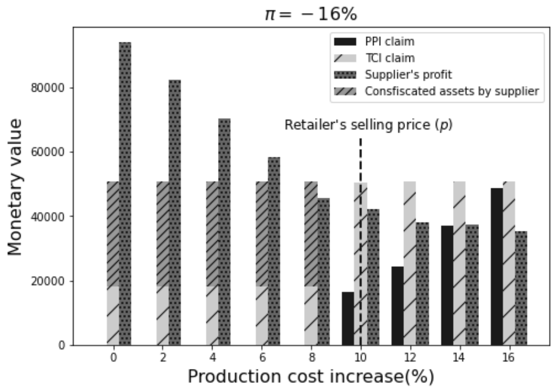
The risk of unsatisfied demand is Imposed when the retailer's risk level is $\delta > 0.5$. In the absence of PPI, a more conservative retailer could decrease the supplier's lost assets significantly. On the other hand, the supplier is vulnerable to meet unsatisfied demand risk. By increasing the risk level, the supplier's lost asset reduction is much more smoother using PPI. Therefore, purchasing PPI mitigates the risk of unsatisfied demand and losing assets simultaneously, even against risk seeker retailer.

Summary of insights:

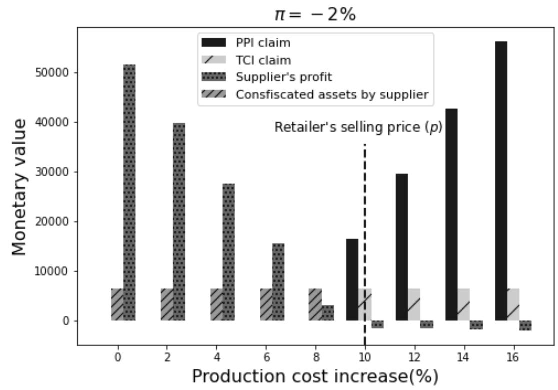
- Working with risk averse retailer can devalue TCI.
- PPI works very well to capture the risk of working with a risky retailer.
- PPI works better than a risk averse retailer.

3.4.3. Benefits of loan insurance to tackle financial disruption

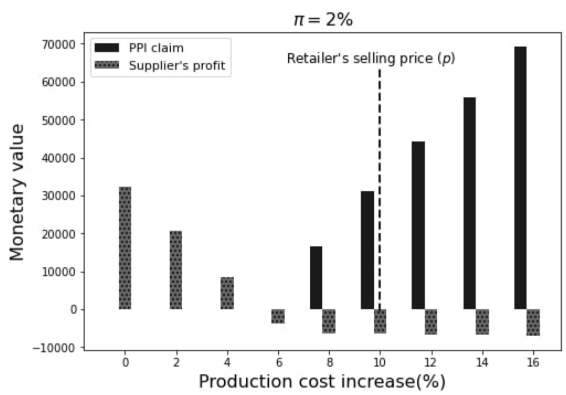
Although a retailer's risk attitude can manage supply chain risks, a disruption in the supply chain might cause policies to change. For example, on 28 February 2022, the WTI crude oil price climbed from \$93.5 to \$107 in only two business days. It might raise production costs and cause supply chain disruptions. The impact of disruption on supplier decisions is investigated in this study. To do so, we raise the unit production cost for various retailer's profit margin values. The consequences of this change in production cost are depicted in Figure 3-11.



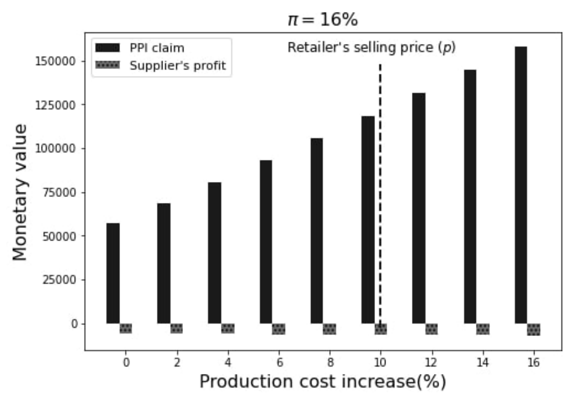
a) low profit margin retailer



b) a little low profit margin retailer



c) a little high profit margin retailer



d) high profit margin retailer

Figure 3-11: supplier's optimal solutions after disruption

Figure 3-11a represents that the supplier's profit decreases by increasing the production cost c. Surprisingly, the supplier's profit remains positive even if the production cost surpasses the retailer's selling price. It confirms the advantages of purchasing insurance for the supply chain if the production cost increases more than 10 percent. In fact, the supplier's lost assets are compensated by the PPI claim and supplier makes profit by receiving TCI claim. However, it increases the insurance company cost. That's why in real world problems, the insurer has the discretion to cancel his participation in the supply chain during the insured period (Yang et al., 2021). Hence, the supplier is damaged significantly if the insurer cancels the insurance contract. To avoid this risk, the supplier can ignore purchasing TCI if the disruption is obvious for others. On the other hand, both the supplier and retailer benefit from TCI and PPI if insurers do not cancel their contracts. In this case, the supplier takes advantage of both retailer's assets and TCI claim if the production cost be less than retailer's selling price. However, the retailer's assets are saved if the production cost surpass the retailer's selling price. In fact, the TCI and PPI covers the value of confiscated assets. In this situation, the supplier purchases both TCI and PPI simultaneously. Clearly, the TCI works better than PPI to raise supplier's profit at first. However, PPI works better gradually if the disruption be more severe.

Figure 3-11b depicts a different story for a better retailer's profit margin. Suppose the retailer's profit margin is -2 percent. The supplier does not take advantage of TCI unless production cost meets retailer's selling price. Surprisingly, if production cost goes up more than the retailer's selling price, the TCI claim covers the value of retailer's lost assets at a moment. However, this coverage is smaller than the last case and it cannot make a profitable supplier. Interestingly, PPI works better than TCI if the retailer is struggling to manage her expenses. Therefore, the cancelability of TCI pales the importance.

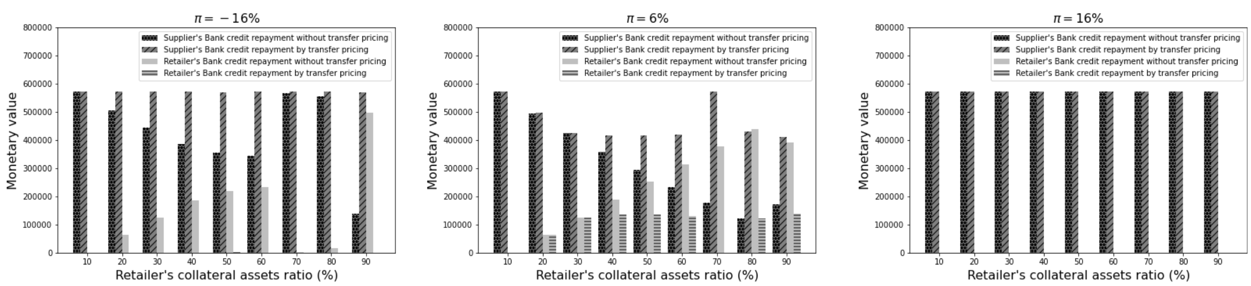
Figure 3-11c shows that the supplier starts requesting PPI claim when production cost is less than the retailer's selling price, mainly because of retailer's profit margin. In the case of more retailer's commission fee, figure 3-11d shows the advantages of PPI to avoid supplier bankruptcy after disruption. Although the supplier's lost does not change too much, the PPI claim grows up significantly.

Summary of insights:

- PPI avoids bankruptcy and TCI preserves supplier profitability in the case of financial disruption.
- Supplier is recommended to avoid purchasing TCI if financial disruption is predictable.
- PPI works better than TCI if the disruption cost is severe.

3.4.4. Benefits of transfer pricing in supply chain finance

In addition to the insurance services, transfer pricing may justify the financial risk of supply chain players. Figure 3-12 discuss the role of transfer pricing in supply chain fairness for different values of retailer's assets. To do so, we analyze the received bank loans of the players by the third contract.



a) low profit margin retailer b) middle profit margin retailer c) high profit margin retailer

Figure 3-12: sensitivity of transfer pricing for supply chain players

According to the observed solutions, transfer pricing and higher values of retailer's collateral assets have no substantial impact on supply chain partners' profits. It does, however, control the risk of taking out bank loans. Borrowing loan from a bank demands not just extra paperwork, but also more stringent requirements. As a result, lowering the amount of bank loan can facilitate supply chain financing, particularly for a retailer with limited collateral assets. In fact, more bank credit raises the risk of financing affordability. Surprisingly, transfer pricing can mitigate the risk of a retailer's financing affordability. Conversely, the supplier may be more vulnerable to the risk of financing affordability. When the retailer's profit margin is -16 percent, Figure 3-12a compares the risk of financing affordability for supply chain players based on the availability of transfer pricing. Clearly, transfer pricing helps a retailer with a negative profit margin tremendously by lower bank credit repayment. In the absence of transfer pricing, a retailer's financing affordability is jeopardized. Notably, transfer pricing literally removes the risk for the retailer. However, this risk is transferred to the profit taker supplier. Figure 3-12b depicts the role of transfer pricing when the retailer's profit margin is 6 percent. In this case, the risk of financing affordability is transferred from retailer to supplier by transfer pricing as well. However, the risk transferring in not as powerful as the negative profit margin retailer. Figure 3-12c confirms that the transfer pricing protects the supplier against the risk of financing affordability being transferred. Therefore, there is no difference caused by transfer pricing in supply chain finance.

Summary of insights:

- Transfer pricing reduces the risk of financing affordability for retailer, while increase the risk for the supplier.

3.5. Conclusion and future studies

In this study, we present three contracts to study hedging tools in supply chain finance including: 1) without insurance services, 2) in the presence of PPI and 3) in the presence of PPI and TCI. Our models allow us to offer an explanation from operations management perspective on the loan insurance in supply chain. They maximize the profit of supplier in cooperating with a retailer who minimize her supply chain finance cost. We also benefit from logistic regression to forecast demand in the presence of financial covariates. It helps the supplier to take optimal decisions against different risk attitude of retailer. Moreover, we develop a solution method to solve the non-convex bilevel models.

We take advantage of Ford Motor Company in cooperating with a retailer in metropolitan Chicago, United States. Our findings show that PPI can avoid supplier's bankruptcy risk, while TCI has another purpose as maintain the supplier profitable as much as possible. We then discuss the advantages of insurance services for different risk attitude of retailer. If the retailer's profit margin is negative, purchasing TCI is recommended to work with a risk seeker retailer. However, working with a risk averse retailer may offset the advantage of TCI. On the other hand, if supplier works with a high profit margin retailer, purchasing PPI is strongly recommended for any cases.

Furthermore, purchasing PPI and TCI s strongly recommended to attack disruption if the retailer's profit is negative. It preserves the supplier's profit at a good level. We also notice that the profit split transfer pricing method can mitigate the risk of financing affordability for retailer if supplier be profitable.

There are so many future research directions to explore. This study states that TCI can always improve the supplier's profitability in cooperating with a low profit margin retailer. This brings the question that why suppliers purchase TCI for cooperating with positive profit margin retailer in practice. One might conjecture that retailer's information asymmetry, various financial and non-financial covariates to forecast demand, taxes, and cancelability of insurance services, play a key role. Furthermore, exploring the TCI and PPI when price is a variable that can be determined by the follower presents an intriguing avenue for future research. We intend to focus on such issues in the future studies. We hope to study the impact of credit rating of supply chain partners on their hedging strategies as well.

References

Alavi, S. H., & Jabbarzadeh, A. (2018). Supply chain network design using trade credit and bank credit: A robust optimization model with real world application. *Computers and Industrial Engineering*. https://doi.org/10.1016/j.cie.2018.08.005

Ashton, J. K., & Hudson, R. (2011). The price, quality and distribution of mortgage payment protection insurance: A hedonic pricing approach.

Babich, V., Marinesi, S., & Tsoukalas, G. (2021). Does crowdfunding benefit entrepreneurs and venture capital investors?. *Manufacturing & Service Operations Management*, *23*(2), 508-524.

Babich, V., & Sobel, M. J. (2004). Pre-IPO operational and financial decisions. *Management Science*, *50*(7), 935-948.

Bard, J. F. (1988). Convex two-level optimization. *Mathematical Programming*. https://doi.org/10.1007/BF01580720

Berlin, M. (2003). Trade Credit: Why Do Production Firms Act as Financial Intermediaries? *Federal Reserve Bank of Philadelphia Business Review*.

Bhavsar, N., & Verma, M. (2021). A subsidy policy to managing hazmat risk in railroad transportation network. *European Journal of Operational Research*.

Campbell, T. A. (2014). What could have been done? Circuit City: A case study of management and employee performance failure. *Performance Improvement*, *53*(4), 16-23.

Cole, R. (2010). *Bank Credit, Trade Credit or No Credit: Evidence from the Surveys of Small Business Finances*. Chicago, IL 60602.

CIBC. (n.d.). Credit card payment protection plan. https://www.cibc.com/en/personal-banking/insurance/creditor/credit-card-payments.html

CNBC (2020), https://www.cnbc.com/2020/12/26/the-10-biggest-retail-bankruptcies-of-2020.html

CNBC (2022), https://www.cnbc.com/2022/06/15/ford-cfo-says-inflation-has-erased-mustang-mach-e-profits-but-isnt-hurting-demand.html?__source=google%7Ceditorspicks%7C&par=google

GlobalAutoIndustry.com. (n.d.). How automotive suppliers can take advantage of trade credit insurance. https://globalautoindustry.com/how-automotive-suppliers-can-take-advantage-of-trade-credit-insurance/

Goolsbee, A. D., & Krueger, A. B. (2015). A retrospective look at rescuing and restructuring General Motors and Chrysler. Journal of Economic Perspectives, 29(2), 3-24.

Dada, M., & Hu, Q. (2008). Financing newsvendor inventory. Operations Research Letters, 36(5), 569-573.

Devalkar, S. K., & Krishnan, H. (2019). The impact of working capital financing costs on the efficiency of trade credit. Production and Operations Management, 28(4), 878-889.

Dittman, J. P. (2015). Protect against bad debt risk.

Ford Motor Company. (n.d.). Ford Dealerships Directory. Retrieved from https://www.ford.com/dealerships/dealer-directory/browse-all/

Gurobi Optimization. (n.d.). Non-Convex Quadratic Optimization. Retrieved from https://www.gurobi.com/resource/non-convex-quadratic-optimization/

Hammami, R., & Frein, Y. (2014). Redesign of global supply chains with integration of transfer pricing: Mathematical modeling and managerial insights. International Journal of Production Economics, 158, 267-277.

Hosseini, S. D., & Verma, M. (2017). A Value-at-Risk (VAR) approach to routing rail hazmat shipments. Transportation research part D: transport and environment, 54, 191-211.

Isidore, C. (2009). GM bankruptcy: End of an era. Retrieved from https://money.cnn.com/2009/06/01/news/companies/gm_bankruptcy/

Jin, W., & Luo, J. (2017). Optimal inventory and insurance decisions for a supply chain financing system with downside risk control. Applied Stochastic Models in Business and Industry. https://doi.org/10.1002/asmb.2219

Jones, P. M. (2010). Trade Credit Insurance. Washington, DC.

Katok, E., & Villa, S. (2021). Centralized or decentralized transfer prices: A behavioral approach for improving supply chain coordination. Manufacturing & Service Operations Management.

Kouvelis, P., & Li, R. (2019). Integrated risk management for newsvendors with value-at-risk constraints. Manufacturing & Service Operations Management, 21(4), 816-832.

Kouvelis, P., & Zhao, W. (2012). Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts. Operations Research. https://doi.org/10.1287/opre.1120.1040

Kouvelis, P., & Zhao, W. (2016). Supply chain contract design under financial constraints and bankruptcy costs. Management Science. https://doi.org/10.1287/mnsc.2015.2248

Kouvelis, P., & Zhao, W. (2018). Who should finance the supply chain? Impact of credit ratings on supply chain decisions. Manufacturing and Service Operations Management. https://doi.org/10.1287/msom.2017.0669

Li, H., Bi, G., Song, W., & Yuan, X. (2022). Trade credit insurance: insuring strategy of the retailer and the manufacturer. International Journal of Production Research, 60(5), 1478-1499.

Li, H., Bi, G., Yuan, X., & Wang, D. (2020). Trade credit insurance in a capital-constrained supply chain. International Transactions in Operational Research, 27(5), 2340–2369. https://doi.org/10.1111/itor.12766

Lin, S., Chen, Y., Li, Y., & Shen, Z. J. M. (2021). Data-Driven Newsvendor Problems Regularized by a Profit Risk Constraint. Production and Operations Management.

OECD (2018), Revised Guidance on the Application of the Transactional Profit Split Method: Inclusive Framework on BEPS: Action 10, OECD/G20 Base Erosion and Profit Shifting Project, OECD Paris.

Motor & Equipment Manufacturers Association 2009. Motor Vehicle Supplier Sector Emergency Financial Assistance Request, Submitted to U.S. Department of Treasury, February 13, 2009.

Nasdaq (2021), Here's Why Ford Motor (NYSE:F) Has A Meaningful Debt Burden, https://www.nasdaq.com/articles/heres-why-ford-motor-nyse%3Af-has-a-meaningful-debt-burden

Pearce, J. (2012). Supplier tactics for dealing with financially distressed corporate customers. Hastings Bus. LJ, 8, 405–444.

Shi, L., Pang, T., & Peng, H. (2021). Optimal strategies of contract-farming supply chain under the cooperative mode of bank-insurance: loan guarantee insurance versus yield insurance. International Transactions in Operational Research.

Statista. (2019). Monthly refunds from payment protection insurance in the United Kingdom (UK) from August 2011 to May 2019 (in million GBP). https://www.statista.com/statistics/746624/monthly-payment-protection-insurance-refunds/

Tang, C. S., Yang, S. A., & Wu, J. (2018). Sourcing from suppliers with financial constraints and performance risk. Manufacturing & Service Operations Management, 20(1), 70-84.

U.S. courts (2020), Table F-2 in https://www.uscourts.gov

Wang, J., Zhao, L., & Huchzermeier, A. (2021). Operations-finance interface in risk management: Research evolution and opportunities. Production and Operations Management, 30(2), 355-389.

Yang, S. A., Bakshi, N., & Chen, C. J. (2021). Trade credit insurance: Operational value and contract choice. Management Science, 67(2), 875-891.

Yang, S. A., & Birge, J. R. (2018). Trade credit, risk sharing, and inventory financing portfolios. Management Science. https://doi.org/10.1287/mnsc.2017.2799

Yang, S. A., Chen, C. J., & Bakshi, N. (2016). Cancelability in Trade Credit Insurance. SSRN Electronic Journal. https://doi.org/10.2139/ssrn.2735907

Appendix 3A:

The single level model of first contract is developed using KKT conditions as follows.

$$Max\ \ z_b + z_t + \vartheta - y_s r_s - a_s r_f \tag{3A-1}$$

$S.t.$

$$(3-1b), (3-1f), (3-1g), (3-1h), (3-1i), (3-1j), (3-1l), (3-2c), (3-2f), (3-2g), (3-2i), (3-2j), (3-8), (3-13a), (3-13b)$$

$$ci - \sum_{h=1}^{\bar{r}_t} \omega_h\, q_b = y_s - z_b \tag{3A-2}$$

$$z_b \le q_b\left(\sum_{h=1}^{\bar{r}_t} \omega_h - c\right) \tag{3A-3}$$

$$z_t \le q_t\left(\sum_{h=1}^{\bar{r}_t} \omega_h\left(1 + \hat{r}_t^h\right) - c\right) - a_t \tag{3A-4}$$

$$\sum_{h=1}^{\bar{r}_t} \omega_h\,(y_b + y_t + y_s(1 + r_s)) \le p\left(\sum_{h=1}^{\bar{r}_t} \omega_h\, q_t + y_s(1 + r_s)\right) \tag{3A-5}$$

$$y_b = \textstyle\sum_{h=1}^{\bar{r}_t} \omega_h\, q_b(1 + r_b) \tag{3A-6}$$

$$y_t = \sum_{h=1}^{\bar{r}_t} \omega_h\, q_t\left(1 + \hat{r}_t^h\right) \tag{3A-7}$$

$$pi - \sum_{h=1}^{\bar{r}_t} \omega_h q_t\left(1 + \hat{r}_t^h\right) - \sum_{h=1}^{\bar{r}_t} \omega_h\, q_b(1 + r_b) - (a_t + a_r)(1 + r_f) \ge \pi\ \frac{ln(\frac{1-\delta}{\delta}) - \mu_1 \mathcal{S} - \varepsilon}{\mu} \tag{3A-8}$$

$$\lambda_6 + \sum_{h=1}^{\bar{r}_t} \omega_h \, (1 + r_b)\lambda_{12} + \lambda_{14} - \sum_{h=1}^{\bar{r}_t} \omega_h \, (1 + r_b)\lambda_{17} = \sum_{h=1}^{\bar{r}_t} \omega_h \, r_b \qquad (3A-9)$$

$$\lambda_7 + \sum_{h=1}^{\bar{r}_t} \omega_h \left(1 + \hat{r}_t^h\right)\lambda_{13} + \lambda_{14} - \sum_{h=1}^{\bar{r}_t} \omega_h \, (1 + r_b)\lambda_{17} = \sum_{h=1}^{\bar{r}_t} \omega_h \, \hat{r}_t^h \qquad (3A-10)$$

$$\lambda_2 - \lambda_{10} + \lambda_{15} + \lambda_{16} - (1 + r_f)\lambda_{17} = (1 + r_f) \qquad (3A-11)$$

$$\lambda_3 - \lambda_{11} + \lambda_{16} - (1 + r_f)\lambda_{17} = (1 + r_f) \qquad (3A-12)$$

$$\lambda_9 + \lambda_{11} - \lambda_{13} - \lambda_{16} = 0 \qquad (3A-13)$$

$$-\lambda_1 + \lambda_8 + \lambda_{10} - \lambda_{12} - \lambda_{15} - \lambda_{16} = 0 \qquad (3A-14)$$

$$\lambda_4 + \lambda_5 - \lambda_{14} + p\lambda_{15} + p\lambda_{16} + p\lambda_{17} = 0 \qquad (3A-15)$$

$$q_t + q_b - i \le \mathcal{M}(1 - b_1) \qquad (3A-16)$$

$$i - d \le \mathcal{M}(1 - b_2) \qquad (3A-17)$$

$$a_r + pi - y_b \le \mathcal{M}(1 - b_3) \qquad (3A-18)$$

$$y_b - \sum_{h=1}^{\bar{r}_t} \omega_h \, q_b(1 + r_b) \le \mathcal{M}(1 - b_4) \qquad (3A-19)$$

$$a_t + a_r + pi - y_t - y_b \le \mathcal{M}(1 - b_5) \qquad (3A-20)$$

$$y_t - a_t \le \mathcal{M}(1 - b_6) \qquad (3A-21)$$

$$i \le \mathcal{M}(1 - b_7) \qquad (3A-22)$$

$$q_t \le \mathcal{M}(1 - b_8) \qquad (3A-23)$$

$$q_b \le \mathcal{M}(1 - b_9) \qquad (3A-24)$$

$$a_t \le \mathcal{M}(1 - b_{10}) \qquad (3A-25)$$

$$a_r \le \mathcal{M}(1 - b_{11}) \qquad (3A-26)$$

$$y_b \le \mathcal{M}(1 - b_{12}) \qquad (3A-27)$$

$$y_t \leq \mathcal{M}(1-b_{13}) \tag{3A-28}$$

$$-y_t + \sum_{h=1}^{\bar{r}_t} \omega_h\, q_t\big(1+\hat{r}_t^h\big) \leq \mathcal{M}(1-b_{14}) \tag{3A-29}$$

$$a_r^{max} - y_b \leq \mathcal{M}(1-b_{15}) \tag{3A-30}$$

$$y_b - a_r \leq \mathcal{M}(1-b_{16}) \tag{3A-31}$$

$$pi - \sum_{h=1}^{\bar{r}_t} \omega_h q_t(1+\hat{r}_t^h) - \sum_{h=1}^{\bar{r}_t} \omega_h\, q_b(1+r_b) - (a_t + a_r)(1+r_f) - \pi\, \frac{ln(\frac{1-\delta}{\delta}) - \mu_1\mathcal{S} - \varepsilon}{\mu} \leq \mathcal{M}(1-b_{17}) \tag{3A-32}$$

$$\lambda_{14} \leq \mathcal{M}b_1 \tag{3A-33}$$

$$\lambda_4 \leq \mathcal{M}b_2 \tag{3A-34}$$

$$\lambda_{15} \leq \mathcal{M}b_3 \tag{3A-35}$$

$$\lambda_{12} \leq \mathcal{M}b_4 \tag{3A-36}$$

$$\lambda_{16} \leq \mathcal{M}b_5 \tag{3A-37}$$

$$\lambda_{11} \leq \mathcal{M}b_6 \tag{3A-38}$$

$$\lambda_5 \leq \mathcal{M}b_7 \tag{3A-39}$$

$$\lambda_7 \leq \mathcal{M}b_8 \tag{3A-40}$$

$$\lambda_6 \leq \mathcal{M}b_9 \tag{3A-41}$$

$$\lambda_3 \leq \mathcal{M}b_{10} \tag{3A-42}$$

$$\lambda_2 \leq \mathcal{M}b_{11} \tag{3A-43}$$

$$\lambda_8 \leq \mathcal{M}b_{12} \tag{3A-44}$$

$$\lambda_9 \leq \mathcal{M}b_{13} \tag{3A-45}$$

$$\lambda_{13} \leq \mathcal{M}b_{14} \tag{3A-46}$$

$$\lambda_1 \leq \mathcal{M}b_{15} \tag{3A-47}$$

$$\lambda_{10} \leq \mathcal{M} b_{16} \tag{3A-48}$$

$$\lambda_{17} \leq \mathcal{M} b_{17} \tag{3A-49}$$

$$\omega_h, i, q_t, q_b, y_b, y_t, a_t, a_r \geq 0 \tag{3A-50}$$

$$\Upsilon_h, b_1, b_2, \dots, b_{17} \in \{0,1\} \tag{3A-51}$$

Where $\lambda_1, \dots, \lambda_{11}$ and $\lambda_{15}, \dots, \lambda_{17}$ are continuous positive variables and $\lambda_{12}, \lambda_{13}, \lambda_{14}$ are free variables.

Appendix 3B:

The single level model of second contract is developed using KKT conditions as follows.

$$Max\, z_b + z_t + \vartheta - y_s r_s - a_s r_f + m_s - k_s$$

$S.t.$

$$(3-1b), (3-1f), (3-1g), (3-1h), (3-1i), (3-1j), (3-1l), (3-2c), (3-2f), (3-2g), (3-2i), (3-2j), (3-8), (3-13a), (3-13b),$$

$$(3-3b)-(3-3d), (3-4b)-(3-4g), (3-A2)-(3-A8), (3-A13), (3-A16)-(3-A31), (3-A33)-(3-A48)$$

$$\lambda_2 - \lambda_{10} + \lambda_{15} + \lambda_{16} - (1+r_f)\lambda_{21} = (1+r_f) \tag{3B-1}$$

$$\lambda_3 - \lambda_{11} + \lambda_{16} + \lambda_{17} - (1+r_f)\lambda_{21} = (1+r_f) \tag{3B-2}$$

$$\lambda_6 + \sum_{h=1}^{\bar{r}_t} \omega_h (1+r_b)\lambda_{12} + \lambda_{14} - \sum_{h=1}^{\bar{r}_t} \omega_h (1+r_b)\lambda_{21} = \sum_{h=1}^{\bar{r}_t} \omega_h r_b \tag{3B-3}$$

$$\lambda_7 + \sum_{h=1}^{\bar{r}_t} \omega_h \left(1+\hat{r}_t^h\right)\lambda_{13} + \lambda_{14} - \sum_{h=1}^{\bar{r}_t} \omega_h (1+r_b)\lambda_{21} = \sum_{h=1}^{\bar{r}_t} \omega_h \hat{r}_t^h \tag{3B-4}$$

$$-\lambda_1+\lambda_8+\lambda_{10}-\lambda_{12}-\lambda_{15}-\lambda_{16}-\beta\lambda_{18}=0 \tag{3B-5}$$

$$\lambda_4+\lambda_5-\lambda_{14}+p\lambda_{15}+p\lambda_{16}+p\lambda_{21}=0 \tag{3B-6}$$

$$\lambda_{20}-\lambda_{17}=-1 \tag{3B-7}$$

$$\lambda_{21}+\lambda_{18}=1 \tag{3B-8}$$

$$a_r-m_r\leq\mathcal{M}(1-b_{17}) \tag{3B-9}$$

$$k_r-\beta y_b\leq\mathcal{M}(1-b_{18}) \tag{3B-10}$$

$$m_r\leq\mathcal{M}(1-b_{19}) \tag{3B-11}$$

$$k_r\leq\mathcal{M}(1-b_{20}) \tag{3B-12}$$

$$pi-\sum_{h=1}^{\bar{r}_t}\omega_h q_t\left(1+\hat{r}_t^h\right)-\sum_{h=1}^{\bar{r}_t}\omega_h\, q_b(1+r_b)-(a_t+a_r)(1+r_f) \\ -\pi\ \frac{ln(\frac{1-\delta}{\delta})-\mu_1\mathcal{S}-\varepsilon}{\mu}\leq\mathcal{M}(1-b_{21}) \tag{3B-13}$$

$$\lambda_{17}\leq\mathcal{M}b_{17} \tag{3B-14}$$

$$\lambda_{18}\leq\mathcal{M}b_{18} \tag{3B-15}$$

$$\lambda_{19}\leq\mathcal{M}b_{19} \tag{3B-16}$$

$$\lambda_{20}\leq\mathcal{M}b_{20} \tag{3B-17}$$

$$\lambda_{21}\leq\mathcal{M}b_{21} \tag{3B-18}$$

The single level model of third contract is developed using KKT conditions as follows.

$$Max\ \ z_b+z_t+\vartheta-y_s r_s-a_s\left(1+r_f\right)+m_s+x-k_s-k_t$$

$$S.t.$$

$(3-1b),(3-1f),(3-1g),(3-1h),(3-1i),(3-1j),(3-1l),(3-2c),(3$

$-2f),(3-2g),(3-2i),(3-2j),(3-8),(3-13a),(3-13b),(3$

$-3b)-(3-3d),(3-4b)-(3-4g),(3-5b)$

$-(3-5h),(3A-2)-(3A-8),(3A-13),(3A-16)$

$-(3A-31),(3A-33)-(3A-48)$

Chapter IV: Managing random Disruptions in Railway Transportation Networks: A Distributionally Robust Optimization Approach

4.1. Introduction

According to the Association of American Railroads, the U.S. freight rail network spans over 140,000 miles and plays a crucial role in the nation's economy. American railroads moved approximately 1.6 million railcars daily (AAR, 2023), which accounted for roughly 28% of all domestic freight transportation (U.S. Department of Transportation, Federal Railroad Administration, n.d.). This highlights the indispensable role of the railway system in connecting businesses and consumers across the United States and emphasizes its significance as a backbone for economic growth and prosperity.

Railway disruption, whether induced by natural events such as hurricanes or earthquakes or by man-made threats like terrorist attacks, can have a crippling effect on security, economy, public health, and safety (US DHS, 2018). Railroad operations, much like business processes, are subject to disruptions that are not rare (WS DOT, 2014) and (Azad et al., 2016). For example, between the years 2009 and 2013, a total of 61 service disruptions were documented for the Amtrak route between Seattle and Vancouver. These interruptions could manifest in one of two ways: either by the full suspension of train services for a specified time frame, known as "annulment", or by a limited disruption that allowed the train to operate only over a subset of its originally planned route. To mitigate the impact of disruptions, it is crucial to design railway infrastructure to operate efficiently under normal conditions and during disruptions (Snyder et al., 2006). Alternatively, assessing the vulnerability of critical infrastructure to failure can help develop risk mitigation and management strategies. This study aims to provide a comprehensive understanding of the challenges posed by random disruption in railway transportation and help develop more effective risk mitigation and management strategies. More specifically, we address random disruption and its effects on service leg capacity within railway transportation networks. Service legs, defined as the individual segments of a railway journey connecting two consecutive stops, are particularly vulnerable to random disruptions. It is notable that service leg random disruption has been explored in the literature. Azad et al., (2016) offer a noteworthy approach to mitigating these issues by introducing an optimization-based methodology specifically targeting service leg disruptions in railroad networks. Jabbarzadeh et al., (2020) further support this perspective, presenting an optimization approach to effectively plan for service leg disruptions due to random events. Their approach underscores the importance and effectiveness of optimized planning in managing service leg disruptions in railroad networks. Starita et al. (2016) formulated a game-theoretical model aimed at safeguarding railway assets from stochastic disruptions. This model has the potential to optimize passenger flows within the system. Shahbi et al. (2019) introduced an innovative methodology designed to enhance the resilience of urban railway transportation systems for passenger conveyance. Their approach accounts for uncertainties in both passenger flow and train operating schedules.

The crucial role of a railway transportation network in today's world demands active planning for railway operators to avoid disruption of service legs. However, due to the unpredictable nature of many disruption sources, such as weather events, technical issues, or human mistakes, it's nearly impossible to prevent all potential problems. This reality highlights why it's so important for railway operators to have mitigation strategies to lessen the effects of disruptions and to recover quickly. Disruptions in the service legs, causing a partial or complete loss of capacity, can greatly affect the railway system's efficiency and reliability. Therefore, these mitigation strategies ensure that railway system can bounce back quickly from any disruption.

Ke and Verma (2021) developed re-routing, repair, and emergency response as key strategies employed by railway operators to ensure the continued delivery of demand following a disruption. However, within the U.S. railway industry, an additional strategy is often utilized - the renting of railcars, or the practice of "trackage rights". This is an arrangement where one railway company pays another competitor to use their tracks, a common occurrence in the U.S. (Cato Institute, 1997). This strategy provides an effective solution in the face of random disruptions, enabling a railway company to leverage the infrastructure of another, thereby ensuring a continuation of services and minimizing the overall impact of disruptions.

In the present study, our primary focus lies on addressing uncertainties related to repair strategies for freight transportation. This focus on repair strategy uncertainty is particularly relevant because other mitigation approaches are often interdependent with the repair strategy. Evaluating the repair strategy becomes especially important when alternative options, such as rerouting, renting railcars, or incurring third-party costs, might be more expensive than implementing a repair strategy. While there are generally two dimensions of uncertainty to consider—cost and time—we aim to manage time-related uncertainties through the application of project management techniques. Specifically, we will explore the trade-offs between the costs incurred and the time required for recovery. The effectiveness of repair strategy in the face of railway disruptions can be significantly influenced by the uncertain nature of repair costs. If these costs are reasonably low, it might render the use of alternative strategies unnecessary, given that repairs are an inevitable aspect of post-disruption recovery. However, determining these costs can be a challenging task, primarily because it depends on the resources required for repair, which can vary and remain uncertain until the actual repair work commences. To address this uncertainty, we have employed a Distributionally Robust Optimization (DRO) approach in this study. DRO is a mathematical method used to manage decision-making problems under uncertainty. This approach allows us to consider the worst-case scenarios for repair costs, ensuring that our mitigation strategies are robust enough to handle high levels of uncertainty. This approach allows us to make more conservative decisions, thereby improving the robustness and efficiency of our disruption recovery strategies.

The above explanation motivates us to explore following research questions. What would be the key factors to establish an optimal contract between a railway company and its competitor, where the railway company determines the quantity of railcars for rent and the competitor sets the pricing? What would be the ideal policy for managing service leg disruptions, considering available strategies such as re-routing, repair, railcar rental, and emergency response? How to manage repair cost uncertainty, and what impact does this uncertainty have on the resilience of the railway transportation network?

We present a novel methodology that employs game theory optimization techniques. Our model involves two intelligent decision-makers: the railway company and its competitor. In the event of a random disruption, the railway company may find it necessary to rent railcars to meet transportation demand. Meanwhile, the competitor determines the rental pricing. As both parties strive to maximize their respective business objectives, this situation naturally transforms into a game-theoretical problem. This renting strategy is designed to help railway operators to make more resilient decisions, both before and after random disruptions that may result in a decrease in capacity. We use this optimization method to model the impact of disruptions on each service leg within the railway network. By analyzing these models, we can determine effective strategies to minimize the impact of potential future disruptions. These strategies include re-routing trains, performing necessary repairs, renting railcars, and implementing emergency responses. To better manage the unpredictable costs of repairs, we apply a "DRO", which enhances the reliability of our recovery strategies. To illustrate how our method can be applied in a real-world context, we provide a case study based on the railway system of a major North American railroad operator. Our analysis revealed that specific itineraries display a substantial dependency on railcar rentals as a disruption management strategy. Moreover, we observed that the engagement of third-party services remains a necessary strategy, despite the implementation of railcar rentals. This insight suggests that a combination of rental and third-party strategies may be integral to a comprehensive disruption management approach. In addition, the implementation of the DRO approach has proven instrumental in providing robust decisions concerning repair strategies. This suggests that DRO is a valuable tool for managing uncertainties associated with repair costs and can facilitate more informed, effective decisions for disruption management.

The rest of the chapter is organized as follows. In Section 4.2, we go through relevant studies that have been published on this topic. This is followed by Section 4.3, which delineates the problem description and outlines the assumptions underlying our study. In Section 4.4, we elaborate on the proposed methodology. Section 4.5 proposes a DRO to tackle uncertainty. Then, in Section 4.6, we share a case study and the results we got from our analysis. Lastly, Section 4.7 offers a summary of our conclusions, outlines the contributions of our study, and suggests potential directions for future research in this area.

4.2. Literature review

The body of literature addressing railway disruptions can be distilled into two defining categories: *first*, investigating the benefits of competitor collaboration in managing disruptions, and *second*, uncertainty in post-disruption strategies.

4.2.1. Benefits of competitor collaboration in handling disruptions

This section unfolds in three subsections: 1) demand satisfaction using competitor capacity, applying competitor resources; 2) railway contract design, applying cooperative game theory; and 3) damaged railcar logistics, discussing management strategies.

Demand satisfaction using competitor capacity: the current literature on railway disruption management primarily focuses on understanding, analyzing, and mitigating disruptions within individual railway companies, often overlooking the potential capacity of competitors. Peterson & Church (2008) pioneered a framework for modeling the impact of disruptions on the rail network, aimed at identifying vulnerabilities and planning for potential risks. This chapter introduces a systematic approach towards understanding the potential effects of disruptions, such as the failure of a bridge or trestle, and devising strategies to manage and mitigate these risks. Khaled et al. (2015) suggest a disruption mitigation strategy that involves identifying critical rail infrastructures, assessing their criticality level through potential rerouting costs, and developing strategic, preparedness, and response plans. This identification helps railway company prioritize protection efforts and redundancy to enhance network resilience. Jabbarzadeh et al. (2020) took an innovative step in researching an optimization approach for planning rail hazmat shipments amid random disruptions. In addition, Albrecht et al. (2013) examined disruption management in the area of rescheduling rail networks amid maintenance disruptions. Using a Problem Space Search metaheuristic, the study demonstrated how train and maintenance schedules could be efficiently optimized on a large-scale rail network. While the developed studies significantly contributed to the domain of disruption management in railway transportation, they overlooked the potential advantage that could be drawn from a competitor's capacity and trackage rights in tackling disruptions.

Railway contract design: The current body of literature reveals a notable research gap in the application of cooperative game theory to railway transportation disruption management. Rosenthal's (2017) study is one of the few instances where cooperative game theory is applied in railway transportation, specifically for cost allocation in a rapid-transit network. However, the focus on disruption management remains underexplored in this area. Similarly, Bhavsar & Verma (2022) addressed hazardous materials (hazmat) risk in railroad transportation networks using a subsidy policy approach. Despite the significance of their work, the application of cooperative game theory was absent, suggesting an area for further investigation in how cooperative game theory can influence such subsidy policies and disruption management.

Other studies such as Khanmohamadi et al. (2018) and Bagheri et al. (2012) provide critical insights into the risks associated with hazmat transportation by rail through security vulnerability analysis and risk modeling. However, they do not explore the potential application of cooperative game theory in managing these risks and disruptions. This pattern extends to the works of Bianco et al. (2016), Verma et al. (2012), and Verma (2011), where despite significant contributions to railway transportation literature, the exploration of cooperative game theory as a tool for managing disruptions is absent. These findings underscore a clear research gap in this domain, highlighting the need for future studies to explore the potential benefits of cooperative game theory in managing railway disruptions, enhancing system resilience, and improving efficiency.

Damaged railcar logistics: The existing studies on railway transportation disruptions largely concentrates on risk management and mitigation strategies, but mainly for undamaged railcars. Ke & Verma (2021) developed a framework for managing disruption risks in rail-truck intermodal transportation networks, focusing on rail yards. Azad et al., (2016) provided an optimization-based methodology to manage disruption risks in railroad networks, especially in service legs. However, the handling and delivery of damaged railcars post-disruption is not explicitly addressed, leaving a noticeable research gap.

Studies by Uddin & Huynh (2019) and Sarhadi et al., (2017) explored the uncertainty in intermodal freight networks and terminal protection, yet they too overlooked the logistics of damaged railcars after a disruption. Despite the significant impact on network resilience, customer satisfaction, and recovery time, practical strategies for dealing with damaged railcars are not widely discussed. This area, currently not receiving the attention it deserves, points to a significant opportunity in railway transportation research.

4.2.2. Uncertainty in post-disruption strategies

This section comprises two key subsections. *First*, uncertainty in disruption recovery, probes the fluctuating nature of repair times. *Second*, DRO for resilience, addresses the potential of this approach in handling uncertainties.

Uncertainty in disruption recovery: Railway disruption management, with a particular focus on uncertain repair times, has been a critical area of investigation. The literature review indicates the existence of several strategies to mitigate these disruptions, but a gap remains in terms of how uncertainty in repair times affects these strategies.

Studies such as those by Van der Hurk et al. (2018) and Xu and Ng (2020) have developed models focusing on disruption management, but these studies often treated uncertain repair times as deterministic or neglected their impact on mitigation strategies such as rerouting or renting railcars. Conversely, the study conducted by Xu et al., (2021) highlighted network resilience but did not thoroughly investigate the impact of uncertain repair times on the effectiveness of specific disruption mitigation strategies.

Efforts to include repair time uncertainty have been made by Tan et al. (2020), who studied passenger evacuation strategies considering uncertain disruption recovery times. However, this research did not extend to other strategies like rerouting or renting railcars. Conversely, Mohammadi et al. (2017) and Fotuhi and Huynh (2018) expanded their focus to multi-modal and intermodal networks but concentrated less on rail transit disruptions.

The current literature lacks comprehensive understanding of how uncertain repair times affect the effectiveness of different mitigation strategies in rail transit disruptions. This presents a research opportunity for more robust models that consider repair time uncertainty and its impact on strategy selection and efficacy.

DRO for resilience: DRO is an advanced optimization approach that considers uncertainty in the problem formulation. Contrasting traditional Robust Optimization (RO) techniques, which typically manage uncertainty by positing worst-case scenarios, DRO adopts a probabilistic perspective. It presumes that the uncertainty conforms to a certain probability distribution, albeit the exact distribution remains unknown. The objective of DRO is to optimize the most unfavorable expectation over all distributions within an ambiguity set, resulting in solutions that exhibit robustness against a multitude of potential scenarios. DRO is advantageous when exact probability distributions are unknown, offers seamless integration with statistical learning techniques for adaptive responses, and provides superior performance under significant data uncertainty by protecting against the worst-case distribution within a designated ambiguity set. It provides a more realistic representation of uncertainty compared to traditional RO methods, as real-world uncertainties often follow some kind of probabilistic distribution. It Also tends to produce more balanced and less conservative solutions than traditional RO, as it accounts for a range of scenarios rather than just the worst case.

DRO has been applied in various domains, including supply chain management, energy systems, and transportation. However, its application in the area of railway transportation disruptions seems limited.

For instance, while Peng et al. (2021) and Ziaei & Jabbarzadeh (2021) used robust optimization approaches to solve multimodal transportation problems under uncertainty, their models did not use DRO. Similarly, Zheng et al. (2014) used a hyper-heuristic approach for emergency railway transportation planning, and Högdahl & Bohlin (2023) combined simulation-optimization for robust timetabling, but neither study made use of DRO. Zhou et al. (2022) did propose a robust optimization model for high-speed railway train planning considering multiple demand scenarios, but they did not use a distributionally robust approach.

This literature on railway disruptions provides a comprehensive exploration of various dimensions of disruption management, from the aspect of competitor collaboration to methodologies for navigating post-disruption complexities. Despite the substantial advancements in addressing these issues, there remains a notable gap in the thorough integration of progressive optimization methods, such as bi-level optimization and DRO to consider pricing players and repair cost uncertainty, respectively. The integration of these methodologies holds promise in revolutionizing disruption management, yielding solutions that are resilient.

This chapter introduces a pioneering methodology using game theory optimization for the railway sector, focusing on the relationship between railway companies and competitors during disruptions. The approach models the potential impacts of disruptions on various service legs of a railway network, providing robust strategies for mitigation. One significant strategy employed is the DRO to manage unpredictable repair costs, enhancing the reliability of recovery strategies. Through a real-world case study of a North American railway operator, the study underscores the dependency on railcar rentals for certain itineraries, despite the presence of third-party services. This study thereby suggests an integrated approach of rerouting, repairing, rentals and third-party services for optimal disruption management.

4.3. Problem description

The railroad transportation system's physical infrastructure is composed of rail yards and tracks. Tracks create a link between each pair of consecutive yards along a route, forming the service legs of a train's journey. The itinerary available to a railcar for its journey is made up of a sequence of these service legs and intermediate yards, each associated with its own cost and transit time. Figure 4-1 offers a schematic representation of the problem, where a set of individual railcars are transported from the origin yard A to meet the demand at the destination yard E.

However, a random disruption at a virtual node F necessitates the implementation of four recovery strategies to meet demand. The first strategy involves rerouting undamaged railcars via an alternate itinerary, transporting them to the closest yard before the disruption node. The second strategy entails repairing the affected service leg and subsequently forwarding the held railcars to the destination node. The third strategy involves utilizing expensive third-party services to transport the demand to the customer's location, with the third party shipping the lost portion from the origin yard.

The fourth and final strategy involves renting competitor tracks (trackage rights) prior to the disruption to transport the damaged railcars to the destination yard post-disruption. This strategy is not uncommon in North America, as no single railroad company owns all the tracks, and renting competitor tracks to transport railcars across the continent is often necessary. This strategy involves moving the damaged railcars to the nearest competitor's yard and then dispatching them to the destination via the competitor's itineraries and/or train services. The competitor sets the rental cost for their railcars. Similar to the Bhavsar and Verma (2022), we make use of a game theory problem. In this study, the railway company requires rented railcars and the competitor determines the rental price. We posit the railway company as the leader and the competitor as the follower in this game-theoretical framework.

Our study considers weekly demand, assuming that railcars are available at the origin yard at the start of the week and that each demand has a delivery due date.

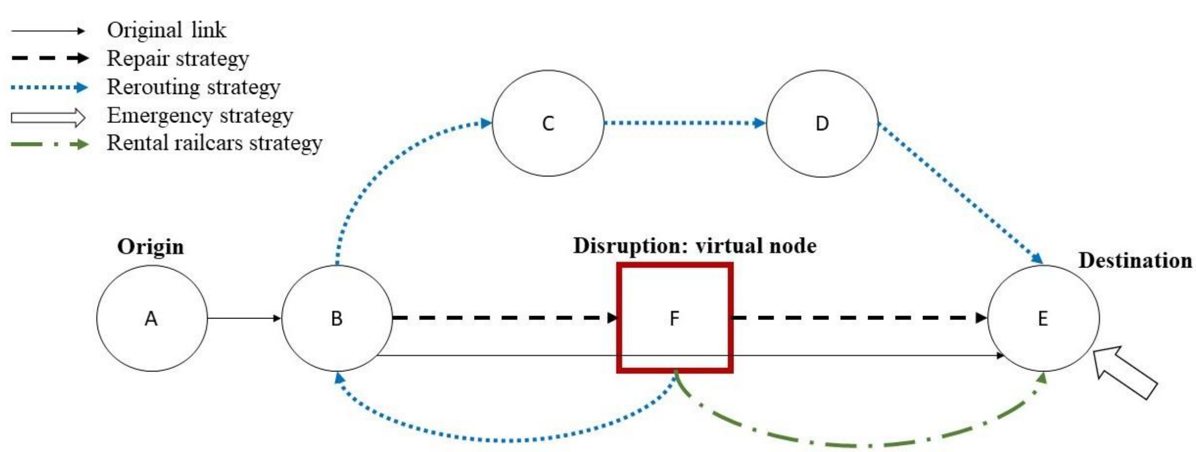


Figure 4-1: schematic of demand satisfaction and recovery strategies

Our approach aims to minimize the transportation cost of railcars and facilitate the recovery of the railway system in the event of a random disruption that could potentially reduce the service legs' capacity.

4.4. Proposed framework

In this section, we initially offer an overview of the proposed framework, specifically designed to handle disruption mitigation and recovery plans. Subsequently, we formulate optimization models for both pre- and post-disruption settings. Finally, we transition the devised bilevel programming problem into a single-level reformulation.

4.4.1. Overview of framework

A summary of the proposed framework is represented in Figure 4-2, including pre- and post-disruption stages. Steps 1-5 form the pre-disruption stage. They identify critical service legs and apply appropriate mitigation strategies. To minimize the cost of transportation for the post-disruption stage, steps 6-8 provide optimal recovery strategies. The details of each stage are represented as follows.

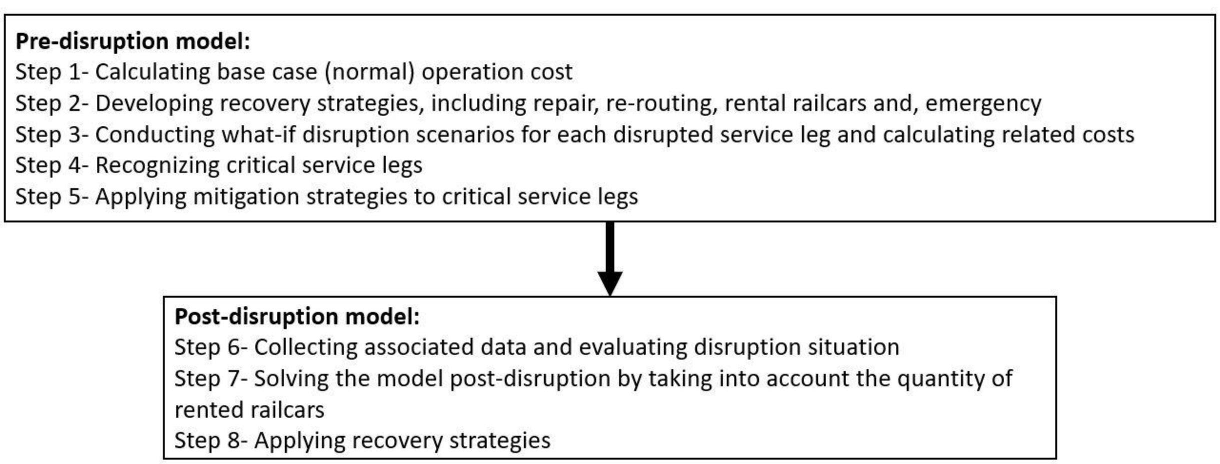


Figure 4-2: proposed framework to manage disruption for rail transportation network

Pre-disruption stage: In *step 1*, we solve a pre-disruption model, presented in Section 4.2, for finding an optimal shipment plan and a benchmark to measure the post-disruption transportation cost. In *step 2*, four different strategies are proposed to reduce the disruption cost, including: repair, re-routing, emergency strategy and renting competitor capacity. In *step 3*, we solve the post-disruption model, presented in Section 4.3, to evaluate the resulting costs of varying levels of disruption for each service leg. *Step 4* makes use of obtained results from last step to determine critical service legs. Particularly, we take advantage of the average disruption cost and the associated standard deviation. *Step 5* decrease the post-disruption cost and improve the recovery capabilities of railroad by implementing the mitigation strategies, including rerouting, repair, renting and emergency, to the critical service legs.

Post-disruption stage: *Step 6* evaluates the level of actual disruption situation. This step can overcome the inefficiencies of step 3 as it covers different l evels of disruption and lost capacity, multiple disrupted service legs and applying mitigation strategies for the railroad. *Step 7* utilize the above steps and re-solve the leader problem of post disruption model after considering the obtained solution for renting railcars in step 3. To do so, the cost of renting railcars would be equal to zero. However, the number of rented railcars in step 3 would be less than the capacity of competitor track. *Step 8* suggests the best recovery strategy for the current disruption situation.

4.4.2. Pre-disruption model

The pre-disruption model minimizes the cost of normal (pre-disruption) operating cost of railroad transportation network, similar to the (Azad et al., 2016). It also obtains the optimal volumes of shipments and number of train services. The associated sets and notations are described as follows.

Sets and indices:

T: set of rail yards, indexed by t;

R_t: set of itineraries using transfer rail yard t;

J: set of demand in a given origin-destination pair, indexed by j;

B_j: set of available itineraries to satisfy demand j before disruption;

I: set of train services, indexed by i;

R_i: set of itineraries using train service i;

Parameters:

dem_j: number of railcars to satisfy demand j;

c_{rj}: railcar transportation cost of demand j on itinerary r;

f_i: operation cost of a train that provides service i;

h_t: capacity of rail yard t for transferring railcars;

u_i: capacity of train service i for carrying railcars on a train;

db_j: delivery due time of demand j before disruption;

td_{rj}: transportation time per railcar of demand j on itinerary r;

Decision variables:

E_{rj}: number of shipped railcars on itinerary r to satisfy demand j

N_i: number of trains that provide train service i

ND_{rj}: $\begin{cases} 1 \text{ if itinerary } r \text{ is used to satisfy demand } j \\ \quad 0 \text{ otherwise} \end{cases};$

The pre-disruption model is represented as follows.

$$Min \sum_{j\in J}\sum_{r\in B_j} c_{rj}E_{rj} + \sum_{i\in I} f_i N_i \qquad (4-1)$$

Subject to:

$$\sum_{j\in J}\sum_{r\in B_j\cap R_t} E_{rj} \le h_t \qquad \forall t \in T \qquad (4-2)$$

$$\sum_{j\in J}\sum_{r\in B_j\cap R_i} E_{rj} \le u_i N_i \qquad \forall i \in I \qquad (4-3)$$

$$\sum_{r\in B_j} E_{rj} = dem_j \qquad \forall j \in J \qquad (4-4)$$

$$E_{rj} \le d_j ND_{rj} \qquad \forall j \in J, r \in B_j \qquad (4-5)$$

$$td_{rj} ND_{rj} \le db_j \qquad \forall j \in J, r \in B_j \qquad (4-6)$$

$$E_{rj}, N_i \ge 0, integer \qquad \forall i \in I, j \in J, r \in B_j \qquad (4-7)$$

$$ND_{rj} \in \{0,1\} \qquad \forall j \in J, r \in B_j \qquad (4-8)$$

The objective function (4-1) minimizes the transportation cost of shipping railcars on itineraries and fixed cost of train services. Constraints (4-2)-(4-3) consider the capacity of rail yards and train services, respectively. Constraint (4-4) satisfy the demand using different itineraries. Constraint (4-5) guarantees that railcars satisfy the demand on itinerary r if the model assign the itinerary for shipping railcars. Constraint (4-6) forces the transportation time to meet the delivery due time. Constraints (4-7)-(4-8) represent the domain of variables.

4.4.3. Post-disruption model

Consider disruption for some service legs and trains passing them, which results in loss of capacity as some or all railcars are destroyed. However, four different recovery strategies are described in Section 4.4.1 to satisfy customer demand. The following set and notations are used for post-disruption mathematical model.

Sets and indices:

K: set of virtual nodes, indexed by k;

R_j: set of available itineraries to satisfy demand j after disruption;

L: set of disrupted service legs, indexed by l;

R_l: set of itineraries using disrupted service leg l;

S: set of service leg repair scenarios, indexed by s;

Parameters:

γ_{jr}: proportion of railcars destroyed, planned to satisfy demand j on itinerary r, ($r \in R_l$);

$\tilde{a}_s$: cost of repair for disrupted service legs under scenario s;

b_j: third party service cost to satisfy demand j;

cap : available capacity of competitor tracks to be rented to satisfy demand j

q_j: maximum competitor tracks revenue per railcar to satisfy demand j if they are not rented;

cc_j: Transportation cost of satisfying demand j for competitor

μ_{rjs}: transportation time per railcar of demand j with repair scenario s on itinerary r that passes through the disrupted service leg ($r \in R_l$);

τ_{rj}: transportation time per railcar of demand j on itinerary r that that does not pass through the disrupted service leg ($r \notin R_l$);

β_j: transportation time per railcar of demand j that that passes through the competitor tracks;

z_j: delivery due time of demand j after disruption;

<u>Decision variables:</u>

X_{rj}: number of railcars of demand j on itinerary r that does not pass through the disrupted service leg ($r \notin R_l$);

Y_{rjs}: number of railcars of demand j on itinerary r that passes through the disrupted service leg after repairing them under scenario s;

V_j: number of railcars of demand j shipped by third party after due date;

P_j: cost of renting a railcar to satisfy demand j using competitor tracks;

W_j: number of rented railcars to satisfy demand j using competitor tracks;

D_j: price of each railcar to satisfy demand j if competitor does not rent them;

M_s: $\begin{cases} 1 \text{ if disrupted service leg is repaired under scenario } s \\ 0 \text{ otherwise} \end{cases}$;

Ω_{rj}: $\begin{cases} 1 \text{ if itinerary } r \text{ that does not pass through the disrupted service leg is used} \\ \text{to satisfy demand } j \\ 0 \text{ otherwise} \end{cases}$;

Ψ_{rjs}: $\begin{cases} 1 \text{ if itinerary } r \text{ that pass through the disrupted service leg is used} \\ \text{to satisfy demand } j \\ 0 \text{ otherwise} \end{cases}$;

Θ_j: $\begin{cases} 1 \text{ if rented railcars are used to satisfy demand } j \\ 0 \text{ otherwise} \end{cases}$;

The post-disruption model is presented as follows where the leader is the railroad company, and the follower is the competitor company who can rent the railcars.

Leader Problem:

$$\underset{X,Y,N,M,V,W}{Min} \sum_{j\in J} \sum_{r\in R_j, r\notin R_l} c_{rj}X_{rj} + \sum_{j\in J} \sum_{r\in R_j\cap R_l} \sum_{s\in S} c_{rj}Y_{rjs} + \sum_{i\in I} f_i N_i + \sum_{s\in S} \tilde{a}_s M_s + \sum_{j\in J} b_j V_j + \sum_{j\in J} P_j\, W_j \qquad (4-9)$$

s.t.

$$\sum_{j\in J} \sum_{r\in R_j\cap R_t, r\notin R_l} c_{rj}X_{rj} + \sum_{j\in J} \sum_{r\in R_j\cap R_l\cap R_t} \sum_{s\in S} c_{rj}Y_{rjs} \leq h_t \qquad \forall t \in T \qquad (4-10)$$

$$\sum_{j\in J} \sum_{r\in R_j\cap R_i, r\notin R_l} c_{rj}X_{rj} + \sum_{j\in J} \sum_{r\in R_j\cap R_l\cap R_i} \sum_{s\in S} c_{rj}Y_{rjs} \leq u_i N_i \qquad \forall i \in I \qquad (4-11)$$

$$\sum_{r\in R_j, r\notin R_l} X_{rj} + \sum_{r\in R_j\cap R_l} \sum_{s\in S} Y_{rjs} \leq \left(1 - \sum_{r\in R_j\cap R_l} \gamma_{jr}\right) dem_j \qquad \forall j \in J \qquad (4-12)$$

$$W_j \leq \left\lfloor \sum_{r\in R_j\cap R_l} \gamma_{jr} \right\rfloor dem_j \qquad \forall j \in J \qquad (4-13)$$

$$\sum_{r\in R_j, r\notin R_l} c_{rj}X_{rj} + \sum_{r\in R_j\cap R_l} \sum_{s\in S} c_{rj}Y_{rjs} + V_j + W_j = dem_j \qquad \forall j \in J \qquad (4-14)$$

$$X_{rj} \leq dem_j \Omega_{rj} \qquad \forall j \in J, r \in R_j \text{ and } r \notin R_l \qquad (4-15)$$

$$Y_{rjs} \leq dem_j \Psi_{rjs} \qquad \forall j \in J, s \in S, r \in R_j \cap R_l \qquad (4-16)$$

$$W_j \leq dem_j \Theta_j \qquad \forall j \in J \qquad (4-17)$$

$$\tau_{rj} \Omega_{rj} \leq z_j \qquad \forall j \in J, r \in R_j \text{ and } r \notin R_l \qquad (4-18)$$

$$\mu_{rjs}\Psi_{rjs} \le z_j \qquad \forall j \in J, s \in S, r \in R_j \cap R_l \qquad (4-19)$$

$$\beta_j \Theta_j \le z_j \qquad \forall j \in J \qquad (4-20)$$

$$\sum_{s \in S} M_s \le 1 \qquad (4-21)$$

$$\Psi_{rjs} \le M_s \qquad \forall j \in J, s \in S, r \in R_j \cap R_l \qquad (4-22)$$

$$W_j \le cap \qquad \forall j \in J \qquad (4-23)$$

$$\Omega_{rj}, \Psi_{rjs}, \Theta_j, M_s \in \{0,1\} \qquad \forall j \in J, s \in S, r \in R_j \qquad (4-24)$$

$$X_{rj}, Y_{rjs}, N_i, V_j, W_j \ge 0, integer \qquad \forall j \in J, s \in S, i \in I, r \in R_j \qquad (4-25)$$

The objective function (4-9) minimizes the transportation cost of the leader including: re-routing cost, transportation cost after repairing, operation cost of train services, repair cost after disruption and cost of renting railcars and transferring products from disrupted service legs to rented railcars. Constraints (4-10)-(4-11) represent the capacity of railyards and train services, respectively. Constraint (4-12) allows the re-routing and emergency strategies to transport undamaged railcars. Constraint (4-13) determines the maximum number of railcars that can be transported by renting railcars strategy. Constraint (4-14) forces the model to satisfy the demand by one of the available strategies, including: re-routing railcars, repairing railcars, using third party, and renting railcars. Constraints (4-15)-(4-17) allow the model to transport the railcars by re-routing, repairing, and renting railcars strategies if they are chosen. Constraints (4-18)-(4-20) ensure that the transportation time for re-routing, repairing, and renting railcars strategy must be less than the delivery time due date. Constraint (4-21) forces the model to offer at most one repair scenario. Constraint (4-22) considers repair strategy for transporting railcars if the associated repair scenario is selected. Constraint (4-23) forces the capacity of using rented railcars strategy. Constraints (4-24)-(4-25) show the domain of variables.

Here, we discuss the competitor decision as the follower. We only take into account the decisions made by the competitor that enable us to meet the leader's demand following a disruption.

Follower Problem:

$$\underset{P,D}{Max} \sum_{j \in J} P_j W_j + \sum_{j \in J} (cap \quad - W_j)\, D_j \tag{4-26}$$

$$P_j \geq cc_j \qquad \forall j \in J \tag{4-27}$$

$$P_j + D_j \geq q_j \qquad \forall j \in J \tag{4-28}$$

$$P_j \leq b_j \qquad \forall j \in J \tag{4-29}$$

$$P_j, D_j \geq 0 \qquad \forall j \in J \tag{4-30}$$

The objective function (4-26) maximizes the revenue of the follower player. The revenue is calculated by lending railcars to the leader and transporting railcars for other customers. Constraint (4-27) forces the renting price to be at least the transportation cost of competitor. Constraint (4-28) ensures the predicted income of a railcar. The predicted income can be found by historical data, expert opinions, regulations, company policies and, etc. Constraint (4-29) restricts the price of lending each railcar to the emergency strategy price. Constraint (4-30) shows the domain of variables in follower model.

4.4.4. Single level reformulation

A two-stage process has been designed to facilitate an enhanced adaptation of the initial optimization program, specifically modified for compatibility with optimization software. The first stage involves developing a closed form solution for the follower problem. Subsequently, we substitute the closed form solution in the leader problem to make a single level reformulation.

4.4.4.1. follower problem solution

We can reformulate the follower problem as follows.

Lemma 1. Given the optimal number of rented railcars W_j^*, the closed form solution can be obtained by (4-31) when $cc_j \leq q_j \leq b_j$.

$$P_j W_j^* = \begin{cases} q_j W_j^* & W_j^* \geq cap - W_j^* \\ cc_j W_j^* & W_j^* < cap - W_j^* \end{cases} \qquad \forall j \in J \qquad (4-31)$$

Proof. we know that for each j, P_j can be anywhere in the range$[cc_j, b_j]$, which is entirely within $[0, q_j]$ because $cc_j \leq q_j \leq b_j$. Now let's reconsider the strategy for assigning P_j and D_j to maximize the objective function:

If $W_j^* \geq cap - W_j^*$, we want to maximize P_j, so we set $P_j = min(q_j, b_j)$ (which is q_j because $b_j \geq q_j$). Then, we set $D_j = q_j - P_j$ which equals 0 because $P_j = q_j$.

If $W_j^* < cap - W_j^*$, we want to minimize P_j, so we set $P_j = max(cc_j, 0)$ (which is cc_j because $cc_j \geq 0$). Then, we set $D_j = q_j - P_j$.

These settings will satisfy all constraints and give the closed-form solution (5-31) for the follower problem.

4.4.4.2. Reduction to a single-level problem utilizing follower closed-form solution

We achieve a reduction from a bi-level to a single-level problem by substituting closed form solution (4-31) within the leader problem. This transformed single-level problem encompasses the decisions of both the upper and lower-level problems. To do so, we can model the problem using a big number $bigm$ and binary decision variable φ_j .

Let's denote φ_j as a binary decision variable such that: $\varphi_j = \begin{cases} 1 & W_j \geq cap - W_j \\ 0 & W_j < cap - W_j \end{cases}$. The large positive number $bigm$, must be greater than or equal to any potential value of W_j; thus $bigm = cap$. Hence, we can rewrite the binary variable constraints as follows.

$$W_j + (1 - \varphi_j)cap \geq cap - W_j \qquad \forall j \in J \qquad (5-32)$$

$$W_j - \varphi_j \; cap < cap - W_j \qquad \forall j \in J \qquad (5-33)$$

Now, we have formulated the conditions of closed form solution (4-31) by the constraints (4-32)-(4-33). To convert the optimal values as well, the following constraints are defined where ψ_j is an auxiliary variable which is equal to $P_j \, W_j$.

$$\psi_j + cc_j\varphi_j \; cap \geq W_j cc_j \qquad \forall j \in J \qquad (5-34)$$

$$\psi_j + q_j(1 - \varphi_j)cap \geq W_j q_j \qquad \forall j \in J \qquad (5-35)$$

Given the constraint (4-32)-(4-34), we can reformulate the post-disruption model as follows.

$$Min \sum_{j\in J} \sum_{r\in R_j, r\notin R_l} c_{rj}X_{rj} + \sum_{j\in J} \sum_{r\in R_j\cap R_l} \sum_{s\in S} c_{rj}Y_{rjs} + \sum_{i\in I} f_i N_i + \sum_{s\in S} \tilde{a}_s M_s + \sum_{j\in J} b_j V_j + \sum_{j\in J} \psi_j \qquad (4-36)$$

s.t.

$$\sum_{j\in J} \sum_{r\in R_j\cap R_t, r\notin R_l} c_{rj}X_{rj} + \sum_{j\in J} \sum_{r\in R_j\cap R_l\cap R_t} \sum_{s\in S} c_{rj}Y_{rjs} \leq h_t \qquad \forall t \in T \qquad (4-37)$$

$$\sum_{j\in J}\sum_{r\in R_j\cap R_i, r\notin R_l} c_{rj}X_{rj} + \sum_{j\in J}\sum_{r\in R_j\cap R_l\cap R_i}\sum_{s\in S} c_{rj}Y_{rjs} \le u_i N_i \qquad \forall i \in I \qquad (4-38)$$

$$\sum_{r\in R_j, r\notin R_l} X_{rj} + \sum_{r\in R_j\cap R_l}\sum_{s\in S} Y_{rjs} \le \left(1 - \sum_{r\in R_j\cap R_l}\gamma_{jr}\right) dem_j \qquad \forall j \in J \qquad (4-39)$$

$$W_j \le \left[\sum_{r\in R_j\cap R_l}\gamma_{jr}\right] dem_j \qquad \forall j \in J \qquad (4-40)$$

$$\sum_{r\in R_j, r\notin R_l} c_{rj}X_{rj} + \sum_{r\in R_j\cap R_l}\sum_{s\in S} c_{rj}Y_{rjs} + V_j + W_j = dem_j \qquad \forall j \in J \qquad (4-41)$$

$$X_{rj} \le dem_j \Omega_{rj} \qquad \forall j \in J, r \in R_j \; and \; r \notin R_l \qquad (4-42)$$

$$Y_{rjs} \le dem_j \Psi_{rjs} \qquad \forall j \in J, s \in S, r \in R_j \cap R_l \qquad (4-43)$$

$$W_j \le dem_j \Theta_j \qquad \forall j \in J \qquad (4-44)$$

$$\tau_{rj}\Omega_{rj} \le z_j \qquad \forall j \in J, r \in R_j \; and \; r \notin R_l \qquad (4-45)$$

$$\mu_{rjs}\Psi_{rjs} \le z_j \qquad \forall j \in J, s \in S, r \in R_j \cap R_l \qquad (4-46)$$

$$\beta_j \Theta_j \le z_j \qquad \forall j \in J \qquad (4-47)$$

$$\sum_{s\in S} M_s \le 1 \qquad (4-48)$$

$$\Psi_{rjs} \le M_s \qquad \forall j \in J, s \in S, r \in R_j \cap R_l \qquad (4-49)$$

$$W_j \le cap \qquad \forall j \in J \qquad (4-50)$$

$$W_j + (1 - \varphi_j) cap \ge cap - W_j \qquad \forall j \in J \qquad (4-51)$$

$$W_j \quad - \varphi_j \; cap < cap - W_j \qquad \forall j \in J \qquad (4-52)$$

$$\psi_j + cc_j \varphi_j \; cap \geq W_j cc_j \qquad \forall j \in J \qquad (4-53)$$

$$\psi_j + q_j(1 - \varphi_j \;)cap \geq W_j q_j \qquad \forall j \in J \qquad (4-54)$$

$$\Omega_{rj}, \Psi_{rjs}, \Theta_j, M_s, \varphi_j \quad \in \{0,1\} \qquad \forall j \in J, s \in S, r \in R_j \qquad (4-55)$$

$$X_{rj}, Y_{rjs}, N_i, V_j, W_j \geq 0, integer \qquad \forall j \in J, s \in S, i \in I, r \in R_j \qquad (4-56)$$

$$\psi_j \geq 0 \qquad \forall j \in J \qquad (4-57)$$

4.5. Implementing a Distributionally Robust Optimization Approach to Address Uncertainties in Repair Time

Railway disruptions, though infrequent, pose significant challenges due to their diverse nature and the scarcity of related data. Traditional approaches like stochastic programming struggle here, requiring extensive data to be effective. Traditional robust optimization, while valuable, often prioritizes decision-maker preferences over actual data. In contrast, DRO adeptly merges real-world statistics with decision-maker risk perspectives. This means that DRO not only bases its strategies on tangible data but also aligns with decision maker risk attitude, ensuring a more holistic and practical response to railway disruptions.

To effectively manage uncertainties associated with repair time, we propose the implementation of a DRO approach. This strategy is designed to offer reliable and optimal solutions, despite the inherent unpredictability of repair costs. For the stochastic repair cost $\tilde{\boldsymbol{a}} = (\tilde{a}_s)$ as $s \in S$, we propose the investigation of a particular ambiguity set, similar to Yang et al., (2023). This set is delineated by the obtainable incomplete data on the characteristics of the distribution $\mathbb{P}$ of $\tilde{\boldsymbol{a}}$, specifically the support, mean, and dispersion parameters.

$$\mathbb{F}_a = \left\{ \mathbb{P} \left| \begin{array}{l} \mathbb{E}_{\mathbb{P}}(\tilde{\boldsymbol{a}}) \in [\underline{\lambda}, \overline{\lambda}] \\ \mathbb{E}_{\mathbb{P}}(|\tilde{a}_s - \lambda_s|) \leq \sigma_s, \qquad \forall s \in S \\ \mathbb{P}(\tilde{\boldsymbol{a}} \in [\underline{\boldsymbol{a}}, \overline{\boldsymbol{a}}]) = 1 \end{array} \right. \right\} \qquad (5-58)$$

Where $\underline{\lambda} = (\underline{\lambda}_s)_{s\in S}$, $\overline{\lambda} = (\bar{\lambda}_s)_{s\in S}$, $\lambda = (\lambda_s)_{s\in S}$, $\sigma = (\sigma_s)_{s\in S}$, $\underline{\boldsymbol{a}} = (\underline{a}_s)_{s\in S}$, $\overline{\boldsymbol{a}} = (\overline{a}_s)_{s\in S}$ with $\lambda \epsilon [\underline{\lambda}, \overline{\lambda}] \subseteq [\underline{a}, \overline{a}]$ are the estimated inputs for the ambiguity set, derived from historical data. Specifically, the first constraint of the ambiguity set $\mathbb{F}_a$ stipulates that the expected repair cost $\mathbb{E}_{\mathbb{P}}(\tilde{\boldsymbol{a}})$ is known to reside within the interval $[\underline{\lambda}, \overline{\lambda}]$. The second constraint utilizes the notion of absolute dispersion, indicating that the marginal dispersion of $\tilde{a}_s$ is proximate to its mean λ_s. It is widely acknowledged that mean absolute deviation is a frequently employed dispersion measure in robust statistics. This measure is less prone to the effects of outliers and deviations from classic model assumptions, rendering it more resilient when the distribution is calculated from historical data (Ghosal & Wiesemann, 2020). The final constraint asserts that the uncertain repair cost $\tilde{\boldsymbol{a}}$ is supported on the rectangular set $[\underline{\boldsymbol{a}}, \overline{\boldsymbol{a}}]$. It is worth highlighting that despite adopting the above specifications, our model can be readily modified to accommodate other variants of the first-order moment ambiguity set, such as the first-order generic moment ambiguity set (Ghosal & Wiesemann, 2020).

Lemma 2. Given the ambiguity set $\mathbb{F}_a$ and the repair strategy decision M_s, the worst-case repair cost $\sup_{\mathbb{P}\in\mathbb{F}_a} (\sum_{s\in S} \tilde{a}_s M_s)$ is equivalent to the following mixed integer programing.

$$Min \sum_{s\in S} \left(\overline{\lambda}_s \eta_s^1 - \underline{\lambda}_s \eta_s^2 + \sigma_s(\eta_s^3 + \eta_s^4) + \eta_5\right) \qquad (4-59)$$

s.t.

$$-\sum_{s\in S}\lambda_s(\eta_s^3-\eta_s^4)+\eta_5\geq\sum_{s\in S}\delta_s \tag{4-60}$$

$$\delta_s\geq\underline{a}_s[M_s-\eta_s^3+\eta_s^4-\eta_s^1+\eta_s^2] \qquad \forall s\in S \tag{4-61}$$

$$\delta_s\geq\overline{a}_s[M_s-\eta_s^3+\eta_s^4-\eta_s^1+\eta_s^2] \qquad \forall s\in S \tag{4-62}$$

$$\eta_s^1,\eta_s^2,\eta_s^3,\eta_s^4\geq 0 \qquad \forall s\in S \tag{4-63}$$

Proof. We restructure the objective function (4-36), incorporating the ambiguity set $\mathbb{F}_a$ as follows:

$$\begin{aligned}Min\sum_{j\in J}\sum_{r\in R_j,r\notin R_l}c_{rj}X_{rj}+\sum_{j\in J}\sum_{r\in R_j\cap R_l}\sum_{s\in S}c_{rj}Y_{rjs}+\sum_{i\in I}f_iN_i+\sum_{j\in J}b_jV_j\\+\sum_{j\in J}\psi_j+\sup_{\mathbb{P}\in\mathbb{F}_a}\left(\sum_{s\in S}\tilde{a}_sM_s\right)\end{aligned} \tag{5-64}$$

Where the term $\sup_{\mathbb{P}\in\mathbb{F}_a}(\sum_{s\in S}\tilde{a}_sM_s)$ considers the worst-case repair cost after disruption under the distribution $\mathbb{P}$ over the ambiguity set $\mathbb{F}_a$. Now, we aim to transform the last term of objective function (4-64) into a tractable reformulation.

$$\sup\int_{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]}\sum_{s\in S}\tilde{a}_sM_s\ d\mathbb{P}(\tilde{\boldsymbol{a}}) \tag{4-65}$$

s.t.

$$\int_{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]}\tilde{a}_s\,d\mathbb{P}(\tilde{\boldsymbol{a}})\leq\overline{\lambda}_s \qquad \forall s\in S \tag{4-66}$$

$$\int_{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]}\tilde{a}_s\,d\mathbb{P}(\tilde{\boldsymbol{a}})\geq\underline{\lambda}_s \qquad \forall s\in S \tag{4-67}$$

$$\int_{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]}(\tilde{a}_s-\lambda_s)\,d\mathbb{P}(\tilde{\boldsymbol{a}})\leq\sigma_s \qquad \forall s\in S \tag{4-68}$$

$$\int_{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]} -(\tilde{a}_s - \lambda_s)\, d\mathbb{P}(\tilde{\boldsymbol{a}}) \leq \sigma_s \qquad \forall s \in S \qquad (4-69)$$

$$\int_{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]} d\mathbb{P}(\tilde{\boldsymbol{a}}) = 1 \qquad \forall s \in S \qquad (4-70)$$

Let $\eta_s^1, \eta_s^2, \eta_s^3, \eta_s^4, \eta_5$ represent the dual variables associated with constraints (4-66)-(4-70). According to the classical strong duality as described by Bertsimas et al., (2019), the dual model corresponding to the linear programing model (4-65)-(4-70) can be expressed as follows:

$$Min \sum_{s\in S} \left(\overline{\lambda}_s \eta_s^1 - \underline{\lambda}_s \eta_s^2 + \sigma_s(\eta_s^3 + \eta_s^4) + \eta_5\right) \qquad (4-71)$$

s.t.

$$\sum_{s\in S} \left(\tilde{a}_s(\eta_s^1 - \eta_s^2) + (\tilde{a}_s - \lambda_s)(\eta_s^3 - \eta_s^4)\right) + \eta_5 \geq \sum_{s\in S} \tilde{a}_s M_s \qquad \forall \tilde{\boldsymbol{a}} \in [\underline{\boldsymbol{a}}, \overline{\boldsymbol{a}}] \qquad (4-72)$$

$$\eta_s^1, \eta_s^2, \eta_s^3, \eta_s^4 \geq 0$$

To find the worst-case solution, the constraint (4-72) can be reformulated as:

$$-\sum_{s\in S} \lambda_s(\eta_s^3 - \eta_s^4) + \eta_5$$

$$\geq \underset{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]}{\text{Max}} \left(\sum_{s\in S} \tilde{a}_s M_s - \tilde{a}_s(\eta_s^3 - \eta_s^4) - \tilde{a}_s(\eta_s^1 - \eta_s^2)\right) \qquad (4-73)$$

$$= \underset{\tilde{\boldsymbol{a}}\in[\underline{\boldsymbol{a}},\overline{\boldsymbol{a}}]}{\text{Max}} \left(\sum_{s\in S} \tilde{a}_s(M_s - \eta_s^3 + \eta_s^4 - \eta_s^1 + \eta_s^2)\right)$$

The inequality (5-73) is equal to the following formula.

$$-\sum_{s\in S}\lambda_s(\eta_s^3-\eta_s^4)+\eta_5 \geq Max\left\{\sum_{s\in S}\underline{a}_s[M_s-\eta_s^3+\eta_s^4-\eta_s^1+\eta_s^2],\sum_{s\in S}\overline{a}_s[M_s-\eta_s^3+\eta_s^4-\eta_s^1+\eta_s^2]\right\} \qquad (4-74)$$

By introducing continuous variable δ_s, we can transform the right hand side of inequality (4-74) to achieve the desired reformulation.

4.6. Case study

In this section, we employ our proposed methodology to a realistic problem instance, drawing from the actual infrastructure of a Class I railroad operator in the mid-western United States as detailed in Verma et al. (2011). We conduct an in-depth exploration of critical service legs and formulate disruption recovery and mitigation strategies. This procedure not only aids managerial decision-making in both the pre-disruption and post-disruption stages but also emphasizes the necessity for a meticulous analysis of each service leg for all trains in the network. By initially outlining the case study specifics and subsequently illustrating the application of our post-disruption model via a specific service leg, we demonstrate the model's potential in generating actionable insights for enhancing network resilience and disruption management.

4.6.1. Case study setting

Figure 4-3 represents the infrastructure of a Class I railroad operator, reconstructed using a Geographic Information System (ESRI, 2008). Parameter coding was carried out in Python, and the problem instances were solved using Gurobi Optimizer version 9.5.1. All computational processes were executed on a laptop computer equipped with an Intel Core i7 2.80GHz and 8 GB memory. Notably, the computational time for each instance solved did not exceed one second of CPU time.

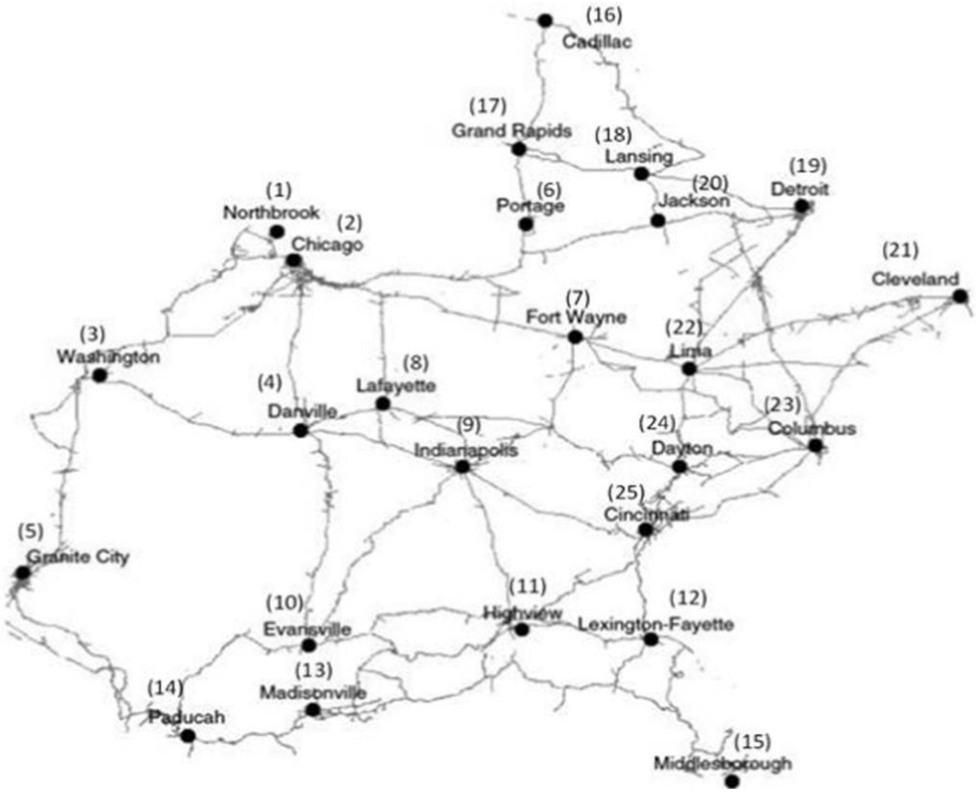


Figure 4-3: representation of the railroad transportation network in the Midwestern United States (adopted from Verma et al., (2011))

The presented network comprises 25 yards, each functioning as both a supply and demand point for the rest. These yards are interconnected via 31 train services, collectively utilizing the 114 service legs - defined as the track segments between consecutive yards - in the network. It's important to note that the train services, described in Verma et al. (2011), were structured to ensure interconnectedness between the yards, subsequently facilitating the definition of itineraries for each supply-demand pair. As a result, among the 600 origin-destination pairs, 1 to 4 itineraries were established for a total of 1338. The network accommodates the movement of 4,440 railcars, with each demand ranging between 1 and 15 railcars, and the delivery due date set at 1.25 times the average transportation time across available itineraries. It is presumed that trains travel at an average speed of 27.7 miles per hour (RPM, 2014), with each train accommodating a maximum capacity of 120 railcars. Cost parameters for train services and yard capacity values are derived from extant scholarly literature (Ahuja et al., 2007; Verma et al., 2011). For instance, we assume a cost of $0.50 per mile to move a railcar, along with an intermediate handling cost of $50 per railcar, while the cost of transportation for the competitor is 5 percent more. The third party cost is 50 percent more than the transportation cost. The fixed cost of a train service is approximated by multiplying the required service completion time by an assumed rate of $100 per hour. For a profitable competitor, the expected revenue is 20 percent more than its transportation cost. Lastly, we contemplate three discrete repair scenarios with repair times of 24, 12, and 6 hours. The cost ranges for each scenario, expressed in thousands of dollars, are as follows: [100,000, 200,000], [200,000, 300,000], and [300,000, 400,000], respectively. The anticipated repair cost fluctuates, falling within a range defined by the average cost of each scenario, plus or minus its standard deviation. The standard deviation for the scenarios is $2,000. This model assumes a project management approach that accelerates completion time by allocating additional resources.

4.6.2. Impact of disruption in a single service leg and train service

To clearly illustrate the functionality of the post-disruption model, we focus on the disruption occurring on the first service leg of the train journey from Chicago to Middlesborough, specifically, the Chicago-Northbrook service leg. This service leg serves 221 itineraries, and we hypothesize a loss of 20% of railcars engaged in this service. This choice, despite the model's capacity to handle any disruption level, aligns with the Federal Railroad Administration's (FRA) 2015 accident statistics. Among 1897 reported railroad accidents, 186 involved railcars, with 105 sustaining partial damage, equating to approximately 22% (FRA, 2015; Azad et al., 2016).

Optimal values ($)		Rerouting strategy	Repair strategy	Emergency strategy	Renting railcars strategy	Utilized trains	Total
Cost	Before disruption	---	---	---	---	90,230	15,411,393
	After disruption	13,301,337	720,782 + 255,000 = 975,782	802,317	1,213,292	77,676	16,370,405
Transported railcars	Before disruption	---	---	---	---	54	4,440
	After disruption	4,066	117	94	163	48	4,440

Table 4-1: optimal Solutions for Train Service: Pre and Post-Disruption Analysis

Table 4-1 details the optimal solutions for pre- and post-disruption scenarios. The first column contemplates the cost and transported railcars. The next two columns display the cost and number of railcars transported for itineraries either bypassing (i.e., rerouting strategy) and using the disrupted service leg (i.e., repair strategy) for both disruption stages. The fourth column represents the solutions for engaging third-party facilities, followed by a strategy involving renting railcars for the disrupted service leg. The sixth column provides the cost and quantity of train services, while the sixth column aggregates the total cost and transported railcars. It should be noted that in the pre-disruption scenario, where the repair, renting of railcars, and emergency service strategies were unnecessary, the associated costs and transported railcars are recorded as zero in their respective fields.

The implementation of recovery strategies clearly increases the post-disruption costs. The primary contributors to this increase are the rerouting of railcars and repair work, involving 4,066 and 117 railcars respectively. The overall expenditure for the repair strategy amounts to $975,782, which encompasses $720,782 for the transit of railcars through the disrupted service leg, and $255,000 for the repair of the disrupted service leg. Furthermore, 94 railcars had to rely on delayed emergency services, while 163 railcars were rented from competitors to address the disruption. A noteworthy observation is the reduction in the number of trains from 54 to 48 due to the deployment of these various strategies.

Our analysis indicates that certain itineraries demonstrate a heightened reliance on emergency strategies. Specifically, the Fort Wayne to Granite City and Grand Rapids to Northbrook itineraries required emergency measures for 11 railcars and Cadillac to Washington requires 10 railcars. Additionally, the Lansing to Northbrook, Lansing to Washington, and Fort Wayne to Washington routes depended on the emergency strategy for 8 railcars. Please see the details of optimal emergency response for disruption in the first service leg of the train journey from Chicago to Middlesborough in Table 4-2.

Itinerary	**Transported railcars by third party**
Northbrook-Granite City	3
Northbrook-Middlesborough	4
Chicago-Granite City	4
Chicago-Paducah	7

Danville-Middlesborough	1
Portage-Northbrook	6
Portage-Washington	1
Fort Wayne-Washington	8
Fort Wayne-Granite City	11
Cadillac-Northbrook	1
Fort Wayne-Highview	1
Cadillac-Washington	10
Grand Rapids-Washington	4
Grand Rapids-Northbrook	11
Lansing-Northbrook	8
Lansing-Washington	8
Dayton-Granite City	6

Table 4-2: optimal transported railcars by third party

Simultaneously, some itineraries heavily depend on railcar rental strategies. For instance, the Danville to Lexington-Fayette itinerary necessitated the rental of 8 railcars from competitors. Likewise, the Northbrook to Madisonville and Chicago to Highview itineraries required 5 rented railcars each. This highlights the variation in strategic response depending on the specifics of the itinerary and disruption.

Interestingly, the post-disruption scenario necessitated an additional train service from Indianapolis to Columbus. Contrastingly, the number of trains operating from Chicago to Middlesborough decreased by two. Further reductions were observed in the services from Paducah to Columbus, Cleveland to Chicago, Chicago to Lexington-Fayette, and Chicago to Columbus, Columbus to Granite City, each declining by one. These changes are detailed in Table 4-3.

Origin	Destination	Number of service legs	Cost per train ($)	Before disruption		After disruption	
				Number of Trains	Trian Services fixed cost	Number of Trains	Trian Services fixed cost
Chicago	Detroit	3	917	2	1834	2	1834
Chicago	Cleveland	4	1343	1	1343	1	1343
Chicago	Columbus	4	1120	2	2240	1	1120
Chicago	Indianapolis	3	897	2	1794	2	1794
Chicago	Lexington-Fayette	4	1337	3	4011	2	2674
Chicago	Middlesborough	6	2893	4	11572	2	5786
Indianapolis	Northbrook	4	1350	1	1350	1	1350
Indianapolis	Middlesborough	4	2090	2	4180	2	4180
Indianapolis	Cadillac	3	1337	2	2674	2	2674
Indianapolis	Cleveland	4	1347	2	2694	2	2694
Indianapolis	Columbus	3	863	1	863	2	1726
Indianapolis	Chicago	4	1273	1	1273	1	1273
Columbus	Chicago	4	1400	2	2800	2	2800
Columbus	Granite City	5	2177	2	4354	1	2177
Columbus	Cadillac	4	1410	2	2820	2	2820
Columbus	Madisonville	4	1960	1	1960	1	1960
Cleveland	Chicago	4	1343	4	5372	3	4029
Cleveland	Paducah	6	2197	2	4394	2	4394
Cincinnati	Detroit	5	1480	1	1480	1	1480
Lexington-Fayette	Chicago	4	1603	5	8015	5	8015

Lexington-Fayette	Grand Rapids	7	2367	2	4734	2	4734
Middlesborough	Fort Wayne	4	1593	1	1593	1	1593
Madisonville	Detroit	5	1930	3	5790	3	5790
Paducah	Columbus	4	1653	3	4959	2	3306
Middlesborough	Cleveland	4	1537	1	1537	1	1537
Middlesborough	Detroit	6	2297	2	4594	2	4594
Total trains				54	90230	48	77676

Table 4-3: optimal solutions of train services before and after disruption

4.6.3. Comprehensive analysis of all service legs

In this section, we provide a comprehensive overview of each service leg and all train services within the network depicted in Fig. 4-3 and Table 4-3, following our in-depth disruption analysis on a single service leg of the train service. We have based our model on the assumption that each service leg, along with one of its train services, experiences disruption every time the model is optimized.

A situation of uncorrelated multiple disruptions, wherein more than one service leg or train service is impacted, can be addressed by identifying the location and impact, denoted by the number of available railcars, of the disruptions, followed by an update to the sets K and L. Again, our model posits that a disruption typically results in a 20% damages.

Table 4-4 reports the transported railcars and average cost per service leg after disruption. The table's first and second columns denote the origin and destination of each service leg, respectively, while the third column corresponds to the sequence of the service leg within the train service.

Column four (CAD) represents the optimal cost of transportation after disruption in the associated service leg. Columns five through eight represent the number of railcars transported post-disruption using rerouting, repair, railcar rental, and emergency response strategies, respectively. we denote rerouted railcars as RR, transported railcars on disrupted service legs as TD, rented railcars from competitors as RC, and transported railcars by third parties as TP. The final column (NO) illustrates the number of train services utilized following the disruption.

Origin	Destination	Order	CAD	RR	TD	RC	TP	NO
Chicago	Middlesborough	First	16,370,406	4,066	117	94	163	48
Cleveland	Chicago	Fourth	16,239,061	4,124	130	69	117	50
Cleveland	Chicago	Third	16,230,513	4,063	198	61	118	51
Lexington-Fayette	Chicago	Fifth	16,187,318	4,071	109	77	183	50
Lexington-Fayette	Chicago	Sixth	16,187,233	4,015	129	89	207	48
Lexington-Fayette	Chicago	Fourth	16,156,203	4,086	121	72	161	51
Chicago	Middlesborough	Fourth	16,100,332	4,290	-	38	112	53
Chicago	Middlesborough	Second	16,094,959	4,222	-	84	134	51
Cleveland	Chicago	Second	16,085,162	4,304	-	49	87	54
Chicago	Lexington-Fayette	Second	16,010,979	4,261	70	-	109	52
Chicago	Middlesborough	Third	15,959,572	4,258	-	61	121	51
Chicago	Lexington-Fayette	First	15,948,866	4,311	-	22	107	52

Madisonville	Detroit	Second	15,915,760	4,189	110	33	108	51
Chicago	Lexington-Fayette	Third	15,907,748	4,298	58	-	84	52
Columbus	Chicago	Second	15,879,639	4,295	-	53	92	52
Indianapolis	Middlesborough	Second	15,847,686	4,338	-	9	93	53
Chicago	Lexington-Fayette	Fourth	15,847,055	4,313	56	-	71	52
Columbus	Chicago	Third	15,810,203	4,327	-	27	86	53
Madisonville	Detroit	First	15,810,127	4,342	-	22	76	52
Chicago	Middlesborough	Fifth	15,807,201	4,321	-	21	98	50
Chicago	Detroit	First	15,801,453	4,268	-	61	111	53
Paducah	Columbus	First	15,768,227	4,350	-	24	66	52
Chicago	Indianapolis	Second	15,762,509	4,306	-	32	102	52
Lexington-Fayette	Chicago	Third	15,762,250	4,287	-	66	87	52
Indianapolis	Cleveland	First	15,755,927	4,318	-	27	95	50
Indianapolis	Middlesborough	First	15,747,823	4,346	-	12	82	52
Chicago	Detroit	Second	15,746,549	4,302	-	55	83	53
Indianapolis	Middlesborough	Third	15,742,484	4,361	-	7	72	52
Columbus	Chicago	Fourth	15,741,744	4,349	-	18	73	54
Middlesborough	Detroit	Third	15,741,272	4,282	-	30	128	52
Chicago	Middlesborough	Sixth	15,722,493	4,355	-	13	72	52
Cleveland	Chicago	First	15,716,782	4,399	-	-	41	54
Columbus	Chicago	First	15,715,180	4,357	-	14	69	52
Chicago	Indianapolis	Third	15,713,546	4,322	-	15	103	52

Paducah	Columbus	Second	15,711,023	4,344	-	27	69	52
Lexington-Fayette	Chicago	Second	15,695,689	4,339	-	23	78	53
Madisonville	Detroit	Third	15,693,043	4,352	-	15	73	53
Indianapolis	Columbus	Second	15,684,227	4,351	-	9	80	52
Columbus	Madisonville	Second	15,681,385	4,350	-	16	74	55
Columbus	Granite City	Second	15,680,440	4,313	-	28	99	53
Columbus	Granite City	First	15,680,026	4,322	-	28	90	53
Indianapolis	Cadillac	Second	15,671,616	4,306	-	40	94	51
Columbus	Madisonville	First	15,669,011	4,355	-	19	66	54
Indianapolis	Columbus	Third	15,653,715	4,363	-	9	68	53
Columbus	Cadillac	Second	15,653,429	4,340	-	-	100	53
Chicago	Indianapolis	First	15,653,104	4,314	-	32	94	53
Indianapolis	Middlesborough	Fourth	15,652,875	4,384	-	7	49	54
Lexington-Fayette	Chicago	First	15,642,020	4,359	-	7	74	53
Middlesborough	Detroit	Fourth	15,640,642	4,332	-	17	91	52
Middlesborough	Detroit	Second	15,640,223	4,333	-	9	98	52
Middlesborough	Detroit	First	15,639,711	4,362	-	-	78	52
Paducah	Columbus	Third	15,634,447	4,370	-	16	54	53
Columbus	Cadillac	Third	15,629,797	4,368	-	-	72	52
Indianapolis	Northbrook	First	15,624,250	4,373	-	-	67	52
Middlesborough	Cleveland	First	15,622,274	4,370	-	-	70	52
Cincinnati	Detroit	First	15,619,203	4,373	-	7	60	53
Cleveland	Paducah	Third	15,606,552	4,348	-	28	64	53

Indianapolis	Cleveland	Second	15,600,089	4,354	-	22	64	53
Cleveland	Paducah	First	15,597,306	4,365	-	17	58	52
Lexington-Fayette	Grand Rapids	Third	15,595,364	4,374	-	-	66	54
Madisonville	Detroit	Fourth	15,589,644	4,367	-	14	59	53
Indianapolis	Chicago	Third	15,588,123	4,359	-	7	74	54
Columbus	Granite City	Third	15,579,511	4,349	-	6	85	53
Indianapolis	Cadillac	First	15,579,141	4,347	-	40	53	52
Indianapolis	Chicago	Fourth	15,575,122	4,369	-	7	64	54
Middlesborough	Detroit	Fifth	15,574,670	4,361	-	17	62	53
Chicago	Detroit	Third	15,567,658	4,403	-	6	31	54
Indianapolis	Northbrook	Second	15,566,176	4,377	-	-	63	54
Cleveland	Paducah	Fourth	15,565,416	4,385	-	12	43	53
Cincinnati	Detroit	Third	15,565,405	4,373	-	2	65	53
Indianapolis	Chicago	Second	15,564,789	4,350	-	7	83	54
Indianapolis	Columbus	First	15,563,452	4,383	-	-	57	53
Chicago	Cleveland	Second	15,558,593	4,361	-	-	79	54
Cleveland	Paducah	Fifth	15,557,589	4,394	-	12	34	54
Chicago	Cleveland	First	15,553,090	4,356	-	-	84	54
Chicago	Columbus	Second	15,551,929	4,353	-	12	75	54
Cleveland	Paducah	Second	15,549,405	4,388	-	5	47	53
Chicago	Columbus	Fourth	15,545,781	4,361	-	12	67	54
Columbus	Madisonville	Fourth	15,541,238	4,397	-	8	35	55
Cincinnati	Detroit	Fourth	15,540,110	4,362	-	2	76	53
Chicago	Columbus	First	15,539,219	4,359	-	12	69	54

Columbus	Granite City	Fourth	15,538,747	4,391	-	6	43	53
Indianapolis	Northbrook	Third	15,537,656	4,390	-	6	44	54
Columbus	Cadillac	First	15,533,876	4,404	-	-	36	53
Cincinnati	Detroit	Second	15,532,463	4,373	-	7	60	53
Lexington-Fayette	Grand Rapids	Fourth	15,529,195	4,377	-	-	63	54
Indianapolis	Cleveland	Third	15,526,000	4,378	-	11	51	53
Columbus	Madisonville	Third	15,524,815	4,391	-	8	41	54
Lexington-Fayette	Grand Rapids	Second	15,520,118	4,389	-	-	51	53
Indianapolis	Chicago	First	15,519,776	4,380	-	2	58	54
Lexington-Fayette	Grand Rapids	Fifth	15,518,914	4,367	-	-	73	54
Chicago	Columbus	Third	15,518,547	4,377	-	12	51	54
Middlesborough	Fort Wayne	Second	15,516,779	4,383	-	-	57	53
Chicago	Cleveland	Third	15,513,413	4,398	-	-	42	54
Middlesborough	Fort Wayne	First	15,507,775	4,394	-	-	46	53
Middlesborough	Cleveland	Second	15,507,710	4,405	-	-	35	54
Cincinnati	Detroit	Fifth	15,502,042	4,403	-	-	37	54
Chicago	Cleveland	Fourth	15,501,129	4,418	-	-	22	54
Lexington-Fayette	Grand Rapids	Sixth	15,498,726	4,384	-	8	48	54
Middlesborough	Cleveland	Third	15,496,001	4,414	-	-	26	55
Paducah	Columbus	Fourth	15,494,459	4,409	-	-	31	53
Indianapolis	Cadillac	Third	15,488,260	4,386	-	12	42	53

Columbus	Cadillac	Fourth	15,483,364	4,401	-	-	39	54
Madisonville	Detroit	Fifth	15,482,811	4,418	-	-	22	53
Middlesborough	Fort Wayne	Fourth	15,482,157	4,420	-	-	20	54
Cleveland	Paducah	Sixth	15,478,895	4,422	-	2	16	54
Lexington-Fayette	Grand Rapids	First	15,478,094	4,419	-	-	21	54
Lexington-Fayette	Grand Rapids	Seventh	15,475,733	4,391	-	19	30	53
Indianapolis	Cleveland	Fourth	15,473,304	4,399	-	4	37	53
Middlesborough	Cleveland	Fourth	15,462,340	4,417	-	-	23	55
Middlesborough	Fort Wayne	Third	15,460,546	4,410	-	-	30	53
Middlesborough	Detroit	Sixth	15,456,087	4,420	-	-	20	54
Columbus	Granite City	Fifth	15,453,628	4,425	-	-	15	53
Indianapolis	Northbrook	Fourth	15,441,454	4,415	-	6	19	54
Chicago	Middlesborough	First	16,370,406	4,066	117	94	163	48
Cleveland	Chicago	Fourth	16,239,061	4,124	130	69	117	50
Cleveland	Chicago	Third	16,230,513	4,063	198	61	118	51
Lexington-Fayette	Chicago	Fifth	16,187,318	4,071	109	77	183	50
Lexington-Fayette	Chicago	Sixth	16,187,233	4,015	129	89	207	48
Lexington-Fayette	Chicago	Fourth	16,156,203	4,086	121	72	161	51
Chicago	Middlesborough	Fourth	16,100,332	4,290	-	38	112	53
Chicago	Middlesborough	Second	16,094,959	4,222	-	84	134	51

Cleveland	Chicago	Second	16,085,162	4,304	-	49	87	54
Chicago	Lexington-Fayette	Second	16,010,979	4,261	70	-	109	52
Chicago	Middlesborough	Third	15,959,572	4,258	-	61	121	51
Chicago	Lexington-Fayette	First	15,948,866	4,311	-	22	107	52
Madisonville	Detroit	Second	15,915,760	4,189	110	33	108	51
Chicago	Lexington-Fayette	Third	15,907,748	4,298	58	-	84	52
Columbus	Chicago	Second	15,879,639	4,295	-	53	92	52
Indianapolis	Middlesborough	Second	15,847,686	4,338	-	9	93	53
Chicago	Lexington-Fayette	Fourth	15,847,055	4,313	56	-	71	52
Columbus	Chicago	Third	15,810,203	4,327	-	27	86	53
Madisonville	Detroit	First	15,810,127	4,342	-	22	76	52
Chicago	Middlesborough	Fifth	15,807,201	4,321	-	21	98	50
Chicago	Detroit	First	15,801,453	4,268	-	61	111	53
Paducah	Columbus	First	15,768,227	4,350	-	24	66	52
Chicago	Indianapolis	Second	15,762,509	4,306	-	32	102	52
Lexington-Fayette	Chicago	Third	15,762,250	4,287	-	66	87	52
Indianapolis	Cleveland	First	15,755,927	4,318	-	27	95	50
Indianapolis	Middlesborough	First	15,747,823	4,346	-	12	82	52
Chicago	Detroit	Second	15,746,549	4,302	-	55	83	53
Indianapolis	Middlesborough	Third	15,742,484	4,361	-	7	72	52

Columbus	Chicago	Fourth	15,741,744	4,349	-	18	73	54
CAD: cost (after disruption); **RR**: rerouted railcars; **TD**: transported railcars on disrupted service legs; **RC**: Rented railcars from competitor; **TP**: Transported railcars by third party; **NO**: Number of train services								

Table 4-4: Optimal solutions after disruption

In this analysis, we identify critical service legs. Assuming disruption occurs in the service leg represented by the first three columns, we then compute the post-disruption transportation cost. This allows us to evaluate the cost implications of disruptions for each service leg. Consequently, we can identify the most critical service leg. This identification was made by determining the difference in costs before and after the disruption. This difference was then utilized to calculate an average and its corresponding standard deviation. Service legs with a cost exceeding Δ standard deviations from the calculated average were deemed critical due to the excessive expense associated with a disruption on those routes. The value of Δ is determined by the decision maker. Figure 4-4 visually delineates these critical service legs within the context of our case study where $\Delta = 2$. For instance, a disruption at the fourth service leg of the Cleveland-Chicago itinerary proves to be particularly costly. When evaluating the average transportation cost across all service legs, this specific leg's post-disruption cost is more than two standard deviations above the average. This significant deviation underscores its criticality. As illustrated in Figure 4-4, we have highlighted this service leg, represented by the line stretching from Fort Wayne to Chicago.

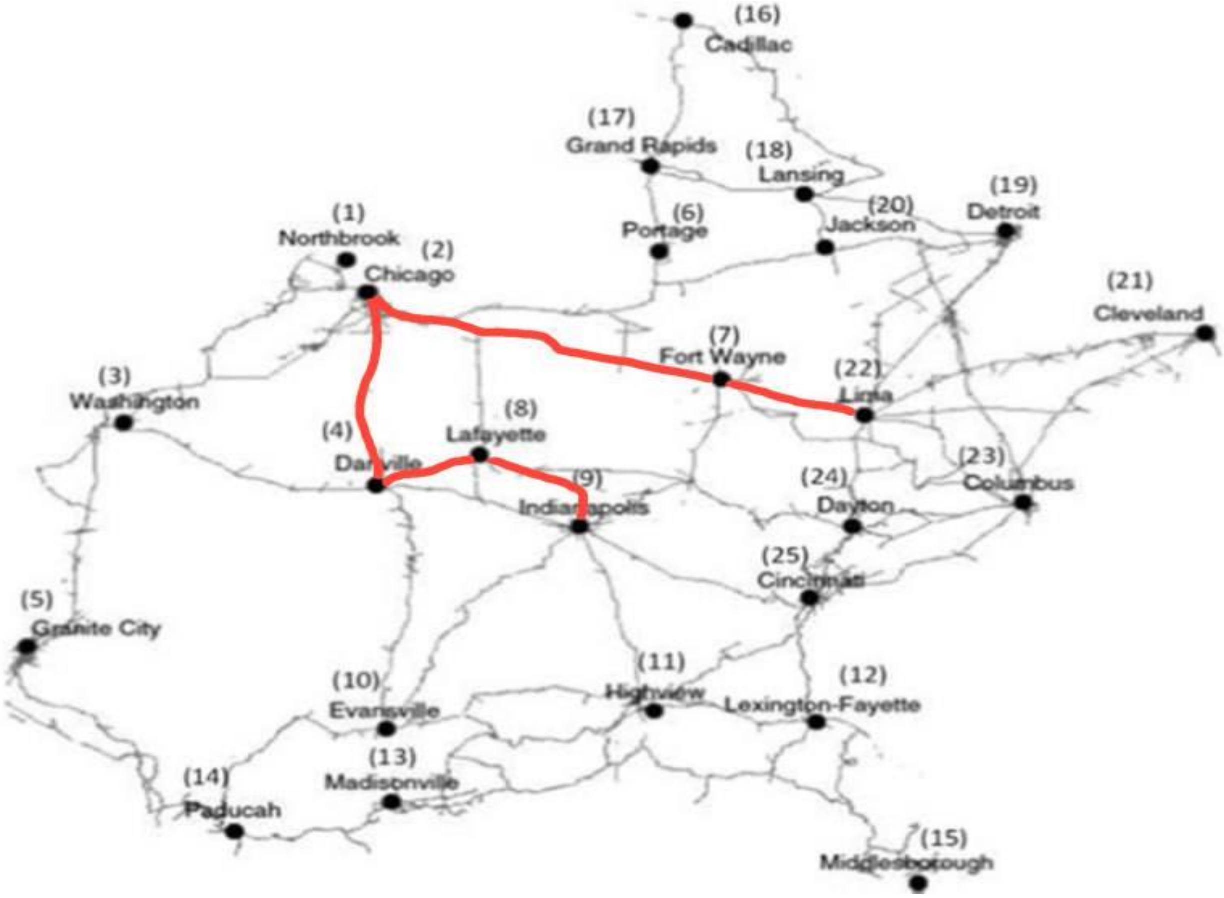


Figure 4-4: critical service legs

4.6.4. Robustness of proposed DRO

In this section, we undertake a comprehensive evaluation of the merit of integrating uncertainty and distributional robustness into our model. This assessment is primarily carried out by Distributionally Robust Optimization (DRO), Deterministic Models (DM), and Simulation-Based Models (SBM).

The DM optimizes based solely on the nominal values of repair costs for service legs post-disruption. We determine the nominal values by taking the average of the highest and lowest repair costs. It simplifies complex situations by eliminating all elements of uncertainty. In contrast, the

SBM represents a more complex approach. It is constructed to approximate the expected transportation cost following a disruption, given the simulated repair costs for service legs. We employ the Monte Carlo simulation to model the cost of the repair strategy post-disruption, utilizing the SBM method. However, the model's memory limitations restrict its ability to process large-scale instances via a Monte Carlo simulation approach. Therefore, a sample size of 200 of repair cost was chosen at random, where the problem is solved for each disrupted service leg. Following this process, we can derive an average transportation cost after disruption based on simulated repair costs for 114 disrupted service legs.

We compiled the results from each of the three models (DRO, DM, and SBM) to comparatively assess the efficiency of DRO. Table 4-4 delineates the total post-disruption transportation cost for each service leg, according to each approach.

Upon reviewing the objective function values presented in the second to fourth columns of Table 5-5, it becomes evident that DRO tends to yield more conservative solutions. This outcome results in an increased volume of railcar transportation through rerouting, railcar renting, and emergency strategies compared to DM and SBM. This trend is anticipated, given that the DRO aims to minimize the worst-case scenario of repair costs for service legs, incorporating all potential distributions within the ambiguity sets. Simultaneously, DM found the optimal objective function for nominal values of repair cost, and the SBM minimizes expected transportation costs based on a given sample set. However, the fifth and sixth columns indicate that the DM tends to provide a more conservative solution compared to the SBM in our case study. The next two columns represent the difference of DRO objective function with DM and SBM, respectively.

Table 4-5 contrasts the results from DRO, DM, and SBM approaches. The initial three columns detail the disruption location, while the subsequent columns display the post-disruption transportation costs using DRO, DM, and SBM. Notably, in most instances, the DRO cost is higher, shown by the last two columns. The SBM objctive function is calculated as the average of objective function values, with the repair cost being determined through simulation. This suggests that accounting for worst-case repair costs typically results in higher transportation costs. This elevated repair cost can potentially lead to shifts in the post-disruption transportation strategy for the impacted railcars. Our DRO approach exhibits a risk averse performance as it is evidenced by the optimal values that are slightly superior to those generated by the DM and the SBM. Moreover, the DRO approach contributes a robust solution that specifies the worst-case scenario. This robustness is particularly valuable as it provides resilience against potential disruptions in the railway transportation network. The DRO model often advocates for a more costly approach compared to the strategies suggested by DM and SBM. It signifies that DRO tends to favor strategies ensuring robustness, even if they come with a higher initial cost, demonstrating its behavior towards mitigating risk.

Origin	**Destination**	**Order**	**Objective (DRO)**	**Objective (DM)**	**Objective (SBM)**	**DRO Vs. DM**	**DRO Vs. SBM**
Cleveland	Chicago	Third	16,236,075	16,213,616	16,214,390	22,459	21,685
Madisonville	Detroit	Second	15,920,760	15,900,127	15,900,029	20,633	20,731
Chicago	Middlesborough	First	16,375,406	16,359,485	16,356,084	15,921	19,321
Lexington-Fayette	Chicago	Fifth	16,192,329	16,171,887	16,177,318	20,441	15,010
Chicago	Lexington-Fayette	Fourth	15,852,055	15,822,448	15,838,032	29,607	14,023
Cleveland	Chicago	Fourth	16,243,768	16,235,272	16,232,971	8,496	10,797

Chicago	Lexington-Fayette	Second	16,016,102	15,990,184	16,007,340	25,918	8,762
Lexington-Fayette	Chicago	Sixth	16,192,643	16,197,918	16,184,220	-5,276	8,422
Chicago	Lexington-Fayette	Third	15,912,921	15,930,607	15,905,363	-17,686	7,558
Lexington-Fayette	Chicago	Fourth	16,160,994	16,173,607	16,154,180	-12,613	6,814
Cleveland	Chicago	Second	16,085,129	16,085,085	16,083,913	45	1,216
Indianapolis	Middlesborough	First	15,747,988	15,747,823	15,747,068	165	920
Indianapolis	Columbus	Third	15,654,561	15,653,715	15,653,718	846	843
Columbus	Chicago	Second	15,879,540	15,879,639	15,878,789	-99	751
Indianapolis	Columbus	Second	15,684,628	15,683,941	15,683,941	687	687
Chicago	Indianapolis	Third	15,713,514	15,712,844	15,712,844	670	670
Cleveland	Chicago	First	15,717,201	15,716,854	15,716,703	348	498
Indianapolis	Middlesborough	Third	15,742,369	15,741,878	15,741,878	491	491
Columbus	Granite City	Second	15,680,596	15,680,108	15,680,108	488	488
Middlesborough	Cleveland	Third	15,495,848	15,495,303	15,495,381	545	468
Columbus	Cadillac	Third	15,629,797	15,629,347	15,629,347	450	450
Lexington-Fayette	Chicago	First	15,641,699	15,641,291	15,641,291	409	409
Lexington-Fayette	Grand Rapids	Seventh	15,476,125	15,475,780	15,475,780	345	345
Columbus	Chicago	First	15,715,180	15,714,841	15,714,841	339	339
Indianapolis	Chicago	Second	15,564,974	15,564,672	15,564,672	302	302

Columbus	Madisonville	Second	15,681,622	15,681,370	15,681,321	253	301
Middlesborough	Cleveland	Second	15,507,710	15,507,555	15,507,412	155	299
Columbus	Madisonville	Third	15,524,443	15,524,769	15,524,146	-326	297
Chicago	Middlesborough	Fifth	15,807,201	15,806,916	15,806,916	285	285
Chicago	Cleveland	Third	15,512,761	15,512,499	15,512,499	262	263
Indianapolis	Middlesborough	Fourth	15,652,788	15,652,542	15,652,542	246	246
Indianapolis	Middlesborough	Second	15,847,568	15,847,568	15,847,332	0	236
Middlesborough	Fort Wayne	First	15,508,084	15,507,848	15,507,848	236	236
Indianapolis	Cadillac	Third	15,488,517	15,488,400	15,488,298	117	218
Madisonville	Detroit	Fourth	15,589,656	15,589,468	15,589,468	189	189
Middlesborough	Fort Wayne	Third	15,460,726	15,460,546	15,460,546	180	180
Indianapolis	Chicago	Fourth	15,575,187	15,575,008	15,575,008	179	179
Indianapolis	Cadillac	Second	15,671,779	15,671,616	15,671,616	163	163
Chicago	Lexington-Fayette	First	15,948,776	15,948,645	15,948,660	132	117
Chicago	Columbus	Second	15,552,035	15,551,929	15,551,929	106	106
Columbus	Madisonville	First	15,669,011	15,668,928	15,668,949	83	62
Paducah	Columbus	Third	15,634,465	15,634,361	15,634,407	104	58
Chicago	Indianapolis	Second	15,762,546	15,762,504	15,762,504	43	43
Indianapolis	Cleveland	Fourth	15,473,304	15,473,267	15,473,267	38	38
Middlesborough	Fort Wayne	Fourth	15,481,658	15,481,639	15,481,639	19	20
Cleveland	Paducah	Fifth	15,557,735	15,557,735	15,557,718	0	18
Cincinnati	Detroit	Fifth	15,502,042	15,502,042	15,502,025	0	17
Indianapolis	Cleveland	First	15,755,927	15,755,927	15,755,918	0	10
Columbus	Cadillac	Second	15,653,430	15,653,421	15,653,421	9	9

Middlesborough	Cleveland	First	15,622,384	15,622,375	15,622,375	9	9
Columbus	Chicago	Third	15,810,210	15,810,202	15,810,202	9	9
Lexington-Fayette	Grand Rapids	Fourth	15,529,195	15,529,191	15,529,191	4	4
Indianapolis	Cleveland	Third	15,526,008	15,526,008	15,526,006	0	2
Lexington-Fayette	Chicago	Third	15,762,250	15,762,250	15,762,250	0	0
Middlesborough	Detroit	Second	15,640,223	15,640,223	15,640,223	0	0
Middlesborough	Detroit	Fifth	15,574,659	15,574,659	15,574,659	0	0
Chicago	Columbus	First	15,539,173	15,539,173	15,539,173	0	0
Columbus	Chicago	Fourth	15,741,723	15,741,723	15,741,723	0	0
Indianapolis	Cleveland	Second	15,600,089	15,600,089	15,600,089	0	0
Chicago	Indianapolis	First	15,653,104	15,653,104	15,653,104	0	0
Middlesborough	Detroit	Fourth	15,640,642	15,640,642	15,640,642	0	0
Cincinnati	Detroit	First	15,619,207	15,619,207	15,619,207	0	0
Cleveland	Paducah	First	15,597,294	15,597,294	15,597,294	0	0
Lexington-Fayette	Grand Rapids	Third	15,595,364	15,595,364	15,595,364	0	0
Indianapolis	Chicago	Third	15,587,621	15,587,621	15,587,621	0	0
Cleveland	Paducah	Fourth	15,565,416	15,565,416	15,565,416	0	0
Cincinnati	Detroit	Third	15,565,405	15,565,405	15,565,405	0	0
Lexington-Fayette	Grand Rapids	Fifth	15,518,674	15,518,674	15,518,674	0	0
Chicago	Columbus	Third	15,518,547	15,518,547	15,518,547	0	0
Middlesborough	Fort Wayne	Second	15,517,042	15,517,042	15,517,042	0	0

Columbus	Cadillac	Fourth	15,483,434	15,483,434	15,483,434	0	0
Columbus	Granite City	Fifth	15,452,850	15,452,850	15,452,850	0	0
Cincinnati	Detroit	Second	15,532,463	15,532,463	15,532,464	0	-1
Chicago	Detroit	First	15,801,458	15,801,458	15,801,461	0	-3
Lexington-Fayette	Grand Rapids	Second	15,520,118	15,520,127	15,520,127	-9	-9
Indianapolis	Northbrook	Third	15,536,857	15,536,861	15,536,868	-4	-10
Columbus	Madisonville	Fourth	15,540,929	15,540,942	15,540,942	-13	-13
Columbus	Cadillac	First	15,533,752	15,533,767	15,533,768	-15	-16
Chicago	Detroit	Second	15,746,463	15,746,475	15,746,490	-12	-28
Lexington-Fayette	Grand Rapids	Sixth	15,498,726	15,498,726	15,498,766	0	-41
Middlesborough	Detroit	Third	15,741,272	15,741,272	15,741,322	0	-49
Paducah	Columbus	First	15,768,273	15,768,227	15,768,332	47	-59
Columbus	Granite City	Fourth	15,538,747	15,538,747	15,538,809	0	-62
Lexington-Fayette	Grand Rapids	First	15,478,094	15,478,178	15,478,164	-84	-70
Middlesborough	Cleveland	Fourth	15,461,646	15,461,725	15,461,725	-79	-79
Cleveland	Paducah	Second	15,549,594	15,549,328	15,549,677	266	-83
Chicago	Middlesborough	Second	16,094,959	16,095,043	16,095,043	-84	-84
Paducah	Columbus	Second	15,711,023	15,710,952	15,711,114	71	-90
Chicago	Detroit	Third	15,567,654	15,567,827	15,567,745	-173	-90
Indianapolis	Northbrook	Second	15,565,656	15,565,757	15,565,757	-101	-101
Chicago	Middlesborough	Third	15,959,766	15,959,868	15,959,868	-102	-102
Indianapolis	Northbrook	First	15,624,160	15,624,265	15,624,265	-105	-105

Madisonville	Detroit	First	15,809,947	15,810,084	15,810,088	-138	-142
Chicago	Cleveland	Second	15,558,081	15,558,232	15,558,232	-151	-151
Lexington-Fayette	Chicago	Second	15,695,699	15,695,851	15,695,851	-153	-153
Chicago	Middlesborough	Fourth	16,099,796	16,099,796	16,099,968	0	-171
Chicago	Columbus	Fourth	15,545,616	15,545,788	15,545,788	-172	-172
Columbus	Granite City	First	15,679,769	15,679,962	15,679,962	-193	-193
Cleveland	Paducah	Sixth	15,478,951	15,479,166	15,479,171	-215	-220
Paducah	Columbus	Fourth	15,494,596	15,494,833	15,494,835	-236	-238
Columbus	Granite City	Third	15,579,502	15,579,769	15,579,769	-267	-267
Indianapolis	Cadillac	First	15,579,211	15,579,499	15,579,479	-288	-268
Chicago	Cleveland	Fourth	15,501,084	15,501,402	15,501,402	-318	-318
Indianapolis	Columbus	First	15,563,456	15,563,784	15,563,784	-328	-328
Middlesborough	Detroit	Sixth	15,456,210	15,456,550	15,456,550	-340	-340
Madisonville	Detroit	Third	15,692,499	15,693,021	15,692,867	-522	-368
Cleveland	Paducah	Third	15,606,590	15,606,964	15,606,964	-375	-374
Chicago	Middlesborough	Sixth	15,722,081	15,722,466	15,722,466	-385	-385
Indianapolis	Chicago	First	15,519,776	15,520,176	15,520,176	-401	-401
Cincinnati	Detroit	Fourth	15,539,575	15,540,110	15,540,110	-535	-535
Indianapolis	Northbrook	Fourth	15,441,464	15,442,069	15,442,069	-605	-605
Madisonville	Detroit	Fifth	15,482,611	15,483,367	15,483,222	-756	-611
Middlesborough	Detroit	First	15,639,568	15,640,277	15,640,277	-708	-708
Chicago	Cleveland	First	15,553,090	15,553,906	15,553,906	-816	-816

Table 4-5: comparison results of the DRO, DM and SBM

In this study, to compare the obtained solutions of DRO, DM and SBM, we aim to investigate a single service leg. Again, we analyze the potential disruption in the initial service leg from Chicago to Middlesborough. As delineated in Table 4-6, a comprehensive analysis has been conducted on the DRO, DM, and SBM methods. The 'SBM-Min' row represents the lowest transportation cost post-disruption, as derived from the simulation outcomes. Conversely, 'SBM-Max' indicates the highest cost incurred in the simulation for transportation subsequent to the disruption. We make use of SBM-Max and SBM-Min to compare DRO with the worst case and best case in simulated real world.

				Number of railcars				
Origin	Destination	Order		Rerouting	Repair	**Renting** railcars	Emergency	**Objective** function
			DRO	4,066	117	163	94	16,370,405
			DM	4,068	115	163	94	16,368,030
			SBM-Max	4,066	117	163	94	16,386,971
Chicago	Middlesborough	First	SBM-Min	4,068	115	163	94	16,357,710

Table 4-6: comparison strategies of the DRO, DM and SBM for a service leg

The DRO method is inherently conservative in its approach. When utilizing the DRO approach, the model alters the transportation modes for two affected railcars post-disruption, in contrast to the DM and SBM-Min methods. This change is largely due to the inclusion of the worst-case repair cost in decision-making. By considering this, decision-makers prioritize allocating additional resources to expedite the disrupted service leg's repair. Consequently, this repair tactic allows for the transportation of 117 railcars through the affected service leg. On the other hand, when the worst-case scenario isn't factored in, as in the DM and SBM-Min models, a more time consuming repair process is chosen. This leads to a minor decrease in the number of transported railcars, totaling 115, to meet to the delivery due date.

4.6.5. Benefits of expanding the time window

Delivery due dates play a critical role in determining post-disruption transportation costs. If there's a longer window before the due date, the company can choose cost-effective repair methods or reroute railcars through more economical routes. This choice can help the railway avoid expensive emergency responses, leading to significant cost savings. However, extended delivery times might lead to customer dissatisfaction. In this study, we examine transportation strategies post-disruption for different delivery due dates.

In this section, we undertake a detailed sensitivity analysis focusing on the delivery due time post-disruption. Previously, our models operated under the assumption that the delivery due time equates to 1.25 times the regular transportation duration. Our intention now is to explore various delivery due time values to better comprehend their implications.

For the purpose of this analysis, we consistently examine the same service leg as elucidated in subsection 6.2. The outcomes of this study are graphically illustrated in Figure 4-5. This visualization details the number of railcars transported through differing strategies post-disruption.

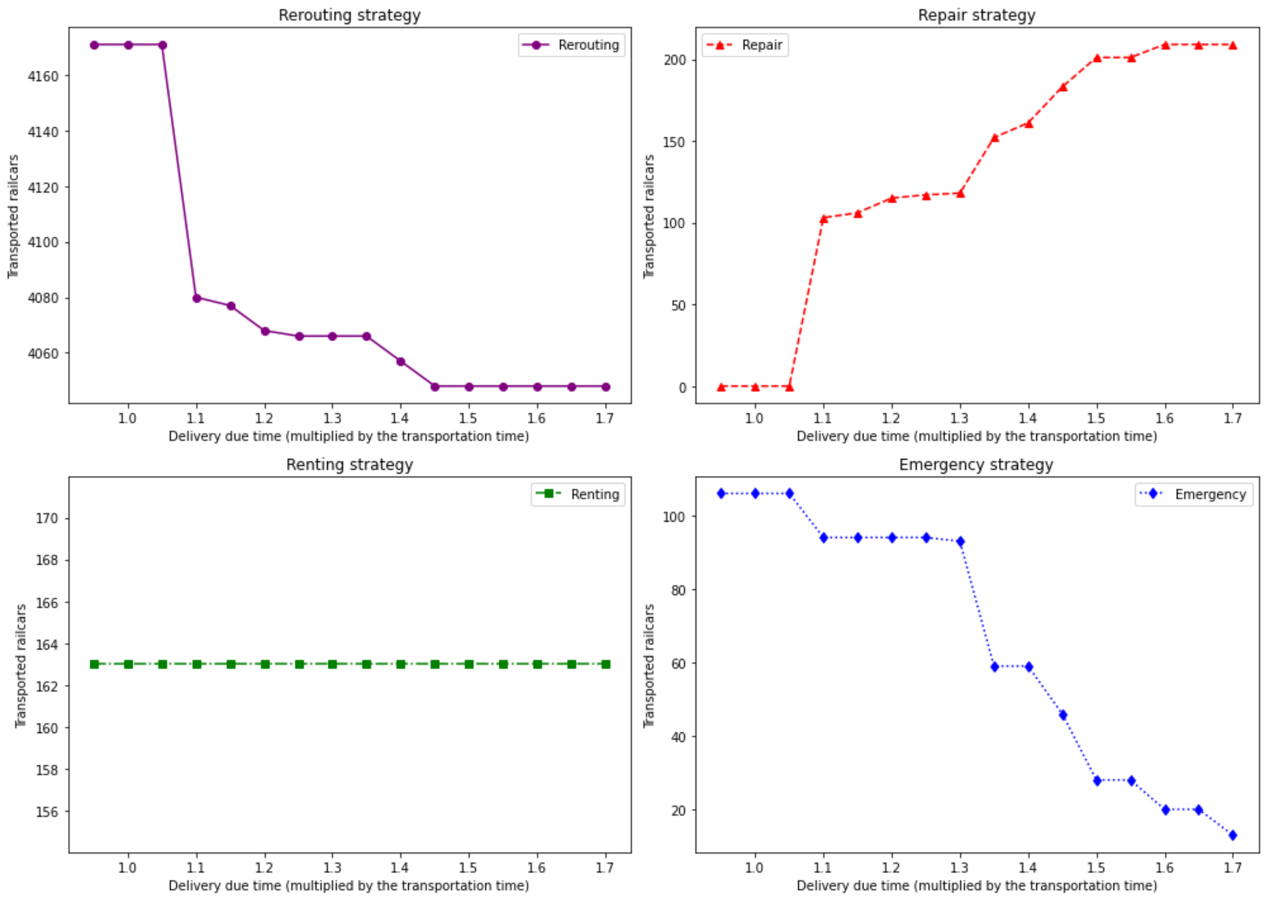


Figure 4-5: transported railcars for different delivery due times

Our findings suggest a noticeable decline in the number of railcars associated with rerouting as the delivery due time window broadens. Specifically, by the culmination of the assessed range, the rerouting cost has diminished approximately by 3%. Railcars utilizing the repair strategy depict an intriguing trend. They commence from a baseline of zero and subsequently exhibit a significant surge. This uptick signifies that with the elongation of the time window, the feasibility and advantage of implementing repairs amplify. Therefore, with an extended time window, a strategic blend of rerouting and repairs may be warranted to optimize costs.

The costs associated with renting railcars remain consistent across all examined sensitivity levels. This consistency hints at the resilience of the renting strategy, which remains impervious to fluctuations in the time window. Thus, renting offers a stable and predictable alternative, serving as a buffer in the face of changing dynamics. There is a marked decline in emergency costs as the delivery due time window expands. Specifically, by the end of our examined range, there's a dramatic reduction of approximately 87.7%. This underlines that elongating delivery due time can significantly mitigate disruption costs.

It is imperative for managerial decision-makers to adopt a holistic strategy. A balanced amalgamation of rerouting, timely repairs, and consistent renting emerges as the optimal approach. If decision-makers negotiate with customers to extend the delivery due time following disruptions, or agree to a minor penalty for delivery delays, it can substantially reduce the railway company's reliance on costly third-party solutions. However, even with an extended delivery window, they still need to lease the necessary railcars from competitors. Our model can greatly assist decision-makers in determining the optimal number of railcars to rent.

4.7. Conclusion

In this study, we introduced a novel disruption risk management methodology for a railroad transportation network. This methodology considers various strategies, including rerouting, repairing, renting competitor railcars, and leveraging a third party for emergency response. Uniquely, we incorporated the strategy of renting railcars from competitor infrastructures, thereby reducing post-disruption transportation costs. We pioneered the development of a game-theoretical model to address random disruptions in the field. In this model, an innovative approach was proposed to ascertain the optimal solution for the railway company (leader), utilizing the closed-form solution of the competitor (follower). To manage cost uncertainty in the repair strategy, a distributionally robust optimization approach was employed.

We tested the methodology on a mid-western United States railroad network. The findings highlighted its utility in identifying critical service legs to mitigate disruption impact and facilitate efficient recovery. Notably, the novel strategy of renting railcars enhanced railroad transportation resilience, often reducing third-party transportation costs significantly. By assuming disruptions in each service leg, our findings indicate that the renting strategy can substantially decrease the transportation costs post-disruption. Additionally, we discovered that DRO can alter the optimal railcar transportation strategy. Furthermore, our solutions suggest that negotiations to extend the delivery date can notably reduce post-disruption transportation costs.

Despite these advances, certain assumptions underpin our model, including the immediate availability of all train services for both the railway company and the competitor at the week's beginning. Additionally, scheduling of train services was not considered. Such assumptions could make the model formulation complex as train services often vary throughout the week. Future research could explore different types of risks, such as terrorist attacks. These studies may need to adapt the model to account for complexities like correlation between disruptions and recovery efforts. Scenarios with disruptions occurring during the recovery period may also warrant investigation, necessitating new recovery strategies and an understanding of their relationship with existing strategies. While our model has made significant progress in disruption risk management, further exploration will enhance its robustness and applicability, thus improving the resilience of our transportation infrastructures.

References:

Albrecht, A. R., Panton, D. M., & Lee, D. H. (2013). Rescheduling rail networks with maintenance disruptions using problem space search. Computers & Operations Research, 40(3), 703-712.

Association of American Railroads. (2023). Rail safety fact sheet. Retrieved from https://www.aar.org/wp-content/uploads/2023/03/AAR-Freight-Rail-Safety-Inspections-Fact-Sheet.pdf

Azad, N., Hassini, E., & Verma, M. (2016). Disruption risk management in railroad networks: An optimization-based methodology and a case study. Transportation Research Part B: Methodological, 85, 70-88.

Bagheri, M., Saccomanno, F., & Fu, L. (2012). Modeling hazardous materials risks for different train make-up plans. Transportation Research Part E: logistics and transportation review, 48(5), 907-918.

Bertsimas, D., Sim, M., & Zhang, M. (2019). Adaptive distributionally robust optimization. Management Science, 65(2), 604-618.

Bhavsar, N., & Verma, M. (2022). A subsidy policy to managing hazmat risk in railroad transportation network. European Journal of Operational Research, 300(2), 633-646.

Bianco, L., Caramia, M., Giordani, S., & Piccialli, V. (2016). A game-theoretic approach for regulating hazmat transportation. Transportation Science, 50(2), 424-438.

Cato Institute. (1997). Trackage Rights and the Future of Regulation. Retrieved from https://www.cato.org/sites/cato.org/files/serials/files/regulation/1997/4/reg20n2g.html

ESRI, 2008. www.esri.com

Fotuhi, F., & Huynh, N. (2018). A reliable multi-period intermodal freight network expansion problem. Computers & Industrial Engineering, 115, 138-150.

Ghosal, S., & Wiesemann, W. (2020). The distributionally robust chance-constrained vehicle routing problem. Operations Research, 68 (3), 716–732.

Högdahl, J., & Bohlin, M. (2023). A Combined Simulation-Optimization Approach for Robust Timetabling on Main Railway Lines. Transportation Science, 57(1), 52-81.

Jabbarzadeh, A., Azad, N., & Verma, M. (2020). An optimization approach to planning rail hazmat shipments in the presence of random disruptions. Omega, 96, 102078.

Khaled, A. A., Jin, M., Clarke, D. B., & Hoque, M. A. (2015). Train design and routing optimization for evaluating criticality of freight railroad infrastructures. Transportation Research Part B: Methodological, 71, 71-84.

Ke, G. Y., & Verma, M. (2021). A framework to managing disruption risk in rail-truck intermodal transportation networks. Transportation Research Part E: Logistics and Transportation Review, 153, 102340.

Khanmohamadi, M., Bagheri, M., Khademi, N., & Ghannadpour, S. F. (2018). A security vulnerability analysis model for dangerous goods transportation by rail–Case study: Chlorine transportation in Texas-Illinois. Safety science, 110, 230-241.

Mohammadi, M., Jula, P., & Tavakkoli-Moghaddam, R. (2017). Design of a reliable multi-modal multi-commodity model for hazardous materials transportation under uncertainty. European Journal of Operational Research, 257(3), 792-809.

Peng, Y., Yong, P., & Luo, Y. (2021). The route problem of multimodal transportation with timetable under uncertainty: Multi-objective robust optimization model and heuristic approach. RAIRO-operations Research, 55, S3035-S3050.

Peterson, S. K., & Church, R. L. (2008). A framework for modeling rail transport vulnerability. Growth and Change, 39(4), 617-641.

Rosenthal, E. C. (2017). A cooperative game approach to cost allocation in a rapid-transit network. Transportation Research Part B: Methodological, 97, 64-77.

Sarhadi, H., Tulett, D. M., & Verma, M. (2017). An analytical approach to the protection planning of a rail intermodal terminal network. European Journal of Operational Research, 257(2), 511-525.

Shahabi, A., Raissi, S., Khalili-Damghani, K., & Rafei, M. (2021). Designing a resilient skip-stop schedule in rapid rail transit using a simulation-based optimization methodology. *Operational Research*, *21*, 1691-1721.

Snyder, L. V., Scaparra, M. P., Daskin, M. S., & Church, R. L. (2006). Planning for disruptions in supply chain networks. In Models, methods, and applications for innovative decision making (pp. 234-257). Informs.

Starita, S., & Scaparra, M. P. (2016). Optimizing dynamic investment decisions for railway systems protection. European Journal of Operational Research, 248(2), 543-557.

Tan, Z., Xu, M., Meng, Q., & Li, Z. C. (2020). Evacuating metro passengers via the urban bus system under uncertain disruption recovery time and heterogeneous risk-taking behaviour. Transportation research part C: emerging technologies, 119, 102761.

U.S. Department of Transportation. (n.d.). Freight Rail Overview. Federal Railroad Administration. Retrieved from https://railroads.dot.gov/rail-network-development/freight-rail-overview

Uddin, M., & Huynh, N. (2019). Reliable routing of road-rail intermodal freight under uncertainty. Networks and Spatial Economics, 19, 929-952.

US DHS, 2018. Department of homeland security, http://www.dhs.gov/what-critical-infrastructure Accessed December 12, 2018.

Van der Hurk, E., Kroon, L., & Maróti, G. (2018). Passenger advice and rolling stock rescheduling under uncertainty for disruption management. Transportation Science, 52(6), 1391-1411.

Verma, M., Verter, V., & Gendreau, M. (2011). A tactical planning model for railroad transportation of dangerous goods. Transportation Science, 45(2), 163-174.

Verma, M. (2011). Railroad transportation of dangerous goods: A conditional exposure approach to minimize transport risk. Transportation research part C: emerging technologies, 19(5), 790-802.

Verma, M., Verter, V., & Zufferey, N. (2012). A bi-objective model for planning and managing rail-truck intermodal transportation of hazardous materials. Transportation research part E: logistics and transportation review, 48(1), 132-149.

Washington State Department of transportation, 2014. Landslide Mitigation Action Plan, Washington State Department of transportation url: http://www. wsdot.wa.gov/NR/rdonlyres/8B3B653E- 5C50- 4E2B- 977E- AE5AB36751B7/0/LandslideMitigationActionPlan.pdf

Xu, L., & Ng, T. S. A. (2020). A robust mixed-integer linear programming model for mitigating rail transit disruptions under uncertainty. Transportation Science, 54(5), 1388-1407.

Xu, L., Ng, T. S., & Costa, A. (2021). Optimizing disruption tolerance for rail transit networks under uncertainty. Transportation science, 55(5), 1206-1225.

Yang, Y., Yin, Y., Wang, D., Ignatius, J., Cheng, T. C. E., & Dhamotharan, L. (2023). Distributionally robust multi-period location-allocation with multiple resources and capacity levels in humanitarian logistics. European Journal of Operational Research, 305(3), 1042-1062.

Zhou, W., Kang, J., Qin, J., Li, S., & Huang, Y. (2022). Robust optimization of high-speed railway train plan based on multiple demand scenarios. Mathematics, 10(8), 1278.

Ziaei, Z., & Jabbarzadeh, A. (2021). A multi-objective robust optimization approach for green location-routing planning of multi-modal transportation systems under uncertainty. Journal of Cleaner Production, 291, 125293.

Zheng, Y. J., Zhang, M. X., Ling, H. F., & Chen, S. Y. (2014). Emergency railway transportation planning using a hyper-heuristic approach. IEEE Transactions on Intelligent Transportation Systems, 16(1), 321-329

Chapter VI: Conclusion and future studies

In this thesis, we have focused on a comprehensive exploration of disruptions in supply chains, with a particular focus on financial challenges, and random disruptions in railway transportation. Through three distinct but interconnected research pieces, the study not only take into account the complexities of these disruptions but also offers innovative strategies to manage and mitigate their impacts effectively.

The journey begins with an in-depth analysis of financial disruptions within supply chain network planning. Here, we developed a mixed-integer programming model that integrates external financing approaches—specifically trade and bank credit—into the supply chain financial planning problem. This model uniquely addresses the post-COVID era challenges, such as heightened interest rate uncertainty and supplier reliability issues, which have significantly impacted business profitability. Our approach, tailored for large buyers with substantial bargaining power, leverages the credit rating of suppliers to establish a more resilient financial planning framework. This segment of the thesis not only highlights the interplay between different financing options but also sheds light on how these choices impact supply chain profitability under uncertain demand. We make use of a hybrid robust optimization to tackle the interest rate and credit rate uncertainties.

Progressing to the second facet of our research, we turn our attention to the contract design problem between suppliers and retailers, a critical aspect of mitigating financial disruptions. We innovatively introduce three types of contracts, each designed to hedge against financial risks in supply chain finance. Our models, applied in a real-world context with Ford Motor Company, reveal the protective role of PPI and TCI against bankruptcy risks and profitability dips. This part of the study not only provides practical solutions to current challenges but also opens intriguing avenues for future research, especially in understanding the role of TCI across different profit margins and the influence of financial and non-financial factors in demand forecasting.

The final piece of the thesis presents a novel disruption risk management methodology for railway transportation networks, addressing random disruptions through a suite of strategies such as repairing, rerouting, emergency response and renting competitor railcars. The adoption of a game-theoretical model and a distributionally robust optimization approach marks a significant advancement in managing cost uncertainties and optimizing disruption management strategies. Applied to a mid-western U.S. railroad network, our methodology demonstrates substantial cost reductions and enhanced resilience in transportation operations by renting railcar strategy.

This thesis provides a multifaceted nature of disruptions in supply chains and transportation networks. It offers frameworks and methodologies that are not only academically significant but also highly relevant in real-world problems. As we look to the future, the findings and methodologies developed in this thesis will undoubtedly continue to inform and shape strategies and policies in this vital field. For example, investing on resources to strengthen unstable infrastructures might reduce disruptions in both material and financial flows. Also, factors like retailers' information asymmetry, financial and non-financial predictors, taxes, and insurance cancelability might be influential in designing financial contracts in supply chain. Additionally, addressing uncertain covariates in supply chain finance contracts could be a compelling field.

Finally, exploring pricing strategies for rental railcars, especially in the case of intentional disruptions such as terrorist attacks, would be intriguing.