An Empirical Study of Goodwin Growth Models

AN EMPIRICAL STUDY OF GOODWIN GROWTH MODELS

BY

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To my parents, sister and Sreeta

Abstract

Three dynamic macroeconomic models have been econometrically tested for ten OECD countries. One of them is a classic model developed by R.M. Goodwin on the lines of Lotka-Volterra predator prey model of mathematical biology, the second is an extension developed by Desai incorporating inflation and the third is a model developed by van der Ploeg incorporating a generalized production function. We start with estimating parameters and equilibrium of the Goodwin growth model, we then discuss data issues and reasons of poor estimates in a well-known paper by Harvie. Next we analyze the impact of inflation on the bargaining power of the agents and on the equilibrium through Desai's model. Finally, we estimate CES production function and equilibrium estimates of the van der Ploeg model. We find that the estimates of the van der Ploeg model are within the bounds of observed values for 6 out of 10 countries examined. Moreover, the error of the model is much less than suggested by previous studies. This implication is important and encouraging for further econometric estimation of generalized dynamical models in macroeconomics.

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Notation and abbreviations

$Y(t)$	total real output
$K(t)$	total real capital stock
$L(t)$	total labour employed
$N(t)$	total labour force
w	wage rate
r	capital rent
ω	wage share
λ	employment rate
a_t	productivity
α	productivity growth rate
β	population growth rate
δ	depreciation rate
m	money wages
$(\frac{\dot{p}}{p})^e$	expected inflation
$\frac{\dot{p}}{p}$	actual inflation

Chapter 2 Specific

ν	capital output ratio
γ, ρ	terms in Phillips curve
z_t	rate of growth of real wages

Chapter 3 Specific

π	mark up over cost
γ_2, ρ_2, η_2	terms in phillips curve
τ_2	adjustment term for expected inflation
η_3	adjustment term for inflation
σ	capital output ratio
σ^*	long term capital output ratio

Chapter 4 Specific

λ_l	labour aumented technological growth
σ	elasticity of substitution
π	distribution parameter
γ_3, ρ_3, η_3	terms in Phillips curve
γ, ρ_1, ρ_2	terms in Phillips curve
$\bar{\pi}$	mark up over cost

σ^*	long term capital output ratio
χ_2	adjustment term for inflation
Y_0,K_0,L_0	geometric average of the total real output, capital and labour employed
r_0,w_0	arithmetic average of the capital rent and the wage rate

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Chapter 1

Introduction

Das Kapital and The Communist Manifesto have been among the most influential texts written in 19th century, affecting both academic debate and policy making. Marx's work on class struggle and alternating ups and downs in capitalism has been an instrumental idea in many theoretical models later in the 20th century. One of the most prominent macro models, stemming from the work of Marx is Goodwin's model of growth cycles. Taking analogy from the predator prey models in biology, Goodwin modeled the relationship between wage share and employment rate. Within this model, capitalists bear the brunt of their expansion (due to increased profitability) by increasing employment and consequently losing bargaining power to the labor. This shift of bargaining power leads to increasing wage share (thus reducing profitability). With employment at its peak, firms are forced to layoff to reduce declining profitability. Now, the balance of power starts shifting back from labor to capitalists. This behavior of economic agents repeats itself to form closed loop Goodwin cycles. Mathematically, the model derives its dynamic relationship by imposing a functional form to the production function and behavioral characteristics of capitalists and workers.

In the next section we will briefly discuss the mathematical details for the Goodwin model, following the explanation in [11]

1.1 Goodwin model

Goodwin [10] explained the dynamic relationship between wage share and employment by using a set of differential equations. His model assumes a Leontief Production function given by:

$$Y(t) = \min\left\{\frac{K(t)}{\nu}, a(t)L(t)\right\} \tag{1.1}$$

where Y is real output, K is real capital accumulated in the economy, L is the employed labor force, a is the labor productivity and ν is a constant capital output ratio. He also assumed exponential growth function for both productivity and total labour force with α and β represents the corresponding growth rate parameters:

$$a(t) = a(0)e^{\alpha t} \tag{1.2}$$

$$N(t) = N(0)e^{\beta t} \tag{1.3}$$

The ratio of L and N is defined as the employment rate in an economy. Along the way, he also assumed complete utilization of resources, that is,

$$Y(t) = \frac{K(t)}{\nu} = a(t)L(t) \tag{1.4}$$

Finally Goodwin assumed two behavioral relationships, first; (i) growth rate of real wages is linearly proportional to the employment rate λ (i.e. linear Phillips curve) and (ii) capitalists use all the profit as investment in capital, that is

$$\dot{\mathrm{w}} = (\gamma + \rho\lambda)\mathrm{w} = \Phi(\lambda)\mathrm{w} \tag{1.5}$$

$$\dot{K} = (Y - \mathrm{w}L) - \delta K \tag{1.6}$$

where, δ is the depreciation rate. Using the above assumptions and defining $\omega = \frac{\mathrm{w}L}{Y}$ as the wage share in the economy, one can derive the following set of equations to describe the relationship of wage share and employment rate:

$$\frac{\dot{\omega}}{\omega} = \Phi(\lambda) - \alpha \tag{1.7}$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\nu} - (\alpha + \beta + \delta) \tag{1.8}$$

For a given initial condition (ω_0,λ_0), the solution of the above two differential equations is a closed cycle around the equilibrium point

$$\bar{\lambda} = \Phi^{-1}(\alpha) \tag{1.9}$$

$$\bar{\omega} = 1 - (\alpha + \beta + \delta)\nu \tag{1.10}$$

We see from equation (1.9) and (1.10) that a positive shock in productivity will have to be compensated by higher profit share and thus lower wage share. This increase in productivity will also lead to higher equilibrium employment rate. A positive shock to bargaining coefficient ρ will lead to firing of labour and thus the equilibrium employment rate declines. On the other hand, an increase in the labor force growth rate will add to higher profit share for the capitalists. An illustration of this solution is given in the Figure 1.1.

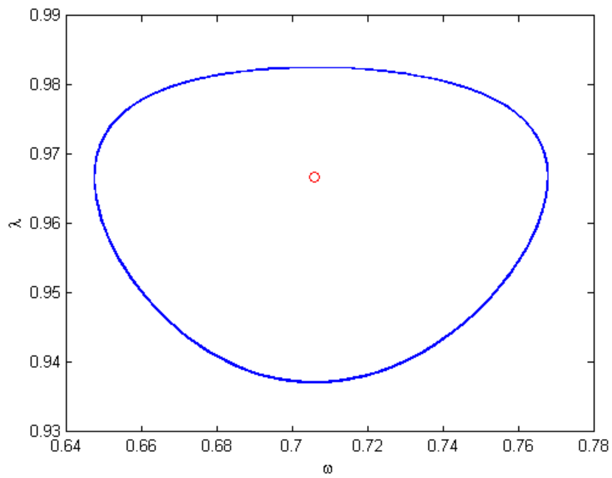


Figure 1.1: Goodwin Simulations

1.2 Literature Review

Ever since the classic paper by Goodwin in 1967, there has been a large amount of research on theoretical developments for the Goodwin model. Although the list of theoretical work is long, I will try to summarize some classic work in this regard. Desai [4] introduced three extensions of the model to incorporate inflation, expected inflation and variable capacity utilization. Later in a series of papers [28; 29; 30], van der Ploeg introduced the impact of savings by households, substitutibility between labor and capital and cost minimizing direction of technical change. There has also been numerous works to understand the impact of government policy within the framework of the Goodwin model [31; 26; 1; 32; 3]. Problems in this space have varied between choice of policy - Keynesian/Classical, role of policy lag and types of debt - private/public. Recently Huu and Costa-Lima [13] introduced a stochastic version of the Goodwin model with Brownian noise in the productivity factor. However, amidst all these developments on the theoretical front, there has been little focus on empirical understanding and its applications. Since this thesis is focused on empirical developments, we will next discuss how the literature has developed in this regard.

One of the first studies that tried to analyze the Goodwin model in the context of real data was done by Atkinson [2]. Although not a full fledged econometric study, it did try to relate the period of the post war trade cycle with the estimated period of the Goodwin model. Major breakthrough in the area came with Desai's [5] estimation of the Goodwin model, where he laid the foundation on how to estimate such dynamic models. He rejected the empirical validity for the assumptions of the Goodwin model by testing for more generalized models for which the Goodwin model is a special case. Goldstein [9] took a slightly different approach to understand such dynamical models. He used multivariate vector auto regression(VAR) specification to understand dynamic interaction between unemployment and profit share of income. He also extended the model to include structural shifts in it. In a celebrated paper [12], Harvie tested the Goodwin model for 10 OECD countries. He compared the equilibrium wage share and employment rate for the Goodwin model with the observed average values. Although he observed qualitative evidence of the cyclical relationship as proposed by Goodwin, there was poor quantitative evidence of the Goodwin model being close to reality. Unfortunately there were some serious reporting errors in his estimation methodology and the problem with data construction. We will discuss these issues in detail in a later chapter. In this regard, Mohun and Veneziani [21] have discussed the appropriate data for econometric estimation for the Goodwin model and the problems with Harvie's estimation. Although they did not do econometric estimation, they compared the qualitative cycles and trends for non-farm payroll data for US using 2 different data-sets. Next, Dibeh et al. [7] used the

Bayesian inference method to estimate the parameters of the system of the differential equation for France and Italy. Rather than looking at the underlying components of the Goodwin model like Harvie, they estimate the parameters of the resultant two differential equations. They tweak the classic Goodwin model by introducing a sinusoidal wave to act as exogenous periodic variation. Although the estimates are very close to real data, they do not comment on the theoretical properties such as structural stability of the new stochastic system. Flaschel extended the literature by analyzing the wage share-employment rate relationship using modern econometric techniques [8].He used a Hodrick-Prescott filter to decompose the state variables into trend and cycles for the US economy. He found considerable evidence of the closed Goodwin cycles that is more prominent than looking at raw data. Further he used a non-parametric bivariate P-spline regression to understand the relationship between wage share and employment rate and the dynamics of unemployment-inflation rate. An important contribution in this study is the separation of long phase cycles and business cycles through this method. Molina and Medina [22] extended the work done by Harvie [12] for 67 developed and developing countries. These countries could be divided into three groups, one which depicted Goodwin cycles, the second with movement in opposite direction as predicted by Goodwin due to demand pushed cycles, and a third without any cyclical movement. Later Tarassow [27] used bivariate VAR model for quantifying the relationship between wage share and employment rate in the US economy. The core of the paper is the implementation of impulse response functions and variance decomposition of forecast error to understand the propagation of shocks in one variable to the other using both the raw data and HP filtered data. Massy et.al. [20] introduced multiple sine-cosine terms to the state equations in the Goodwin model in order to better explain the fluctuations in real data for 16 countries. Although addition of harmonics definitely improved the fit, there is no discussion on the theoretical properties or the impact on structural stability of the model. Recently Moura and Ribeira [23] took a non-conventional route to estimate the Goodwin model and an extension proposed by Desai [6] for the Brazilian economy from 1981-2009. The non-conventionality lies in the data construction for workers share and employment rate. They use the Gompertz - Pareto distribution on individual income database for Brazil to find the workers share and profit share. Moreover since the methodology to calculate unemployment changed over the years, they redefined unemployment as when the average individual income is equal to or below 20% of the national minimum salary. Using these two new data series, they estimate classic Goodwin and its extension. Although there is clear evidence of qualitative cycles, they do not find quantitative evidence to support the Goodwin model or its extension.

1.3 Main Results

The structure of the thesis is as follows. In Chapter 2, we estimate underlying parameters and equilibrium of the classic Goodwin model[10] for 10 OECD countries. In the process, we also discuss various econometric and data issues in the previous work by Harvie[12] in this regard. We will evaluate the performance of the model in this and later chapters by comparing the equilibrium estimates with the observed historical average values. For the Goodwin model, we find that of the two state variables, only equilibrium estimate for employment rate is close to the observed historical average. Next, in Chapter 3 we analyze the impact of inflation on the bargaining power of the agents through a generalized model proposed by Desai[4] and we also estimate the equilibrium values produced by the model. Since this is a generalized version of the Goodwin model, we find mixed results for the impact of inflation on the bargaining power of the employees. For some countries, real and nominal bargaining remain the same, while for others, there is a significant difference. However, there is almost no improvement in the estimates of the equilibrium values. Finally in Chapter 4, we estimate the parameters and the equilibrium estimates of 2 models incorporating a generalized production function with constant elasticity of substitution(CES). One of these models is as proposed by van der Ploeg[29] and the other is an extension to understand the impact of inflation with the CES production function. The estimation methodology for the CES production function stems from the systems approach proposed in [18]. We find impressive improvement in the equilibrium estimates of the wage share and employment rate, both of which lie within the observed historical range for 6 of the 10 countries examined. This implication is important and encouraging for further econometric estimation of generalized dynamical models in macroeconomics.

Chapter 2

Goodwin Model Estimation

2.1 Data Construction and Sources

We use data for Australia, Canada, Denmark, Finland, France, Italy, Norway, UK and US from 1960-2010 from the AMECO database. Because of German unification in 1990-91, we see a jump in all the variables of Germany. Thus, we cut short the series for Germany till 1990.

We defined output as GDP at factor cost that is, net of taxes and subsidies on production and imports. Thus, real output Y equals

$$Y = \frac{\text{GDP at current prices - net taxes on production and imports}}{\text{GDP Deflator}} \tag{2.11}$$

For the estimation of the Goodwin model, we have to divide total real income Y into total real wages and total real profits. Wages can be gauged from the 'Compensation of Employees' variable in the database, but this does not include the income of the self employed, which can be significantly high (see figure 2.2). Since we can not find segregation of proprietor's income into labor and capital, we follow Klump et al. [16] to use compensation per employee as proxy for labor income of the self employed. Thus,

$$W = \text{Total real wages} = (1 + \frac{\text{Self Employed}}{\text{Total Employees}}) * \frac{\text{Compensation of Employees}}{\text{GDP Deflator}} \tag{2.12}$$

The leftovers from total real income Y is thus defined as total real profits $\Pi = Y - W$. For definition of labor productivity and wage rate, we first define total employment L as (total employees + self employed) and total labor force N as (total employment + total unemployed). Now productivity and wage rate become

$$a = \frac{Y}{L} \text{ and } \mathrm{w} = \frac{W}{L} \tag{2.13}$$

For variables using real capital stock K, we have net capital stock (2005 prices) within the database. We divide it by output Y to get capital output ratio ν. Capital rent r can then be defined as

$$r = \frac{\Pi}{K} \tag{2.14}$$

For depreciation rate δ, we use the definition from the manual of AMECO database, that is,

$$\delta = \frac{\text{Consumption of fixed capital at current prices}}{\text{Price deflator for gross fixed capital formation} * \text{Net capital stock (2005 Prices)}} \tag{2.15}$$

Now our two state variables, the employment rate λ and the wage share ω, are defined as

$$\lambda = \frac{\text{Total employees+ self employed}}{\text{Total employees+Self employed+Total unemployed}} = \frac{L}{N} \tag{2.16}$$

$$\omega = \frac{\text{Total real wages}}{\text{Real output}} = \frac{W}{Y} \tag{2.17}$$

2.2 Econometric Setup

The estimate of the equilibrium employment rate $\bar{\lambda}$ in equation (1.9) depends on the estimation of Phillips curve and the productivity growth rate α, and the estimate of the equilibrium wage share $\bar{\omega}$ in equation (1.10) depends on α, β, δ and ν. We can estimate the parameters for productivity growth rate and population growth rate using the log regression of the variables on the time trend, that is,

$$log(a_t) = log(a_0) + \hat{\alpha}t + \epsilon_{1t} \tag{2.18}$$

$$log(N_t) = log(N_0) + \hat{\beta}t + \epsilon_{2t} \tag{2.19}$$

We estimate the coefficients for the Phillips curve using the bounds testing approach suggested by Pesaran et. al. [25]. We will start with estimating an unrestricted error correction model (ECM) of the form

$$\Delta z_t = \phi_0 + \sum_{i=1}^{p} \phi_{1i}\Delta z_{t-i} + \phi_1 \Delta\lambda_{t-1} + \phi_2 z_{t-1} + \phi_3 \lambda_{t-1} + \epsilon_{3t} \tag{2.20}$$

where rate of growth of wages is defined as

$$z_t = \ln(\mathrm{w}_t) - \ln(\mathrm{w}_{t-1}) \tag{2.21}$$

and the lag p is determined using Bayesian Information Criterion. It's important at this step to ensure that the errors are not serially correlated and the model is "dynamically stable". If that is the case, then we perform an "F-test" of the hypothesis $H_0 : \phi_2 = \phi_3 = 0$ against the alternative that H_0 is not true. The distribution of the test statistic is non-standard and critical values for small samples were given in [24]. If we find critical values above the threshold, then we'll estimate the long run "levels model" and restricted ECM to find long run equilibrium relationship and short run dynamic effects.

2.3 Critique of Harvie

Before describing our results, we take a slight detour to discuss about some of the estimation and reporting issues in [12]. To start with, [12] had a reporting error in the estimation of the Phillips curve and this error propagated through the study. Coefficients for the employment rate in Table A2.3 (of [12]) are out of place. The estimates for the coefficient of the employment rate (and its lags), and the lags of wage growth rate are not in unison, with the former being much larger.[1] This error propagated further, leading to inappropriate equilibrium estimates of employment rate in Table 2 of [12]. The mistakenly large estimates of the parameters in the Phillips curve effectively killed the impact of productivity growth on employment rate. Harvie's estimates of employment rate were downward biased, with over 10 percentage point error for few countries. For example, correct estimate of employment rate for UK was 94.7% while Harvie reported as 84.7% - a full 10 percentage points

[1]We thank Dr. Harvie for informing in one of the email conversations that the coefficients of the employment rate were supposed to be in percentages but mistakenly used as numbers.

less. Table 2.1 shows the difference between the estimate of [12] and the corresponding table had there been no error. His estimate of time period of the Goodwin cycle is also incorrect due to same reporting problem and consequent miscalculation. The correct time period of the Goodwin cycle should have been between 10-22 years for his data, however it was reported to be between 1 and 2 year for all the countries.

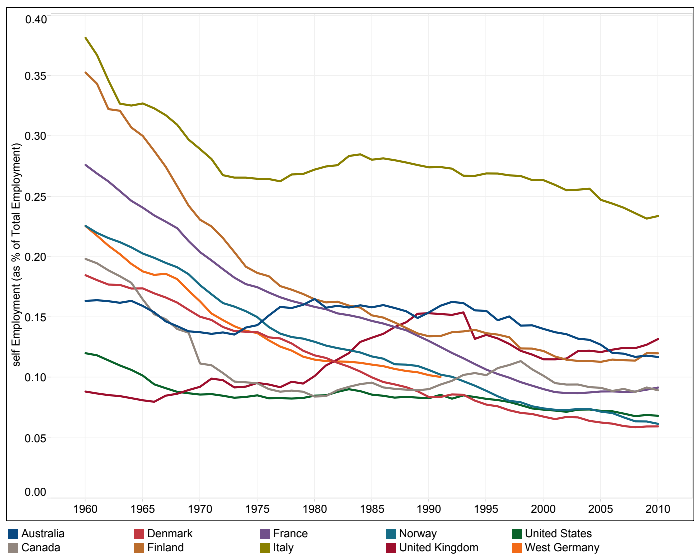


Figure 2.2: Self Employment (as pct. of Total Employment)

Secondly, the definition of wage share in his study does not segregate the income of the self employed into labor and capital income. Including proprietor's income as part of 'net operating surplus' is a gross underestimation of the wage share. We present Fig 2.2 to understand the percentage of the labor force that was self employed in the sample countries. As of 2010, while Italy has around a quarter of its population as self employed, 3 of the other 8 countries have over 10% of total employment as self employed. This effect was more prominent in early part of the data, when 8 of the 10 countries had over 15% of total employment as self employed. Including their total income as part of profits is therefore inappropriate.

Thirdly, his methodology was inconsistent in defining the total income of the economy. While defining wage share, Harvie used (Compensation of Employees + Net Operating Surplus) as a proxy for total income, leaving out consumption of fixed capital and taxes on production and imports from the GDP. However, for defining productivity and in the derivation of equilibrium values, he used GDP as a proxy for total income.

Variable	Australia	Canada	Finland	France	Germany	Greece	Italy	Norway	UK	US
α	1.66%	1.60%	3.03%	3.64%	3.29%	4.01%	4.60%	2.62%	2.21%	1.11%
β	2.26%	2.59%	0.80%	0.76%	0.41%	0.36%	0.49%	1.34%	0.37%	2.06%
σ	2.50	1.57	3.14	1.80	2.49	3.03	3.35	3.67	2.57	1.78
γ	0.62	0.59	0.32	0.55	0.85	0.46	0.71	1.18	0.19	0.08
ρ	0.67	0.65	0.36	0.62	0.66	0.53	0.82	1.22	0.22	(0.08)
λ_{Harvie}	92.9%	90.3%	88.9%	88.5%	130.5%	86.1%	86.9%	96.7%	84.7%	
$\lambda_{correct}$	95.4%	92.8%	97.3%	94.3%	135.5%	93.5%	92.5%	98.9%	94.7%	
diff	2.47%	2.45%	8.42%	5.87%	5.02%	7.50%	5.61%	2.15%	10.09%	
T_{Harvie}	1.32	1.06	2.09	1.18	1.13	1.73	1.49	1.2	2.42	
$T_{Correct}$	13.1	10.5	20.1	11.5	11.1	16.6	14.5	11.9	22.9	

Table 2.1: Corrected Table (Table 2 of [12])

In the results that follow, we address these problems with the methodology as in [12]

2.4 Result

2.4.1 Summary Statistics

Average wage share for the period varied between 61.48% for Norway to 71.47% for UK and average employment rate varied between 92.80% for Italy to 97.31% for Norway. Norway is a curious case with highest average employment rate (and least variability) and lowest average wage share (and highest variability). Finland remains the only exception with over 4% volatility in the employment rate. Major contribution of this standard deviation is attributed to a slump in the Finnish economy during early 1990's. During this time, the employment rate in Finland dropped by over 12 percentage point in 4 years from 1990 to 1994 and wage share continued to decline throughout the 90's. On the other hand, the United States has among the most stable wage share and employment rate when compared with any of its European counterpart. (Table 2.2)

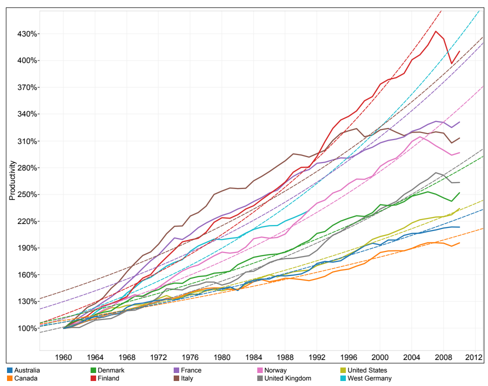
	Wage Share		Employment Rate	
Country	mean	std	mean	std
Australia	65.17%	(3.66%)	94.57%	(2.82%)
Canada	67.24%	(2.68%)	92.64%	(2.14%)
Denmark	68.43%	(2.28%)	95.54%	(2.58%)
Finland	69.97%	(5.39%)	93.75%	(4.18%)
France	70.94%	(3.79%)	93.61%	(3.29%)
Italy	68.14%	(4.40%)	92.80%	(1.93%)
Norway	61.48%	(5.92%)	97.31%	(1.51%)
United Kingdom	71.47%	(2.15%)	94.38%	(3.11%)
United States	65.52%	(1.72%)	94.16%	(1.55%)
West Germany	68.38%	(1.80%)	97.19%	(2.30%)

Table 2.2: Summary Statistics

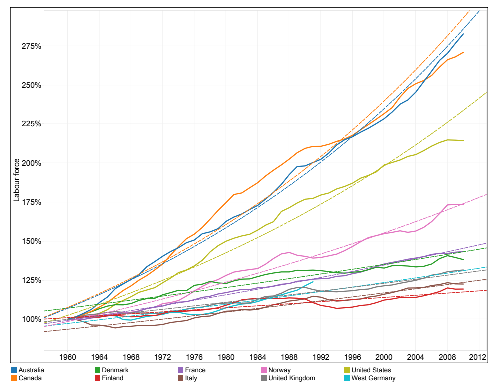
2.4.2 Estimation results

Table 2.3 shows the results for the labour force growth estimate for equation (2.19). Australia and Canada top the list with roughly 2% growth rate of labor force followed by US which has 1.65%. All the European economies considered except Norway, face the problem of ageing population with the growth rate less than three quarters of a percent. Similarly, table 2.4 depicts the estimates of the productivity growth equation 2.18. Productivity growth has been between 1.3% for Canada to

2.9% for Finland. Here again, there is a clear difference between European and non-European nations. Within the assumption of exponential growth in productivity, all the European nations have higher productivity growth rate compared to the three non-European countries. Figure 2.3 also gives a graphical interpretation of labour force growth and productivity growth rate.



(a) Productivity



(b) Labour Force

Figure 2.3: Both productivity and labour force are presented as proportion of their value in 1960. The dotted lines are the exponential trend lines

Country	Australia	Canada	Denmark	Finland	France	Italy	Norway	UK	US	Germany
$\ln N_0$	8.39^+	8.89^+	7.74^+	7.73^+	9.92^+	9.92^+	7.32^+	10.07^+	11.20^+	10.14^+
β	0.020^+	0.020^+	0.006^+	0.003^+	0.008^+	0.006^+	0.011^+	0.005^+	0.016^+	0.006^+
Rsquare	0.989	0.965	0.900	0.786	0.995	0.938	0.978	0.963	0.979	0.822
adjRsquare	0.989	0.964	0.898	0.782	0.995	0.937	0.977	0.963	0.978	0.816
Fstat	$4,470^+$	$1,359^+$	443^+	180^+	$10,774^+$	746^+	$2,134^+$	$1,286^+$	$2,248^+$	138^+
LBQstat	123^+	182^+	138^+	121^+	82^+	65^+	104^+	81^+	148^+	51^+
JBStat	4.21	4.45	1.72	0.92	1.75	13.34^+	2.88	3.17	1.76	1.33
ARCHstat	40.89^+	43.32^+	36.40^+	38.52^+	18.61^+	39.11^+	30.06^+	24.15^+	36.54^+	19.88^+

Table 2.3: Estimate of Labour Force growth given by equation (2.19). (+ indicates significance level of 1%)

To estimate the parameters of the Phillips Curve given by equation (1.5), we will first check if the variables in consideration are stationary. Thus, in Table 2.5, we present the estimates of augmented Dickey-Fuller test (ADF test) for real wage

Country	Australia	Canada	Denmark	Finland	France	Italy	Norway	UK	US	Germany
$\ln N_0$	-3.10$^+$	-3.18$^+$	-1.57$^+$	-4.14$^+$	-3.81$^+$	-3.81$^+$	-1.36$^+$	-4.21$^+$	-3.21$^+$	-3.77$^+$
α	0.015$^+$	0.013$^+$	0.018$^+$	0.029$^+$	0.022$^+$	0.021$^+$	0.023$^+$	0.021$^+$	0.016$^+$	0.027$^+$
Rsquare	0.988	0.968	0.976	0.975	0.906	0.829	0.978	0.990	0.986	0.937
adjRsquare	0.987	0.967	0.975	0.975	0.904	0.825	0.978	0.990	0.985	0.935
Fstat	$3,902^+$	$1,462^+$	$1,969^+$	$1,932^+$	474^+	237^+	$2,174^+$	$4,905^+$	$3,370^+$	448^+
LBQstat	37^+	85^+	74^+	99^+	138^+	135^+	63^+	46^+	69^+	76^+
JBStat	2.60	1.29	14.33$^+$	5.59	4.84	5.63	17.52$^+$	0.70	1.91	2.56
ARCHstat	25.27$^+$	35.25$^+$	22.21$^+$	29.19$^+$	44.51$^+$	43.21$^+$	39.70$^+$	14.33$^+$	16.23$^+$	21.97$^+$

Table 2.4: Estimate of Productivity growth given by equation (2.18). (+ indicates significance level of 1%)

growth, employment rate, productivity growth, inflation and nominal wage growth for the 10 countries in consideration. At a broad level we can say that real wage growth and productivity growth is stationary while employment rate, inflation and nominal wage growth are non-stationary for most of the countries.

Country	real Wage Growth	employment Rate	productivity Growth	inflation	nominal Wage Growth
Australia	0.001	0.449	0.001	0.260	0.153
Canada	0.001	0.510	0.001	0.140	0.220
Denmark	0.404	0.535	0.001	0.410	0.200
Finland	0.001	0.073	0.001	0.160	0.424
France	0.063	0.655	0.050	0.695	0.675
Italy	0.073	0.341	0.013	0.607	0.530
Norway	0.001	0.549	0.002	0.001	0.114
UK	0.001	0.293	0.001	0.223	0.138
US	0.001	0.402	0.001	0.438	0.096
Germany	0.147	0.432	0.046	0.157	0.256

Table 2.5: p-values for Augmented dicky-fuller test to check for unit root

Since real wage growth and employment rate have different order of integration (former is stationary, the latter is not), we can not use standard time series models. We will therefore use the bounds testing procedure proposed by [25] which allows us to test for the existence of linear long run relationship when variables have different order of integration. The effective unrestricted error correction model is given by equation (2.22), where optimal lag comes to zero for all the countries.

$$\Delta z_t = \phi_0 + \phi_1 \Delta\lambda_{t-1} + \phi_2 z_{t-1} + \phi_3 \lambda_{t-1} + \epsilon_{3t} \tag{2.22}$$

We now test the restriction that estimated coefficients on lagged variables are zero i.e. $\phi_2 = \phi_3 = 0$. Table 2.6 shows the F-statistic for the joint restriction $\phi_2 = \phi_3 = 0$. Since it has non-standard distribution, [24] provides a set of critical values when all variables are I(0) and when all variables are I(1). These two values serve as lower and upper bounds respectively, and when the F-statistic exceeds the upper critical bound we reject the null hypothesis of no-cointegration. The critical values for 50 observations at 1%, 5% and 10% levels corresponding to I(0) and I(1) values are [7.560 8.685], [5.220 6.070] and [4.190 4.940] respectively [24]. We can see that F-statistic is above the I(1) critical value at the significance level 1% for all the countries except Germany where it is true at the 10% level, thus rejecting the null hypothesis of no-cointegration for all countries. Next we present lbq test to check for no serial correlation in the errors for equation (2.22). This is important to make sure that the model is well specified. We observe in Table 2.7 that the p-values are greater than 10% for all countries and up till lag 5, thus implying no serial correlation.

Country	Australia	Canada	Denmark	Finland	France	Italy	Norway	UK	US	Germany
F statistics	15.548	17.154	33.071	21.107	12.574	8.519	21.421	13.830	8.019	5.651

Table 2.6: F-test for $\phi_2 = \phi_3 = 0$ restriction in equation (2.22). Critical Values at 10% level for I(0) and I(1) are 4.190 and 4.940 respectively

Since we have established that the variables are co-integrated, we will now present

Country	lag 1	lag 2	lag 3	lag 4	lag 5
Australia	0.967	0.926	0.938	0.948	0.970
Canada	0.912	0.242	0.358	0.520	0.664
Denmark	0.742	0.946	0.279	0.334	0.463
Finland	0.714	0.841	0.432	0.594	0.732
France	0.555	0.838	0.859	0.508	0.453
Italy	0.313	0.594	0.719	0.827	0.872
Norway	0.940	0.935	0.795	0.846	0.922
UK	0.948	0.997	0.869	0.323	0.425
US	0.687	0.642	0.794	0.598	0.298
Germany	0.795	0.603	0.786	0.525	0.282

Table 2.7: p-values for serial correlation in the errors of the unrestricted ECM given by equation (2.22)

the long term estimates of the linear Phillips curve(equation (2.23)) in Table 2.8.

$$z_t = \hat{\gamma} + \hat{\rho}_1 \lambda_t + \epsilon_{4t} \tag{2.23}$$

All the countries have negative constant and positive slope. Thus employment has significantly positive impact at 10% level of significance for all the countries except Canada and the values range from 11% for Canada to 75% for Germany. Italy has a higher coefficient 98% but it may be plagued by bias due to auto-correlation in the errors.

Serial correlation in the long term model (Table 2.9) suggests that the model is appropriate for all the countries except Italy, where errors are serially correlated at 5% level of significance.

As a final check, we present the estimates of the restricted error correction model given by equation (2.24) in the Table 2.10. If the model is correct, the coefficient of the lagged error terms should be negative and significant. Model diagnostic tests for

Country	Variable	$\hat{\gamma}$	ρ_1	AdjR2
Australia	Coeff	-0.215	0.242	0.086
	pValue	0.031	0.022	
Canada	Coeff	-0.095	0.115	0.010
	pValue	0.283	0.230	
Denmark	Coeff	-0.330	0.367	0.216
	pValue	0.001	0.000	
Finland	Coeff	-0.258	0.303	0.274
	pValue	0.000	0.000	
France	Coeff	-0.491	0.549	0.755
	pValue	0.000	0.000	
Italy	Coeff	-0.891	0.982	0.507
	pValue	0.000	0.000	
Norway	Coeff	-0.574	0.609	0.039
	pValue	0.100	0.090	
UK	Coeff	-0.108	0.135	0.045
	pValue	0.131	0.076	
US	Coeff	-0.227	0.257	0.086
	pValue	0.031	0.022	
Germany	Coeff	-0.699	0.747	0.673
	pValue	0.000	0.000	

Table 2.8: Long term estimates for Phillips curve given by equation (2.23)

no-autocorrelation and homoscedasticity are accepted for all countries except Italy.

$$\Delta z_t = \phi_{10} + \phi_{11}\Delta\lambda_{t-1} + \phi_{12}\hat{\epsilon}_{4(t-1)} + \epsilon_{5t} \tag{2.24}$$

Finally we present the equilibrium estimates of the Goodwin model in Table 2.11. In this table productivity and population growth rate parameters α and β are taken from Tables 2.4 and 2.3 respectively. Depreciation parameter δ is the average of the historical depreciation calculated using equation (2.15). Similarly, capital output ratio ν is the historical average of the ratio of real capital stock and real output. γ and ρ are the parameters of the linear Phillips curve estimated in Table 2.23. The variables

Country	lag 1	lag 2	lag 3	lag 4	lag 5
Australia	0.155	0.363	0.560	0.646	0.743
Canada	0.407	0.173	0.196	0.307	0.438
Denmark	0.397	0.298	0.061	0.109	0.181
Finland	0.493	0.571	0.174	0.284	0.393
France	0.090	0.237	0.406	0.210	0.311
Italy	0.015	0.010	0.011	0.021	0.035
Norway	0.868	0.852	0.745	0.800	0.891
UK	0.085	0.168	0.289	0.078	0.097
US	0.057	0.134	0.224	0.358	0.322
Germany	0.287	0.342	0.483	0.576	0.276

Table 2.9: p-values for serial correlation in the errors of the long term Phillips curve give by equation (2.23)

ω_G and λ_G are the final equilibrium estimates of the Goodwin model derived by using equations (1.10) and (1.9). Since the model has a closed cycle, we also present the time period T of the cycle given by

$$T = \frac{2\pi}{\sqrt{(\alpha - \gamma)(1/\nu - \alpha - \beta)}} \tag{2.25}$$

The estimates for the equilibrium employment rate λ_G is well within the bound of observed values during the time of study. The difference of the estimated equilibrium from the observed mean for employment rate is under 1 percentage point for all the countries except Canada and Finland, where the difference is 1.06 percentage point. Compared with the estimated values by Harvie, this is a motivating improvement. However, the simplistic assumption of the model manifests itself in the estimates of equilibrium wage share ω_G. Except Finland, we didn't see any other country having equilibrium value within the long term observed cycle. With the unrealistic assumptions on the constant capital output ratio, this should not be surprising. However, the

Country	Variable	constant	$\Delta\lambda_{t-1}$	$\hat{\epsilon}_{4(t-1)}$	AdjR2
Australia	Coeff	0.000	0.023	-0.826	0.390
	pValue	0.944	0.941	0.000	
Canada	Coeff	0.000	0.034	-0.873	0.425
	pValue	0.859	0.884	0.000	
Denmark	Coeff	-0.001	0.343	-1.097	0.586
	pValue	0.654	0.176	0.000	
Finland	Coeff	0.000	0.375	-0.953	0.473
	pValue	0.888	0.076	0.000	
France	Coeff	-0.001	0.103	-0.679	0.330
	pValue	0.436	0.663	0.000	
Italy	Coeff	-0.001	-0.263	-0.580	0.245
	pValue	0.754	0.570	0.000	
Norway	Coeff	0.000	0.902	-0.983	0.469
	pValue	0.963	0.388	0.000	
UK	Coeff	0.000	0.278	-0.762	0.355
	pValue	0.981	0.279	0.000	
US	Coeff	0.000	-0.249	-0.583	0.279
	pValue	0.841	0.175	0.000	
Germany	Coeff	-0.001	0.192	-0.766	0.237
	pValue	0.777	0.676	0.003	

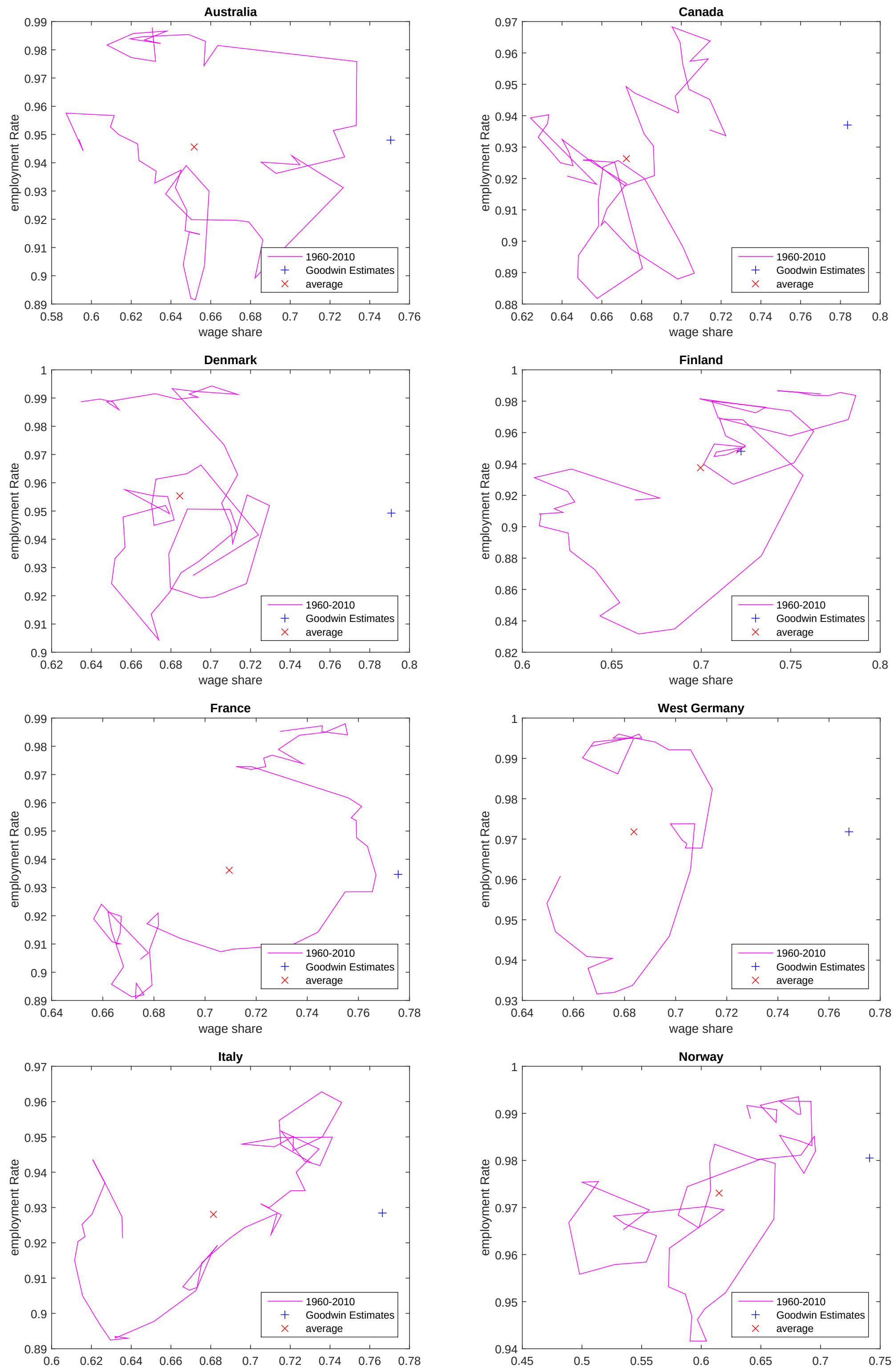
Table 2.10: Restricted Errorcorrection Model

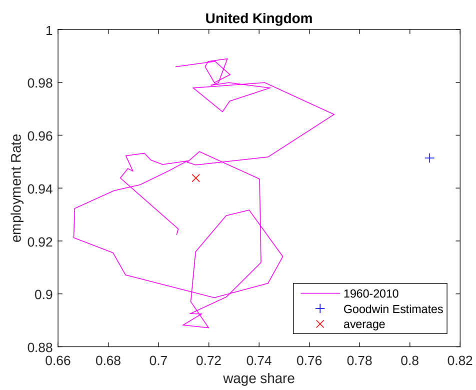
error has reduced drastically and is between 2-12 percentage point, compared with 20-100 percentage point reported by Harvie. The analysis is encouraging not only from the qualitative perspective but also quantitative. Even such a stylized version of a real economy can make important conclusions about evolution and growth in an economy. This encourages us to look at a more generalized versions of the Goodwin model given by [29] and [4], where the assumptions of constant capital output ratio, perfect complementarity of labour and capital and real bargaining are relaxed.

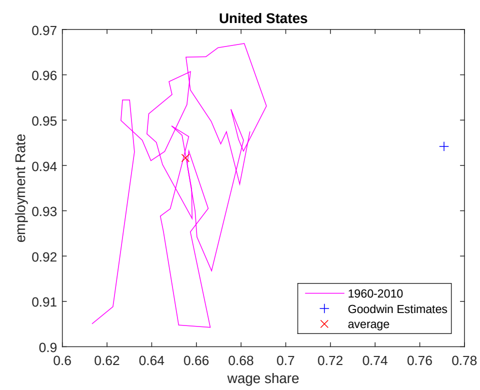
Country	α	β	δ	ν	γ	ρ	ω_G	λ_G	T
Australia	1.47%	1.97%	5.22%	2.881	-0.215	0.242	75.06%	94.80%	25.70
Canada	1.26%	2.01%	4.29%	2.864	-0.095	0.115	78.36%	93.71%	36.58
Denmark	1.81%	0.58%	4.97%	2.842	-0.330	0.367	79.07%	94.92%	20.18
Finland	2.88%	0.30%	5.19%	3.314	-0.258	0.303	72.24%	94.80%	25.13
France	2.21%	0.76%	3.77%	3.326	-0.491	0.549	77.56%	93.46%	18.17
Italy	2.08%	0.56%	4.66%	3.206	-0.891	0.982	76.63%	92.85%	13.46
Norway	2.28%	1.14%	4.65%	3.208	-0.574	0.609	74.11%	98.04%	16.92
UK	2.05%	0.52%	3.72%	3.053	-0.108	0.135	80.78%	95.15%	34.10
US	1.55%	1.65%	5.21%	2.725	-0.227	0.257	77.08%	94.41%	24.00
Germany	2.70%	0.60%	3.62%	3.358	-0.699	0.747	76.78%	97.17%	15.42

Table 2.11: Summary of the parameter values and the estimated equilibrium

Figure 2.4: Estimated equilibrium and observed mean for Goodwin Model




United Kingdom
employment Rate
wage share
1960-2010
Goodwin Estimates
average


United States
employment Rate
wage share
1960-2010
Goodwin Estimates
average

Chapter 3

Desai Model

In his classic paper[4], Sir Meghnad Desai introduced inflation and expected rate of inflation into the Goodwin model. In this chapter, we will first discuss the theoretical setup of the final model in his paper, followed by econometric setup for estimating the parameters as discussed in [5] and finally we will show the equilibrium estimates.

3.1 Model Setup

The model tries to address two important shortcomings of the Goodwin model: wage bargain in terms of real wages and the constant capital output ratio. To address the first case, [4] incorporates expected inflation in his wage bargaining setup through equation (3.26), where $\frac{\dot{m}}{m}$ is nominal wage growth rate and $(\frac{\dot{p}}{p})^e$ is expected inflation rate. The adjustment between expected and actual inflation is given by equation

(3.27) and finally cost push inflation is given by equation (3.28).

$$\frac{\dot{m}}{m} = \gamma_2 + \rho_2\lambda + \eta_2(\frac{\dot{p}}{p})^e \tag{3.26}$$

$$\frac{d}{dt}(\frac{\dot{p}}{p})^e = \tau_2\left\{(\frac{\dot{p}}{p}) - (\frac{\dot{p}}{p})^e\right\} \tag{3.27}$$

$$\frac{\dot{p}}{p} = \eta_3(\ln\omega + \ln\pi) \tag{3.28}$$

The assumption of constant capital output ratio is replaced by assuming cyclical variability dependent on employment rate given by equation (3.29)

$$\sigma = \sigma^*\lambda^{-\mu} \tag{3.29}$$

Using these changes we can derive the state equations as (see Appendix A).

$$\frac{d}{dt}[\frac{\dot{\omega}}{\omega}] = (\gamma_2 - \alpha)\tau_2 + \rho_2\tau_2\lambda + \rho_2\dot{\lambda} + (\eta_2 - 1)\tau_2\eta_3(\ln\omega + \ln\pi) - (\eta_3 + \tau_2)\frac{\dot{\omega}}{\omega} \tag{3.30}$$

$$\frac{\dot{\lambda}}{\lambda} = -\frac{\alpha+\beta+\delta}{1-\mu} + \frac{(1-\omega)\lambda^\mu}{\sigma^*(1-\mu)} \tag{3.31}$$

It can be seen that the coefficients of some of the terms in wage share equation are not the same as given by [5]. This will impact the equilibrium estimates of wage share and employment rate. Now, following [5] we can write these as a system of first order differential equation by setting

$$y_1 = \ln\omega \tag{3.32}$$

$$y_2 = \dot{y}_1 \tag{3.33}$$

$$x_1 = \ln\lambda \tag{3.34}$$

so that,

$$\dot{y}_2 = a_0 + a_1 y_1 + a_2 y_2 + a_3 exp(x_1) + a_4 \dot{x}_1 exp(x_1) \tag{3.35}$$

$$\dot{x}_1 = b_0 + b_1(1 - exp(y_1))exp(\mu x_1) \tag{3.36}$$

with,

$$a_0 = -(-\gamma_2 + \alpha)\tau_2 + (\eta_2 - 1)\tau_2\eta_3 ln\pi \tag{3.37}$$

$$a_1 = (\eta_2 - 1)\tau_2\eta_3 \tag{3.38}$$

$$a_2 = -(\eta_3 + \tau_2) \tag{3.39}$$

$$a_3 = \rho_2\tau_2 \tag{3.40}$$

$$a_4 = \rho_2 \tag{3.41}$$

$$b_0 = -\frac{\alpha + \beta + \delta}{1 - \mu} \tag{3.42}$$

$$b_1 = \frac{1}{\sigma^*(1 - \mu)} \tag{3.43}$$

At equilibrium, $\dot{y}_1 = \dot{y}_2 = \dot{x}_1 = 0$ Thus,

$$a_0 + a_1 y_1 + a_3 exp(x_1) = 0 \tag{3.44}$$

$$\implies a_0 + a_1 \ln\omega + a_3\lambda = 0 \tag{3.45}$$

$$\implies \lambda = -\frac{a_0}{a_3} - \frac{a_1}{a_3}\ln\omega \tag{3.46}$$

$$\implies \lambda = \frac{(-\gamma_2 + \alpha)\tau_2 + (1 - \eta_2)\tau_2\eta_3 \ln\pi}{\rho_2\tau_2} + \frac{(1 - \eta_2)\tau_2\eta_3}{\rho_2\tau_2}\ln\omega \tag{3.47}$$

$$\implies \lambda = \frac{(-\gamma_2 + \alpha) + (1 - \eta_2)\eta_3 \ln\pi}{\rho_2} + \frac{(1 - \eta_2)\eta_3}{\rho_2}\ln\omega \tag{3.48}$$

$$b_0 + b_1(1 - exp(y_1))exp(\mu x_1) = 0 \tag{3.49}$$

$$\implies \omega = 1 + \frac{b_0}{b_1}\lambda^{-\mu} \tag{3.50}$$

$$\implies \omega = 1 - (\alpha + \beta + \delta)\sigma^*\lambda^{-\mu} \tag{3.51}$$

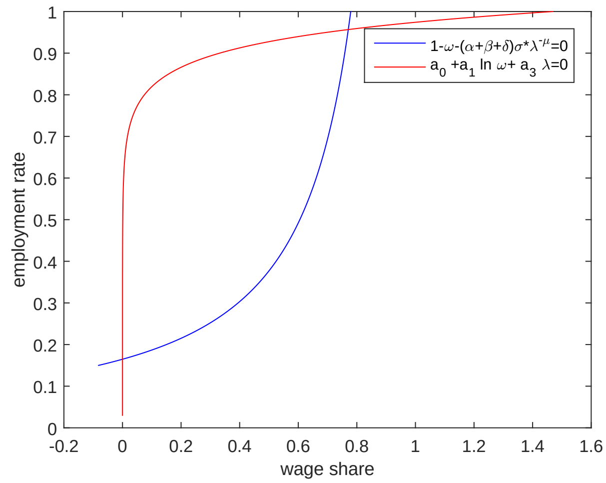


Figure 3.5: parameter values used: $\gamma_2 = -0.3106$, $\rho_2 = 0.3477$, $\eta_2 = 0.9530$, $\alpha = 0.0181$, $\beta = 0.0058$, $\mu = 0.8374$, $\pi = 1.5357$, $\delta = 0.0497$, $\eta_3 = 0.497$, $\tau_2 = 0.874$, $\sigma* = 3$

In Figure 3.5, we illustrate the solution of equations (3.46) and (3.50). The intersection of the two equations give us the required equilibrium estimates. Although there are two equilibrium, we will only consider the higher one as it only relates to reality.

3.2 Econometric Setup

With this theoretical framework in place, we can now discuss the estimation methodology. The first major step in this regard is estimation of modified Phillips curve as given by equations (3.26) and (3.27). The discrete versions of these equations are

$$\Delta \ln(m_t) = \gamma_2 + \rho_2 \lambda_t + \eta_2 \Delta \ln(p^e_t) + \epsilon_{1t} \tag{3.52}$$

$$\Delta \ln(p^e_t) = \tau_2 \Delta \ln(p_t) + (1 - \tau_2) \Delta \ln(p^e_{t-1}) + \epsilon_{2t} \tag{3.53}$$

Since expected inflation is not measurable, we need to eliminate it from the estimation equation. With some rearrangement as described in appendix B, we can get the following

$$\begin{aligned}\Delta^2 \ln(m_t) =& \tau_2\gamma_2 + \tau_2\rho_2\lambda_{t-1} + \rho_2\Delta\lambda_t + \eta_2\tau_2\Delta\ln(p_t) - \tau_2\Delta\ln(m_{t-1}) \\ &+ \tau_2\epsilon_{1t-1} + \Delta\epsilon_{1t} + \eta_2\epsilon_{2t}\end{aligned} \tag{3.54}$$

We can easily estimate this using non-linear least squares. To get an estimate of π and η_3, the two variables in the markup equation, we do the following regression

$$\Delta\ln(p_t) = \eta_3\ln\pi + \eta_3(\ln(m/a)_t - \ln(p_{t-1})) + \epsilon_{3t} \tag{3.55}$$

For variable capital output ratio, we estimate the following equation

$$\Delta\ln\sigma_t = -\mu\Delta\ln\lambda_t + \epsilon_{4t} \tag{3.56}$$

Notice that this is the differenced version of the log transformed equation (3.29). Since both capital output ratio σ and employment rate λ are non-stationary, we use the differenced version to have stationary variables for regression. By now we have all variables except one(σ^*) needed for estimation of equilibrium.

3.3 Results

We estimate the expectation - augmented Phillips curve (equation (3.54)) by using non-linear least squares. The results are presented in Table 3.12. We observe that employment rate has the least impact in Canada and most in Norway with coefficients being 0.26 and 1.32 respectively. Moreover, the coefficient is significant for all the countries and well within 1 for all, except Italy and Norway.

Country	Variable	γ_2	ρ_2	η_2	τ_2	AdjR2	$\tau_2 = 1$	$\eta_2 = 1$	$\tau_2 = 1$ & $\eta_2 = 1$
Australia	Coeff	-0.316	0.350	0.997	0.691	0.523			
	pValue	0.033	0.026	0.000	0.000		0.008	0.982	0.027
Canada	Coeff	-0.229	0.261	0.959	0.634	0.624			
	pValue	0.061	0.048	0.000	0.000		0.000	0.664	0.000
Denmark	Coeff	-0.311	0.348	0.953	0.874	0.475			
	pValue	0.005	0.003	0.000	0.000		0.335	0.600	0.542
Finland	Coeff	-0.352	0.408	0.901	0.678	0.556			
	pValue	0.001	0.000	0.000	0.000		0.003	0.387	0.004
France	Coeff	-0.496	0.556	0.947	1.022	0.611			
	pValue	0.000	0.000	0.000	0.000		0.856	0.263	0.522
Italy	Coeff	-0.920	1.013	0.992	0.846	0.583			
	pValue	0.000	0.000	0.000	0.000		0.188	0.904	0.404
Norway	Coeff	-1.237	1.328	0.298	0.480	0.350			
	pValue	0.000	0.000	0.060	0.000		0.000	0.000	0.000
UK	Coeff	-0.112	0.142	0.954	0.992	0.764			
	pValue	0.150	0.088	0.000	0.000		0.928	0.378	0.649
US	Coeff	-0.245	0.283	0.818	0.713	0.466			
	pValue	0.045	0.029	0.000	0.000		0.017	0.076	0.005
Germany	Coeff	-0.682	0.729	1.024	0.995	0.669			
	pValue	0.000	0.000	0.000	0.000		0.975	0.885	0.989

Table 3.12: Estimation of expected augmented Phillips Curve given by equation (3.54). Column header "$\tau_2 = 1$" and "$\eta_2 = 1$" contain p-value for the t-statistic for the test,and column header "$\tau_2 = 1$ & $\eta_2 = 1$" gives the p-value for the F-test.

Also, the hypothesis $\eta_2 = 1$ can be rejected only for Norway at 5% level and the hypothesis $\tau_2 = 1$ holds true for 5 of the 10 countries examined and strictly less than 1 for the rest. The joint hypothesis that $\eta_2 = \tau_2 = 1$ holds true for Denmark, France, Italy, UK and Germany. Thus, for these countries, the estimates for γ_2 and ρ_2 are similar to what we derived in previous chapter for linear Phillips curve and the assumption of real wage bargaining in basic Goodwin model is not that stringent after all. However, we should definitely not ignore the impact of inflation for meaningful analysis in the countries where this does not hold true.

Moving further, we next estimate the markup equation (3.55). Table 3.13 shows that the markup factor is around 1.6 for all the countries except Norway, where it is 2.2. Also, the adjustment term η_3 is significantly less than 1 for all the countries.

Country	Variable	$\eta_3 ln\pi$	η_3	AdjR2	π
Australia	Coeff	0.195	0.380	0.744	1.673
	pValue	0.000	0.000		
Canada	Coeff	0.181	0.392	0.393	1.589
	pValue	0.000	0.000		
Denmark	Coeff	0.213	0.497	0.748	1.536
	pValue	0.000	0.000		
Finland	Coeff	0.146	0.303	0.600	1.620
	pValue	0.000	0.000		
France	Coeff	0.159	0.379	0.851	1.522
	pValue	0.000	0.000		
Italy	Coeff	0.207	0.435	0.749	1.609
	pValue	0.000	0.000		
Norway	Coeff	0.114	0.141	0.135	2.240
	pValue	0.000	0.005		
UK	Coeff	0.233	0.632	0.888	1.447
	pValue	0.000	0.000		
US	Coeff	0.210	0.450	0.671	1.595
	pValue	0.000	0.000		
Germany	Coeff	0.157	0.348	0.648	1.569
	pValue	0.000	0.000		

Table 3.13: Markup equation 3.55

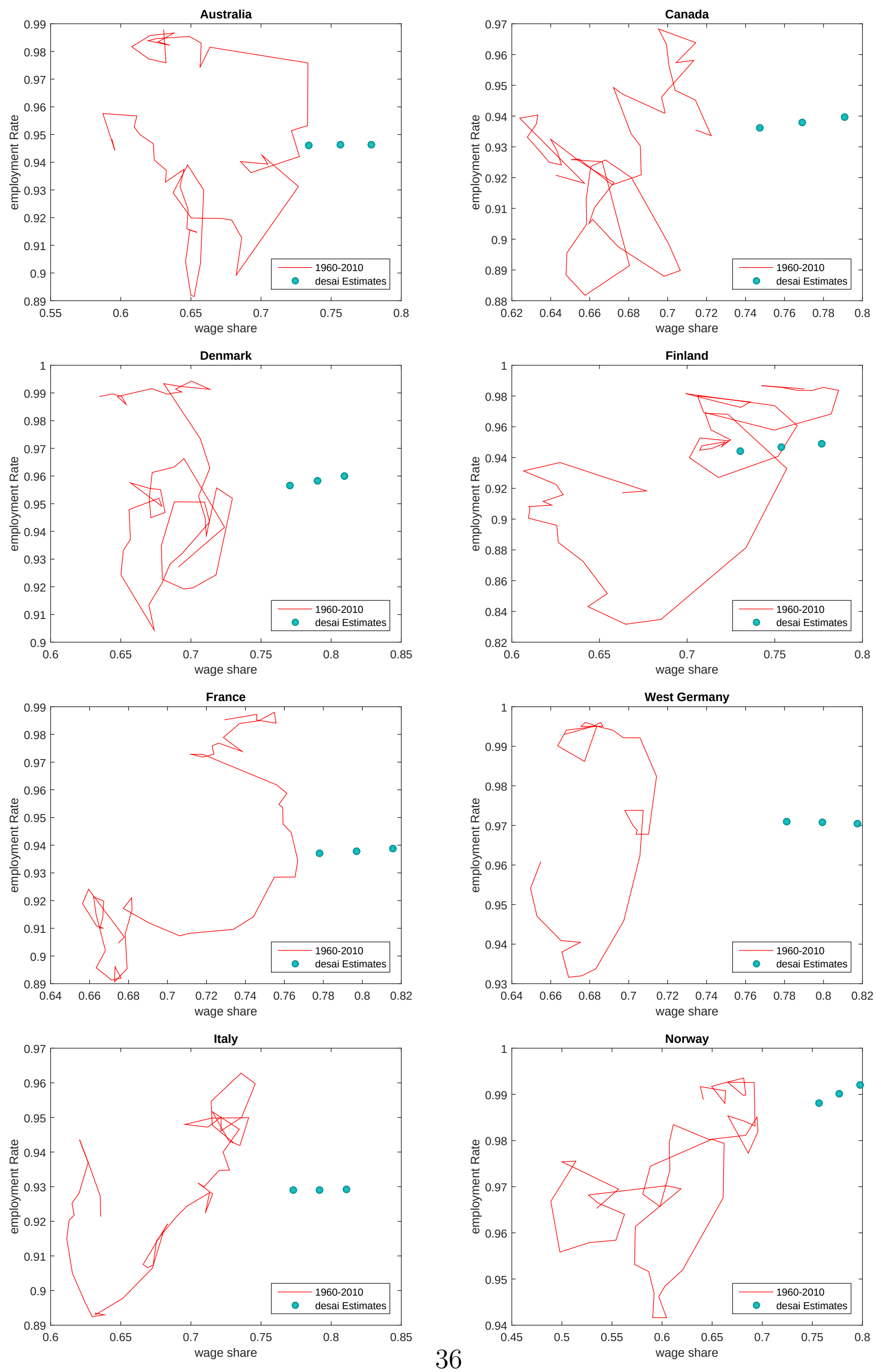
Next we present the estimate of μ in Table 3.14. This estimate also tests for the assumption of constant capital output ratio in the Goodwin model. Since the coefficient estimate varies between 0.4 to 1.8 and it remains significant for 7 of the 10 countries examined, it highlights the strong assumption made by the Goodwin model by assuming constant capital output ratio.

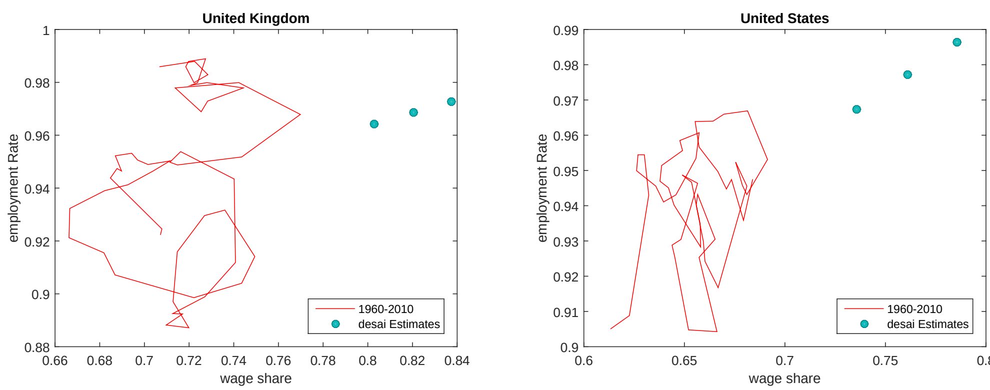
Finally, we are in position to find the equilibrium estimates given by intersection of equations (3.48) and (3.51). Although the equations intersect at two points, we only consider the higher equilibrium estimate as the lower equilibrium is too far off from reality. Since we could not find the estimate of σ^*, we will assume multiple values between 2.5 to 3 to understand its impact. This assumption comes from real data on capital output ratio which lies around 3 for most countries during the period of study. We depict the estimates of wage share and employment rate for this model along with historical line chart in figure 3.6. We find the equilibrium estimates to be far from observed historical average for all the countries except Finland. Thus although the model gave quality insight into the impact of inflation in the bargaining ability of the agents, equilibrium estimates continue to be far off and we need to relax other assumptions to get meaningful estimates of equilibrium wage share and employment rate.

Country	γ_2	ρ_2	η_2	τ_2	η_3	π	μ	β	α
Australia	-0.3160	0.3498	0.9975	0.6913	0.3797	1.673	0.4173	1.97%	1.47%
Canada	-0.2291	0.2611	0.9593	0.6338	0.3918	1.589	1.6353	2.01%	1.26%
Denmark	-0.3106	0.3477	0.9530	0.8740	0.4971	1.536	0.8374	0.58%	1.81%
Finland	-0.3516	0.4083	0.9005	0.6782	0.3028	1.620	1.2289	0.30%	2.88%
France	-0.4956	0.5562	0.9469	1.0216	0.3787	1.522	1.4090	0.76%	2.21%
Italy	-0.9196	1.0130	0.9925	0.8458	0.4355	1.609	0.5095	0.56%	2.08%
Norway	-1.2375	1.3284	0.2976	0.4796	0.1414	2.240	0.4612	1.14%	2.28%
UK	-0.1124	0.1424	0.9539	0.9923	0.6319	1.447	1.1622	0.52%	2.05%
US	-0.2452	0.2831	0.8181	0.7130	0.4501	1.595	1.4115	1.65%	1.55%
Germany	-0.6824	0.7288	1.0241	0.9952	0.3481	1.569	1.8129	0.60%	2.70%

Table 3.14: Desai Model Estimates

Figure 3.6: Estimates from Desai Model for σ^* equal to 2.5,2.75 and 3(right to left)




United Kingdom
employment Rate
wage share
1960-2010
desai Estimates
United States
employment Rate
wage share
1960-2010
desai Estimates

Chapter 4

van der Ploeg Model

4.1 Model Setup

The Goodwin model assumed a Leontief production function and thus assumes no substitution between labour and capital. van der Ploeg [29] incorporated substitutibility between labor and capital by using a CES production function. Within this framework, output is given by

$$Y_t = C[\pi K_t^{-\eta} + (1-\pi)(e^{\lambda_l t} L_t)^{-\eta}]^{\frac{-1}{\eta}} \tag{4.57}$$

where λ_l represents the labor augmented technical growth, C is the efficiency parameter and π is the distribution parameter. Elasticity of substitution can be defined as $\sigma = \frac{1}{1+\eta}$. As η goes to zero (σ goes to 1), the production function converges to be Cobb-Douglas and as η goes to infinity, the production function converges to Leontif, thus requires no substitution.

Using the profit maximization hypothesis, we can derive the optimal wage rate and capital rent as

$$w = \frac{1-\pi}{(Ce^{\lambda_l t})^\eta}(\frac{Y}{L_t})^{\eta+1} \tag{4.58}$$

$$r = \frac{\pi}{C^\eta}(\frac{Y}{K_t})^{\eta+1} \tag{4.59}$$

Moreover, we can also conclude that the optimal capital output ratio and productivity will be

$$\frac{K}{Y} = \frac{1}{C}(\frac{1-\omega}{\pi})^{\frac{-1}{\eta}} \tag{4.60}$$

$$\frac{Y}{L} = Ce^{\lambda_l t}(\frac{\omega}{1-\pi})^{\frac{1}{\eta}} \tag{4.61}$$

Empirically, the most debatable assumption in Goodwin was the constant capital output ratio which has now been replaced as a function of wage share. Under this setting, when capital and labour are complements i.e. $\sigma < 1$ ($\eta > 0$), an increase in wage share will also lead to a higher equilibrium capital output ratio. van der Ploeg[29] also adjusts the wage bargaining equation by adding productivity growth rate i.e. other than employment rate, an increase in productivity growth rate also adds to the bargaining power of the employees. However, his version only catered to real bargaining. As we have seen in the previous chapter, inflation can have substantial impact on the bargaining power of the employees. So, we will also consider a variant of [29], incorporating inflation in the Phillips curve.

4.1.1 Model 1

van der Ploeg [29] incorporated the following bargaining equation

$$\frac{\dot{\mathrm{w}}}{\mathrm{w}} = \Phi(\lambda) + \rho_2 \frac{\dot{a}}{a} = \gamma + \rho_1 \lambda + \rho_2 \frac{\dot{a}}{a} \tag{4.62}$$

As shown in Appendix C, this leads to the following differential equations for wage share and employment rate :

$$\frac{\dot{\omega}}{\omega} = \frac{\Phi(\lambda) - (1-\rho_2)\lambda_l}{1 + \frac{1-\rho_2}{\eta}} \tag{4.63}$$

$$\frac{\dot{\lambda}}{\lambda} = C\pi^{-1/\eta}(1-\omega)^{\frac{1+\eta}{\eta}} - \frac{1}{\eta(1-\omega)}\frac{\dot{\omega}}{\omega} - (\alpha+\beta+\delta) \tag{4.64}$$

Unlike the Goodwin model, where the equilibrium values are non-hyperbolic, the solution of the above two differential equation is a stable sink if $\rho_2\sigma < 1$. The equilibrium estimates will be

$$\bar{\lambda} = \phi^{-1}((1-\rho_2)\lambda_l) \tag{4.65}$$

$$\bar{\omega} = 1 - (\frac{\lambda_l + \beta + \delta}{C})^{\frac{\eta}{1+\eta}} \pi^{\frac{1}{1+\eta}} \tag{4.66}$$

A simulation experiment to illustrate the solution of the two differential equations in the phase plane is shown in Figure 4.7. The simulation has the same initial value and basic parameters as for figure 1.1. Since a linear $\Phi(\lambda)$ can lead to the employment rate above 1, we used its non-linear version for illustration.

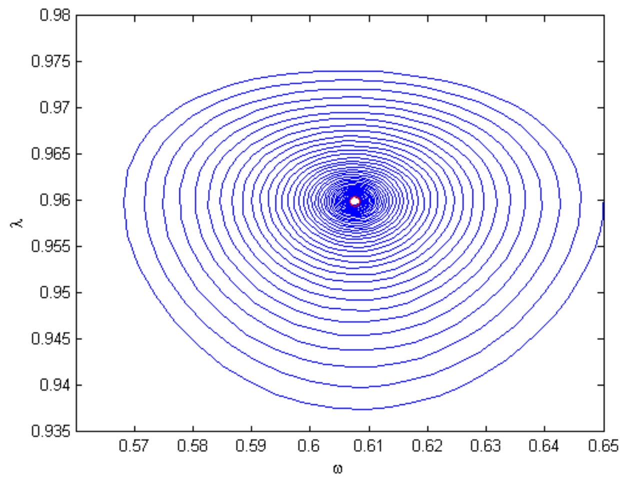


Figure 4.7: van der Ploeg Simulations

4.1.2 Model 2

Here, we will replace real bargaining proposed by [29] to a nominal wage bargaining equation given by equation (4.67) and inflation dynamics given by equation (4.68).

$$\frac{\dot{m}}{m} = \gamma_3 + \rho_3\lambda + \eta_3\frac{\dot{p}}{p} \tag{4.67}$$

$$\frac{\dot{p}}{p} = \chi_2(\ln\omega + \ln\bar{\pi}) \tag{4.68}$$

Thus, we get the following system of differential equations for wage share and employment rate:

$$\frac{\dot{\omega}}{\omega} = \frac{\Phi(\lambda) - \lambda_l - (1-\eta_3)\chi_2\ln(\omega\bar{\pi})}{1+\frac{1}{\eta}} \tag{4.69}$$

$$\frac{\dot{\lambda}}{\lambda} = C\pi^{-1/\eta}(1-\omega)^{\frac{1+\eta}{\eta}} - \frac{1}{\eta(1-\omega)}\frac{\dot{\omega}}{\omega} - (\lambda_l+\beta+\delta) \tag{4.70}$$

and the equilibrium wage share and employment rate is

$$\bar{\omega} = 1 - \left(\frac{\lambda_l + \beta + \delta}{C}\right)^{\frac{\eta}{1+\eta}} \pi^{\frac{1}{1+\eta}} \tag{4.71}$$

$$\bar{\lambda} = \Phi^{-1}(\lambda_l + (1 - \eta_3)\chi_2 \ln(\omega\bar{\pi})) \tag{4.72}$$

Now, the Jacobian of the system given by equations (4.69)-(4.70), around the equilibrium (4.71)-(4.72) has trace and determinant as -

$$Trace = -\frac{(1-\eta_3)\chi_2}{1+\frac{1}{\eta}} - \frac{\lambda\rho_3}{(1-\omega)(1+\eta)} \tag{4.73}$$

$$Determinant = C\rho_3\lambda\pi^{-1/\eta}\omega(1-\omega)^{1/\eta} \tag{4.74}$$

Thus, if $\eta_3 < 1$ and $\sigma < 1$, then the trace and the determinant of the Jacobian is negative and positive respectively. This implies that both the eigen-values are negative, leading to stable equilibrium.

4.2 Econometric Setup

The equilibrium values in the van der Ploeg model are dependent on the estimate of the parameters of the production function. There are numerous econometric techniques to estimate the CES production function such as ordinary least squares (OLS) estimates using the first order conditions, linear approximation as proposed in [17], optimization algorithms for non-linear functions, instrumental variables and the systems approach. Ledesma et. al.[18] discuss in detail the problems and advantages of each of the approaches using simulations. Among many problems discussed, the use of direct non linear optimization on the production function suffers severely from the convergence problem, first order approximation does not give estimate of all the parameters, Kamenta approximation can lead to complex estimates of the labour augmented technological change λ_l. Thus, in this thesis we use the systems approach on lines of seemingly unrelated regression as suggested by [18].

Following Leon-Ledesma [18], normalization of the variables in the production function make the interpretation of the parameters meaningful and the estimation of deep parameters easy and consistent with the basic properties of the CES production function in the context of the growth theory [18]. Thus, equation (4.57) changes to the following form

$$Y = \xi Y_0[\pi_0(\frac{K_t}{K_0})^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)(e^{\lambda_l t}\frac{L_t}{L_0})^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \tag{4.75}$$

where

$$\pi_0 = \frac{r_0K_0}{r_0K_0 + \mathrm{w}_0L_0} \tag{4.76}$$

$$C = \xi Y_0[\frac{r_0K_0^{1/\sigma} + \mathrm{w}_0L_0^{1/\sigma}}{r_0K_0 + \mathrm{w}_0L_0}]^{\frac{\sigma}{\sigma-1}} \tag{4.77}$$

$$\pi = \frac{r_0K_0^{1/\sigma}}{r_0K_0^{1/\sigma} + \mathrm{w}_0L_0^{1/\sigma}} \tag{4.78}$$

For this study, we will normalize the variables (w_t, r_t) with their arithmetic mean and growing variables (Y_t, K_t, L_t) with their geometric mean. We estimate a system of the following three equations - two first order conditions, and the production function:

$$log(r_t) = log(\pi_0\frac{Y_0}{K_0}) + \frac{1}{\sigma}log(\frac{Y_t/Y_0}{K_t/K_0}) + \frac{\sigma-1}{\sigma}log(\xi) \tag{4.79}$$

$$log(\mathrm{w}_t) = log((1-\pi_0)\frac{Y_0}{L_0}) + \frac{1}{\sigma}log(\frac{Y_t/Y_0}{L_t/L_0}) + \frac{\sigma-1}{\sigma}(log(\xi) + \lambda_l(t-t_0)) \tag{4.80}$$

$$log(\frac{Y_t}{Y_0}) = log(\xi) + \frac{\sigma}{\sigma-1}log[\pi_0(\frac{K_t}{K_0})^{\frac{\sigma-1}{\sigma}} + (1-\pi_0)(e^{\lambda_l t}\frac{L_t}{L_0})^{\frac{\sigma-1}{\sigma}}] \tag{4.81}$$

For estimating the Phillips curve specified in model 2 (equation (4.67)), whenever all three variables(wage growth rate, employment rate and inflation) in the nominal Phillips curve are nonstationary, we use Johansen's co-integration test to find the long run relationship between the variables [14; 15]. For all other countries, we use the autoregressive distributed lag (ARDL) bounds testing approach.

Johansen's approach uses a vector error correction model (VECM) to test the hypothesis for co-integration. In the current setting, if we define $y_t = [\frac{\dot{m}}{m} \ \frac{\dot{p}}{p} \ \lambda]$, then a VECM is -

$$\Delta y_t = \Pi(y_{t-1} + C_0) + B_1 \Delta y_{t-1} + B_2 \Delta y_{t-2} + ... + B_q \Delta y_{t-q} + C_1 + \epsilon_t \qquad (4.82)$$

where Π and $B_i's$ are 3x3 matrices. Now, Π can be thought of as a product AB'; where, A and B are 3xr matrices and r$<$ 3 refers to the number of co-integrating relationships. It is also the rank of the matrix Π. The coefficients of A determine the size of the effects of the r error correction terms in 3 equations of VECM. To test for the number of co-integrating relationships given by r, we use Trace tests where the null hypothesis is $H_0 : r \leq r^*$ for some hypothesized value of r^* which ranges from 0-2. We start from testing the hypothesis, $H_0 : r \leq 0$. Suppose if this is accepted, then it implies $\Pi = 0$ and thus there is no co-integration and VECM drops to VAR in differences. Next, if $H_0 : r \leq 0$ is rejected, then we test $H_0 : r \leq 1$ which if accepted implies that we have exactly one co-integrating vector. On the other hand, if $H_0 : r \leq 1$ is also rejected, and so is $H_0 : r \leq 2$ then it implies that Π has full rank. This leads us to the fact that Πy_{t-1} spans the same vector space as y_{t-1}. Therefore, at least one element of y_{t-1} is I(1) which contradicts the assumption in the VECM equation (4.82).

However, since the productivity growth rate is stationary for all the countries, we will use the ARDL bounds testing approach for estimating Phillips curve in Model 1.

4.3 Results

The estimation methodology for the production function was very similar to [18] and the results are presented in Table 4.15. We used the nlsystem function within the RATS software package to estimate the nonlinear system of equations (inputs to the optimization algorithm included 1 million sub iterations, 100 thousand iterations and the convergence criterion of 10^{-5}). Initial values for the optimization were randomly chosen between 0.1 and 0.6 for all the countries. We saw absolute convergence in all except France and Italy. The initialization values were important in these two countries to see convergence and thus we chose it to be same as the final result of Mallick [19]. We find that the elasticity of substitution σ varied between 0.36 and 0.64 and thus it is less than 1 for all the countries examined. This is in line with the previous related research in the literature [19; 16].

Country	σ	λ_l	π_0	$\frac{K_0}{cY_0}$	γ	ρ_1	ρ_2	ω_P	λ_P
Australia	0.377	1.53%	34.8%	2.86	-0.221	0.245	0.232	71.6%	95.0%
Canada	0.375	1.53%	32.8%	2.86	-0.078	0.093	0.192	74.2%	96.4%
Denmark	0.587	1.90%	31.6%	2.81	-0.297	0.323	0.447	73.3%	95.0%
Finland	0.424	2.88%	30.0%	3.30	-0.246	0.285	0.169	71.4%	94.8%
France	0.361	2.60%	29.1%	3.36	-0.384	0.428	0.256	74.5%	94.2%
Italy	0.602	4.11%	31.9%	3.27	-0.467	0.509	0.625	69.0%	94.7%
Norway	0.391	2.68%	38.5%	3.20	-0.571	0.602	0.198	68.9%	98.6%
UK	0.596	2.26%	28.5%	3.04	-0.118	0.138	0.376	75.4%	95.8%
US	0.647	1.83%	34.5%	2.71	-0.246	0.265	0.626	69.8%	95.1%
Germany	0.521	2.79%	31.6%	3.42	-0.445	0.470	0.575	72.6%	97.3%

Table 4.15: van der Ploeg Estimates

Our results for the labor augmented technical progress λ_l is also very similar to results in the literature and remained between 1.5%-2.9% for all countries except Italy where it was 4.11%. The labor augmented technical progress was stronger for European economies compared to non-European, this is similar to what we observed in the case of an exponential productivity growth model of Goodwin. Except Italy, the order of magnitude is also very similar in the two methods. Average capital share π_0 lies between 28.5% for UK to 38.5% for Norway.

4.3.1 Real Phillips Curve - Model 1

As discussed in the previous section, we use the ARDL bound testing approach to estimate the Phillips curve. The Phillips curve used here has an extra term for productivity growth rate. So, our new unrestricted error correction model is the following

$$\Delta z_t = \varphi_0 + \varphi_1 \Delta\alpha_{t-1} + \varphi_2 \Delta\lambda_{t-1} + \varphi_3 z_{t-1} + \varphi_4 \lambda_{t-1} + \varphi_5 \alpha_{t-1} + \epsilon_{5t} \tag{4.83}$$

$$\alpha_t = log(a_t) - log(a_{t-1}) \tag{4.84}$$

and the restriction to be tested is $\varphi_3 = \varphi_4 = \varphi_5 = 0$. Table 4.16 gives the F-statistic of the restriction. The F-statistic is greater than the I(1) threshold given in [24] for 50 observations and k=2, implying that wage growth, productivity growth and employment rate are co-integrated.

Country	Australia	Canada	Denmark	Finland	France	Italy	Norway	UK	US	Germany
F statistics	10.00	13.52	20.60	15.61	9.35	11.15	15.37	6.88	5.51	5.94

Table 4.16: F-test for $H_0 : \varphi_3 = \varphi_4 = \varphi_5 = 0$

Since p-values in Table 4.17 are greater than any acceptable threshold for all the countries, it ensures that the errors of the unrestricted error correction model does not suffer from auto-correlation and the model is well specified.

Country	lag 1	lag 2	lag 3	lag 4	lag 5
Australia	0.971	0.958	0.909	0.918	0.935
Canada	0.732	0.204	0.305	0.458	0.596
Denmark	0.674	0.901	0.727	0.806	0.864
Finland	0.746	0.534	0.435	0.602	0.688
France	0.700	0.926	0.779	0.635	0.585
Italy	0.232	0.214	0.356	0.517	0.643
Norway	0.820	0.643	0.648	0.799	0.886
UK	0.919	0.994	0.941	0.411	0.508
US	0.714	0.872	0.943	0.924	0.538
Germany	0.902	0.828	0.922	0.728	0.580

Table 4.17: p-values of the serial correlation test for Unrestricted Error Correction Model given by equation 4.83

Now since we have proved that variables are co-integrated, we estimate the following long term model

$$z_t = \hat{\gamma}_1 + \hat{\rho}_1 \lambda_t + \hat{\rho}_2 \alpha_t + \epsilon_{6t} \tag{4.85}$$

Table 4.18 shows that all the countries have negative constant and positive slopes for both employment rate and productivity growth rate. Productivity growth rate has the maximum impact for United states where 1% increase in the productivity growth rate increases the growth rate of wages by 0.62% and the minimum for Finland where 1% increase in the productivity growth rate increases growth rate of wages by just 0.17%.

As a final check, we next present the estimates of the restricted error correction model in Table 4.19. The error correction terms are negative and significant for all the countries.

Finally we present the estimates of the final equilibrium estimates derived using

Country	Variable	γ_1	ρ_1	ρ_2	AdjR2
Australia	Coeff	-0.221	0.245	0.232	0.105
	pValue	0.025	0.019	0.166	
Canada	Coeff	-0.078	0.093	0.192	0.023
	pValue	0.382	0.333	0.205	
Denmark	Coeff	-0.297	0.323	0.447	0.386
	pValue	0.001	0.000	0.000	
Finland	Coeff	-0.246	0.285	0.169	0.287
	pValue	0.000	0.000	0.176	
France	Coeff	-0.384	0.428	0.256	0.772
	pValue	0.000	0.000	0.039	
Italy	Coeff	-0.467	0.509	0.625	0.788
	pValue	0.000	0.000	0.000	
Norway	Coeff	-0.571	0.602	0.198	0.029
	pValue	0.104	0.096	0.479	
UK	Coeff	-0.118	0.138	0.376	0.214
	pValue	0.070	0.046	0.002	
US	Coeff	-0.246	0.265	0.626	0.607
	pValue	0.001	0.001	0.000	
Germany	Coeff	-0.445	0.470	0.575	0.804
	pValue	0.000	0.000	0.000	

Table 4.18: Long term estimates in Phillips Curve given by equation 4.18

equation (4.65) and (4.66) in Table 4.15. The employment rate continue to be of the same order and within the bounds for all the countries except Canada. The more interesting case is the improvement in estimates of wage share. For 6 out of 10 countries examined (Australia, Finland, France, Italy, Norway, UK), the estimate of wage share remained within the bounds of the observed values. Moreover, the difference of the estimated equilibrium and the observed mean has reduced from 2-12 percentage point to 1.5-7.5 percentage point.

Country	Variable	constant	$\Delta\alpha_t$	$\Delta\lambda_t$	$\hat{\epsilon}_{6(t-1)}$	AdjR2
Australia	Coeff	0.000	0.072	0.187	-0.846	0.412
	pValue	0.984	0.579	0.597	0.000	
Canada	Coeff	0.000	0.122	-0.044	-0.921	0.418
	pValue	0.827	0.393	0.852	0.000	
Denmark	Coeff	-0.001	-0.146	0.191	-1.245	0.524
	pValue	0.677	0.160	0.481	0.000	
Finland	Coeff	0.000	-0.011	0.346	-1.002	0.494
	pValue	0.945	0.932	0.096	0.000	
France	Coeff	-0.002	-0.064	-0.036	-0.793	0.348
	pValue	0.249	0.538	0.882	0.000	
Italy	Coeff	-0.002	-0.336	-0.184	-1.147	0.430
	pValue	0.433	0.004	0.667	0.000	
Norway	Coeff	-0.002	-0.484	0.410	-0.994	0.498
	pValue	0.762	0.108	0.711	0.000	
UK	Coeff	0.000	-0.238	0.032	-0.720	0.330
	pValue	0.890	0.044	0.906	0.000	
US	Coeff	-0.001	-0.197	-0.446	-0.822	0.280
	pValue	0.670	0.054	0.015	0.001	
Germany	Coeff	-0.002	-0.174	0.118	-1.246	0.342
	pValue	0.545	0.242	0.782	0.000	

Table 4.19: Restricted Errorcorrection Model

4.3.2 Phillips Curve corrected for inflation - Model 2

For this part, we will use two methodologies, Johansen's co-integration test [14; 15] and the bounds testing approach by [25].The variables of interest, wage growth rate, employment rate and inflation are non-stationary for Australia, Canada, Denmark, Finland, France, Italy, UK and Germany (see Table 2.5). Thus, we can use Johansen's cointegration test. However, since Norway and US have stationary inflation and nominal wage growth respectively, we have to use the ARDL bounds testing approach. Table 4.20 suggests that $H_0 : r \leq 0$ is rejected and $H_0 : r \leq 1$ is accepted for Australia, Canada, Denmark, Finland, France, and Germany thus implying that variables are co-integarted for these countries. However, the tests rejected the hypothesis of co-integration for Italy and UK.

Country	Australia	Canada	Denmark	Finland	France	Italy	UK	Germany	cValue
r=0	40.07	44.23	53.60	44.71	39.07	22.77*	26.09*	31.18	29.80
r=1	13.10*	12.53*	13.85*	14.87*	13.98*	12.67	9.57	11.99*	15.49
r=2	5.25	4.32	2.13	4.38	5.52	4.47	3.82	2.86	3.84

Table 4.20: Johansen's test for co-integration (* indicates the value of r for which null hypothesis was accepted)

Since we use the ARDL bounds testing approach approach for Norway and US, the F-stat is 5.21 and 5.65 for them respectively. Both of these values are higher than the critical values for k=2 given in [24]. Next we present the estimates for γ_3, ρ_3 and η_3 for all the countries in Table 4.21. For Australia, Canada, Denmark, Finland, France, and Germany we get the estimate using Johansen's MLE estimate. For the rest, we present the OLS estimates. OLS estimates are consistent for Norway and US since we have shown that the variables are co-integrated. For sake of completeness, we also present the OLS estimates for Italy and UK.

Country	Australia	Canada	Denmark	Finland	France	Italy	Norway	UK	US	Germany
γ_3	-0.143	-0.255	-0.354	-0.112	-0.511	-0.901*	-1.151*	-0.116	-0.159	-0.681*
ρ_3	0.178	0.288*	0.390*	0.142*	0.573*	0.994*	1.245*	0.146*	0.193*	0.716*
η_3	0.790*	0.985*	1.007*	1.068*	0.911*	0.981*	0.184*	0.951*	0.776*	1.334*

Table 4.21: Long term Phillips Curve (* indicates parameter is significant at 10% level)

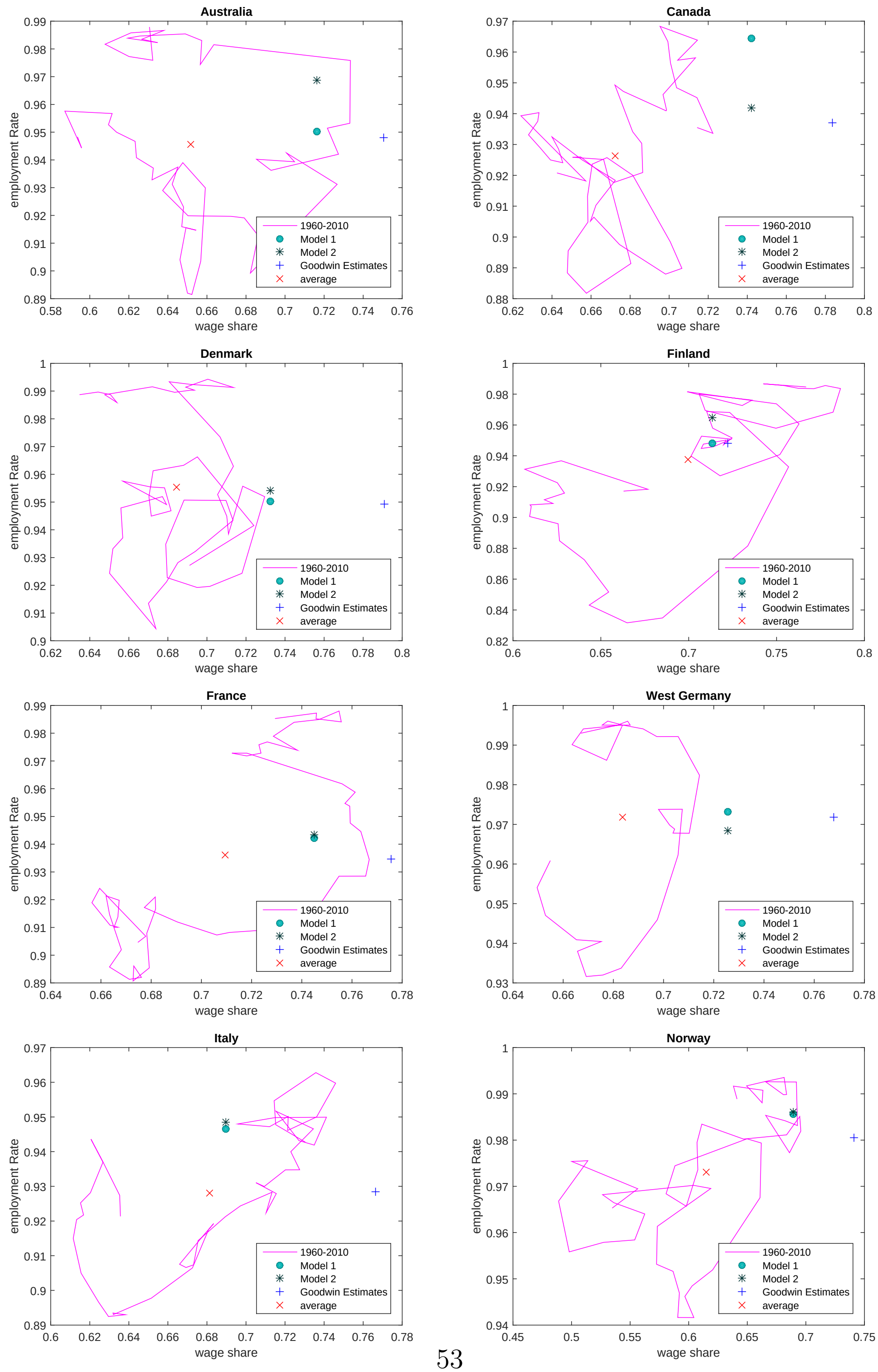
The slope of the employment rate varies from 14% for Finland to 124% for Norway. Norway and Italy show high dependence of nominal wage growth on employment rate compared to it's contemporaries. If we drop these two countries, the range varies from 14% for Finland to 71% for Germany. For Norway, gdp deflator as a measure of inflation shows abnormal volatility after 2000 and replacing the gdp deflator with the cpi as measure of inflation reduces the estimate of the slope but still remains on the higher end and close to 1 (results are available on request). Furthermore, the estimate of the η_3 (impact of inflation on bargaining) is less than or equal to one for all countries except Germany and this is very similar to the slope of expected inflation observed in Desai's model of the previous chapter (see estimate of η_2 in table 3.14). Finally, as is evident from the graphs in Figure 4.8, the estimate of employment rate for both, the van der Ploeg model specified through the real Phillips curve and through the nominal bargaining shows very similar range. The difference in the two estimates of employment rate ranges between -2.27% for Canada to 2.33% for US.

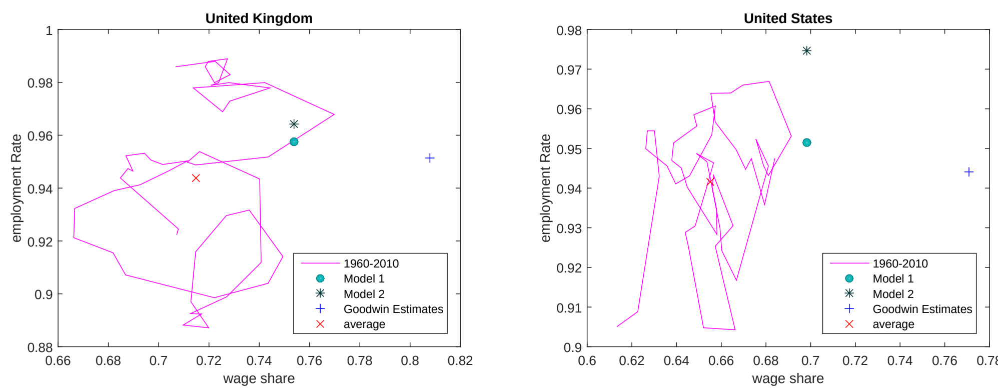
Country	γ_3	ρ_3	η_3	χ_2	π	λ_l	ω_{PE}	λ_{PE}
Australia	-0.143	0.178	0.790	0.380	1.673	0.015	0.716	0.969
Canada	-0.255	0.288	0.985	0.392	1.589	0.015	0.742	0.942
Denmark	-0.354	0.390	1.007	0.497	1.536	0.019	0.733	0.954
Finland	-0.112	0.142	1.068	0.303	1.620	0.029	0.714	0.965
France	-0.511	0.573	0.911	0.379	1.522	0.026	0.745	0.943
Italy	-0.901	0.994	0.981	0.435	1.609	0.041	0.690	0.949
Norway	-1.151	1.245	0.184	0.141	2.240	0.027	0.689	0.986
UK	-0.116	0.146	0.951	0.632	1.447	0.023	0.754	0.964
US	-0.159	0.193	0.776	0.450	1.595	0.018	0.698	0.975
Germany	-0.681	0.716	1.334	0.348	1.569	0.028	0.726	0.968

Table 4.22: van der Ploeg Extended Model

So, unlike in the Goodwin model when capital and labor were not substitutable, here the increased bargaining power will lead capitalists to replace labor by capital and thus reducing labor's bargaining power. This consequently leads to asymptotically stable trajectories. With non-zero elasticity of substitution in all the economies, van der Ploeg's[29] extension incorporating CES production function is much closer to reality than basic Goodwin. However, it still reinforces the Goodwin hypothesis that increased profitability carries the seed of its destruction.

Figure 4.8: Estimated equilibrium and observed mean




United Kingdom
employment Rate
wage share
1960-2010
Model 1
Model 2
Goodwin Estimates
average
United States
employment Rate
wage share
1960-2010
Model 1
Model 2
Goodwin Estimates
average

Chapter 5

Conclusion and Future Work

Through this study we have tested for empirical relevance of a dynamic macroeconomic models proposed by Goodwin [10] and two of its variants proposed by Desai [4] and van der Ploeg [29]. We first tested for the Goodwin model in Chapter 2. Here we discussed multiple issues regarding appropriate data for empirical estimation of such models, this was followed by estimating the underlying parameters and equilibrium values for it. On comparing the equilibrium estimates of the model with the observed average values, we found that although the estimates of the equilibrium employment rate is close to the observed average values, the estimate of the equilibrium wage share is far from reality. Although this should not be surprising given a starkly simple structure of the model, these results are still better than what was observed in the previous studies [12]. Since the Goodwin model incorporates strong assumptions like constant capital-output ratio, perfect complementary factors of production and real bargaining, in the next chapters we relaxed some of these assumptions to make a closer approximation to reality. In Chapter 3, we thus tested for a model by explicitly incorporating inflation in the setup. The specific structure of this model reduces it to the Goodwin model for a particular parameter value. We observe mixed results for the impact of inflation, as for some countries explicitly incorporating inflation would still reduce the model to the basic Goodwin setting. However for others, inflation significantly affects the impact employment would have on bargaining ability of the labours. The equilibrium estimates for wage share continue to be far off from actual observed values for most of the countries. Our main contribution lies in Chapter 4 where the assumption of perfectly complement factors(labour and capital) were relaxed by using a CES production function. We got impressive results as the equilibrium estimate of wage share is within the bounds of the observed values for 6 of the 10 countries examined. This is important because it is the first study we know that has shown confidence in this class of dynamical model not only from qualitative graphs but also quantitative tests. Although promising, these results still need a lot of improvement before we can make accurate forecasts. However, relaxing assumptions in this kind of setup makes the model theoretically complex and empirical estimation a daunting task. Future work in this regard should definitely include the government sector, the household savings and the investments in the money market. However, for any meaningful policy predictions, it is important to econometrically test the assumptions before the theoretical developments

Appendix A

Desai Theoretical Model

Here we will describe the theoretical setup for the Desai Model. The bargaining power in the Desai model is represented through the following setup -

$$\frac{\dot{m}}{m} = \gamma_2 + \rho_2\lambda + \eta_2(\frac{\dot{p}}{p})^e \tag{A.86}$$

$$\frac{d}{dt}(\frac{\dot{p}}{p})^e = \tau_2\{(\frac{\dot{p}}{p}) - (\frac{\dot{p}}{p})^e\} \tag{A.87}$$

$$\frac{\dot{p}}{p} = \eta_3(\ln\omega + \ln\pi) \tag{A.88}$$

Thus, rate of growth of wage share can be written as

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{m}}{m} - \frac{\dot{p}}{p} - \frac{\dot{a}}{a} \tag{A.89}$$

$$= \gamma_2 + \rho_2\lambda + \eta_2(\frac{\dot{p}}{p})^e - \frac{\dot{p}}{p} - \alpha \tag{A.90}$$

$$= (\gamma_2 - \alpha) + \rho_2\lambda + \eta_2[(\frac{\dot{p}}{p})^e - \frac{\dot{p}}{p}] + (\eta_2 - 1)\frac{\dot{p}}{p} \tag{A.91}$$

$$\implies \eta_2[\frac{\dot{p}}{p} - (\frac{\dot{p}}{p})^e] = (\gamma_2 - \alpha) + \rho_2\lambda + (\eta_2 - 1)\frac{\dot{p}}{p} - \frac{\dot{\omega}}{\omega} \tag{A.92}$$

Now, using the derivative operator on equation (A.90)

$$\frac{d}{dt}[\frac{\dot{\omega}}{\omega}] = \rho_2\frac{d\lambda}{dt} + \eta_2\frac{d}{dt}(\frac{\dot{p}}{p})^e - \frac{d}{dt}[\frac{\dot{p}}{p}] \tag{A.93}$$

$$\frac{d}{dt}[\frac{\dot{\omega}}{\omega}] = \rho_2\frac{d\lambda}{dt} + \eta_2\tau_2\{(\frac{\dot{p}}{p}) - (\frac{\dot{p}}{p})^e\} - \eta_3\frac{d}{dt}\ln\omega \text{ (Using (A.87) and (A.88))} \tag{A.94}$$

$$\frac{d}{dt}[\frac{\dot{\omega}}{\omega}] = \rho_2\frac{d\lambda}{dt} + \tau_2[(\gamma_2-\alpha) + \rho_2\lambda + (\eta_2-1)\frac{\dot{p}}{p} - \frac{\dot{\omega}}{\omega}] - \eta_3\frac{\dot{\omega}}{\omega} \text{ (Using (A.92))} \tag{A.95}$$

$$\frac{d}{dt}[\frac{\dot{\omega}}{\omega}] = (\gamma_2-\alpha)\tau_2 + \rho_2\tau_2\lambda + \rho_2\frac{d\lambda}{dt} + (\eta_2-1)\tau_2\frac{\dot{p}}{p} - (\eta_3+\tau_2)\frac{\dot{\omega}}{\omega} \tag{A.96}$$

$$\frac{d}{dt}[\frac{\dot{\omega}}{\omega}] = (\gamma_2-\alpha)\tau_2 + \rho_2\tau_2\lambda + \rho_2\frac{d\lambda}{dt} + (\eta_2-1)\tau_2\eta_3(\ln\omega + \ln\pi) - (\eta_3+\tau_2)\frac{\dot{\omega}}{\omega} \tag{A.97}$$

Next we will derive the differential equation for employment rate.

$$\begin{aligned}
\frac{\dot{\lambda}}{\lambda} &= \frac{\dot{L}}{L} - \frac{\dot{N}}{N} && \text{(A.98)}\\
&= \frac{\dot{Y}}{Y} - \alpha - \beta \text{ since } Y(t) = aL(t) = K(t)/\sigma) && \text{(A.99)}\\
&= \frac{\dot{K}}{K} - \frac{\dot{\sigma}}{\sigma} - \alpha - \beta && \text{(A.100)}\\
&= \frac{1-\omega}{\sigma} - \frac{\dot{\sigma}}{\sigma} - \delta - \alpha - \beta && \text{(A.101)}
\end{aligned}$$

Since,

$$\sigma = \sigma^*\lambda^{-\mu} \tag{A.102}$$

$$\implies \frac{\dot{\sigma}}{\sigma} = -\mu\frac{\dot{\lambda}}{\lambda} \tag{A.103}$$

We can thus write -

$$\frac{\dot{\lambda}}{\lambda} = \frac{1-\omega}{\sigma} - \frac{\dot{\sigma}}{\sigma} - \delta - \alpha - \beta \tag{A.104}$$

$$= \frac{1-\omega}{\sigma^* \lambda^{-\mu}} + \mu \frac{\dot{\lambda}}{\lambda} - \delta - \alpha - \beta \tag{A.105}$$

$$\frac{\dot{\lambda}}{\lambda} = \frac{(1-\omega)\lambda^{\mu}}{\sigma^*(1-\mu)} - \frac{\delta + \alpha + \beta}{1-\mu} \tag{A.106}$$

Appendix B

Desai Econometric Setup

Equation (3.53) can be transformed to

$$\Delta \ln(p^e_t) - \Delta \ln(p^e_{t-1}) = \tau_2(\Delta \ln(p_t) - \Delta \ln(p^e_{t-1})) + \epsilon_{2t} \tag{B.107}$$

Take difference of equation (3.52) and then use equation (B.107) to get

$$\begin{aligned}\Delta^2 \ln(m_t) &= \rho_1 \Delta\lambda_t + \eta_2(\Delta \ln(p^e_t) - \Delta \ln(p^e_{t-1})) + \Delta\epsilon_{1t} && \text{(B.108)}\\ &= \rho_1 \Delta\lambda_t + \eta_2\tau_2(\Delta \ln(p_t) - \Delta \ln(p^e_{t-1})) + \Delta\epsilon_{1t} + \eta_2\epsilon_{2t} && \text{(B.109)}\end{aligned}$$

Multiply equation (3.52) by τ_2 with time index $t-1$ and add to equation (B.109).

$$\begin{aligned}\Delta^2 \ln(m_t) =& \tau_2\gamma_2 + \tau_2\rho_2\lambda_{t-1} + \rho_2\Delta\lambda_t + \eta_2\tau_2(\Delta \ln(p_t) - \Delta \ln(p^e_{t-1}))\\ &+ \tau_2\eta_2\Delta \ln(p^e_{t-1}) - \tau_2\Delta \ln(m_{t-1}) + \tau_2\epsilon_{1t-1} + \Delta\epsilon_{1t} + \eta_2\epsilon_{2t}\end{aligned} \tag{B.110}$$

Appendix C

van der Ploeg Model

$$Y = C[\pi K_t^{-\eta} + (1-\pi)(e^{\lambda_l t} L_t)^{-\eta}]^{\frac{-1}{\eta}} \tag{C.111}$$

let's assume $L_e = e^{\lambda_l t} L$. From profit maximization,

$$\frac{dY}{dL} = w \implies \frac{dY}{dL_e} = we^{-\lambda_l t} = \frac{wL}{e^{\lambda_l t} L} = \frac{W}{L_e} \tag{C.112}$$

Now, differentiating the production function with respect to L_e leads to-

$$\frac{dY}{dL_e} = \frac{1-\pi}{C^\eta} \left\{ \frac{Y}{L_e} \right\}^{1+\eta} \tag{C.113}$$

Thus, using (C.112) and (C.113)

$$\frac{1-\pi}{C^\eta} \left\{ \frac{Y}{L_e} \right\}^{1+\eta} = \frac{W}{L_e} = \frac{WY}{YL_e} = \omega \frac{Y}{L_e} \tag{C.114}$$

$$\implies \omega = \frac{1-\pi}{C^\eta} \left\{ \frac{Y}{L_e} \right\}^{\eta} \tag{C.115}$$

$$\implies \frac{Y}{L_e} = C\left\{\frac{\omega}{1-\pi}\right\}^{\frac{1}{\eta}} \tag{C.116}$$

Thus productivity and rate of growth of productivity is

$$a = \frac{Y}{L} = Ce^{\lambda_l t}\left\{\frac{\omega}{1-\pi}\right\}^{\frac{1}{\eta}} \tag{C.117}$$

$$\frac{\dot{a}}{a} = \frac{1}{\eta}\frac{\dot{\omega}}{\omega} + \lambda_l \tag{C.118}$$

Replacing Y by production function in equation (C.115) and subsequent readjustment gives the optimal K-L ratio as -

$$\frac{K}{L_e} = \left\{\frac{(1-\pi)(1-\omega)}{\omega\pi}\right\}^{\frac{-1}{\eta}} \tag{C.119}$$

Comparing (C.116) and (C.119), we get the optimal K-Y ratio as

$$\nu = \frac{K}{Y} = \left\{\frac{1-\omega}{\pi}\right\}^{\frac{-1}{\eta}}\frac{1}{C} \tag{C.120}$$

$$\frac{\dot{\nu}}{\nu} = \frac{\dot{\omega}}{(1-\omega)\eta} \tag{C.121}$$

We are in good position to derive the two differential equations to explain class struggle within this model -

$$\omega = \frac{w}{a} \implies \frac{\dot{\omega}}{\omega} = \frac{\dot{w}}{w} - \frac{\dot{a}}{a} = \Phi(\lambda) + \rho_2\frac{\dot{a}}{a} - \frac{\dot{a}}{a} = \Phi(\lambda) + (\rho_2 - 1)\frac{\dot{a}}{a} \tag{C.122}$$

Using (C.118)

$$\frac{\dot{\omega}}{\omega} = \Phi(\lambda) + (\rho_2 - 1)(\frac{1}{\eta}\frac{\dot{\omega}}{\omega} + \lambda_l) \tag{C.123}$$

$$\frac{\dot{\omega}}{\omega} = \frac{\Phi(\lambda) - \lambda_l(1-\rho_2)}{(1 + \frac{1-\rho_2}{\eta})} \tag{C.124}$$

Now, the employment growth rate will be -

$$\lambda = \frac{L}{N} \implies \frac{\dot{\lambda}}{\lambda} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N} = \frac{\dot{K}}{K} - \frac{\dot{a}}{a} - \frac{\dot{\nu}}{\nu} - \frac{\dot{N}}{N} = \frac{1-\omega}{\nu} - \frac{\dot{a}}{a} - \frac{\dot{\nu}}{\nu} - \frac{\dot{N}}{N} \tag{C.125}$$

Using (C.120), (C.121) and (C.118)

$$\frac{\dot{\lambda}}{\lambda} = C(1-\omega)\left\{\frac{1-\omega}{\pi}\right\}^{\frac{1}{\eta}} - \delta - \frac{1}{\eta}\frac{\dot{\omega}}{\omega} - \lambda_l - \frac{\dot{\omega}}{(1-\omega)\eta} - \beta \tag{C.126}$$

$$\frac{\dot{\lambda}}{\lambda} = C(1-\omega)\left\{\frac{1-\omega}{\pi}\right\}^{\frac{1}{\eta}} - \delta - \lambda_l - \beta - \frac{1}{\eta(1-\omega)}\frac{\dot{\omega}}{\omega} \tag{C.127}$$

Thus we have derived the two equations describing relationship between wage share and employment rate and explaining the class clashes.

Appendix D

Data Description

Following is the list of preliminary variables taken from the AMECO database for this thesis.

variable Code	**Description**
NWTD	Employees, persons: all domestic industries (National accounts)
NSTD	Number of self-employed-; total economy (National accounts)
NUTN	Total unemployment; Member States: definition -Eurostat
UWCD	Compensation of employees; total economy
OKND	Net capital stock at constant prices; total economy
UKCT	Consumption of fixed capital at current prices; total economy
PIGT	Price deflator gross fixed capital formation; total economy
UTVN	Taxes linked to imports and production minus subsidies
OVGD	Gross domestic product at constant market prices
UVGD	Gross domestic product at current market prices

Bibliography

[1] Asada, T. Stabilization policy in a Keynes-Goodwin model with debt accumulation. *Structural Change and Economic Dynamics 17* (2006), 466–485.

[2] Atkinson, A. B. The Timescale of Economic Models: How Long is the Long Run? *The Review of Economic Studies 36* (1969), 137–152.

[3] Costa-Lima, B., Grasselli, M., Wang, X., and Wub, J. Destabilizing a stable crisis: Employment persistence and government intervention in macroeconomics. *Structural Change and Economic Dynamics 30* (2014), 30–51.

[4] Desai, M. Growth cycles and inflation in a model of class struggle. *Journal of Economic Theory, 6* (1973), 527–545.

[5] Desai, M. An econometric model of the share of wages in national income: UK 1855-1965. *Nonlinear Models of Fluctuating Growth, Berlin, Springer 6* (1984), 253–277.

[6] Desai, M., Henry, B., Mosley, A., and Pemberton, M. A clarification of the Goodwin model of the growth cycle. *Journal Econom. Dynam. Control 30* (2006), 2661–2670.

[7] Dibeh, G., Luchinsky, D., Luchinskaya, D., and Smelyanskiy, V. A Bayesian estimation of a stochastic predator-prey model of economic fluctuations. *Proc. of SPIE "Noise and Stochastics in Complex Systems and Finance"* (2007).

[8] Flaschel, P. The Goodwin distributive cycle after fifteen years of new observations. In *Topics in Classical Micro- and Macroeconomics*. Springer Berlin Heidelberg, 2009, pp. 465–480.

[9] Goldstein, J. P. Predator - prey model estimates of the cyclical profit squeeze. *Metroeconomica 50* (1999), 139–173.

[10] Goodwin, R. M. A growth cycle. In *Socialism, Capitalism and Economic Growth*. Cambridge University Press, 1967, pp. 54–58.

[11] Grasselli, M. R., and Costa-Lima, B. An analysis of the Keen model for credit expansion, asset price bubbles and financial fragility. *Math Finan Econ 6* (2012), 191–210.

[12] Harvie, D. Testing Goodwin: growth cycles in ten oecd countries. *Cambridge Journal of Economics 24* (2000), 349–376.

[13] Huu, A. N., and Costa-Lima, B. Orbits in a stochastic GoodwinLotka-Volterra model. *Journal of Mathematical Analysis and Applications 419* (2014), 48–67.

[14] Johansen, S. Statistical analysis of cointegrating vectors. *Journal of Economic Dynamics and Control 12* (1988), 231–254.

[15] Johansen, S. Estimation and hypothesis testing of cointegrating vectors in gaussian vector autoregressive models. *Econometrica 59* (1991), 1551–1580.

[16] Klump, R., McAdam, P., and Willman, A. Factor substitution and factor augmenting technical progress in the U.S. *Review of Economics and Statistics 89* (2007), 183–192.

[17] Kmenta, J. On estimation of the CES Production function. *International Economic Review 8* (1967), 180–189.

[18] Len-Ledesma, M. A., McAdam, P., and Willman, A. Identifying the elasticity of substitution with biased technical change. *American Economic Review 100* (2010), 1330–1357.

[19] Mallick, D. The role of the elasticity of substitution in economic growth: A cross-country investigation. *Labour Economics 19* (2012), 682–694.

[20] Massy, I., Avila, A., and Garcia-Molina, M. Quantitative evidence of Goodwins non-linear growth cycles. *Applied Mathematical Sciences 7* (2013), 1409–1417.

[21] Mohun, S., and Veneziani, R. Goodwin cycles and the U.S. economy,1948-2004. *MPRA Paper 30444* (2006).

[22] Molina, M., and Medina, E. Are there Goodwin employment-distribution cycles? International empirical evidence. *Cuadernos de Economa 29* (2010), 1–29.

[23] Moura, N. J., and Ribeiro, M. Testing the Goodwin growth-cycle macroeconomic dynamics in Brazil. *Physica A 392* (2013), 2088–2103.

[24] Narayan, P. The saving and investment nexus for China: Evidence from cointegration tests. *Applied Economics 37* (2005), 1979–1990.

[25] Pesaran, M. H., Shin, Y., and R.J.Smith. Bounds testing approaches to the analysis of level relationships. *Journal of Applied Econometrics 16* (2001), 289–326.

[26] Takeuchi, Y., and Yamamura, T. Stability analysis of the Kaldor model with time delays: monetary policy and government budget constraint. *Nonlinear Analysis: Real World Applications 5* (2004), 277–308.

[27] Tarassow, A. The empirical relevance of Goodwins business cycle model for the U.S. economy. *Munich Personal RePEc Archive 22271* (2010).

[28] van der Ploeg, F. Economic growth and conflict over the distribution of income. *Journal of Economic Dynamics and Control 6* (1983), 253–279.

[29] van der Ploeg, F. Classical Growth cycles. *Metroeconomica 37* (1985), 221–230.

[30] van der Ploeg, F. Growth cycles, induced technical change, and perpetual conflict over the distribution of income. *Journal of Macroeconomics 9* (1987), 1–12.

[31] Wolfstetter, E. Fiscal policy and the classical growth cycle. *Zeitschrift fr Nationalkonomie 42* (1982), 375–393.

[32] Yoshida, H., and Asada, T. Dynamic analysis of policy lag in a Keynes-Goodwin model: Stability, instability, cycles and chaos. *Journal of Economic Behavior & Organization 62* (2007), 441–469.